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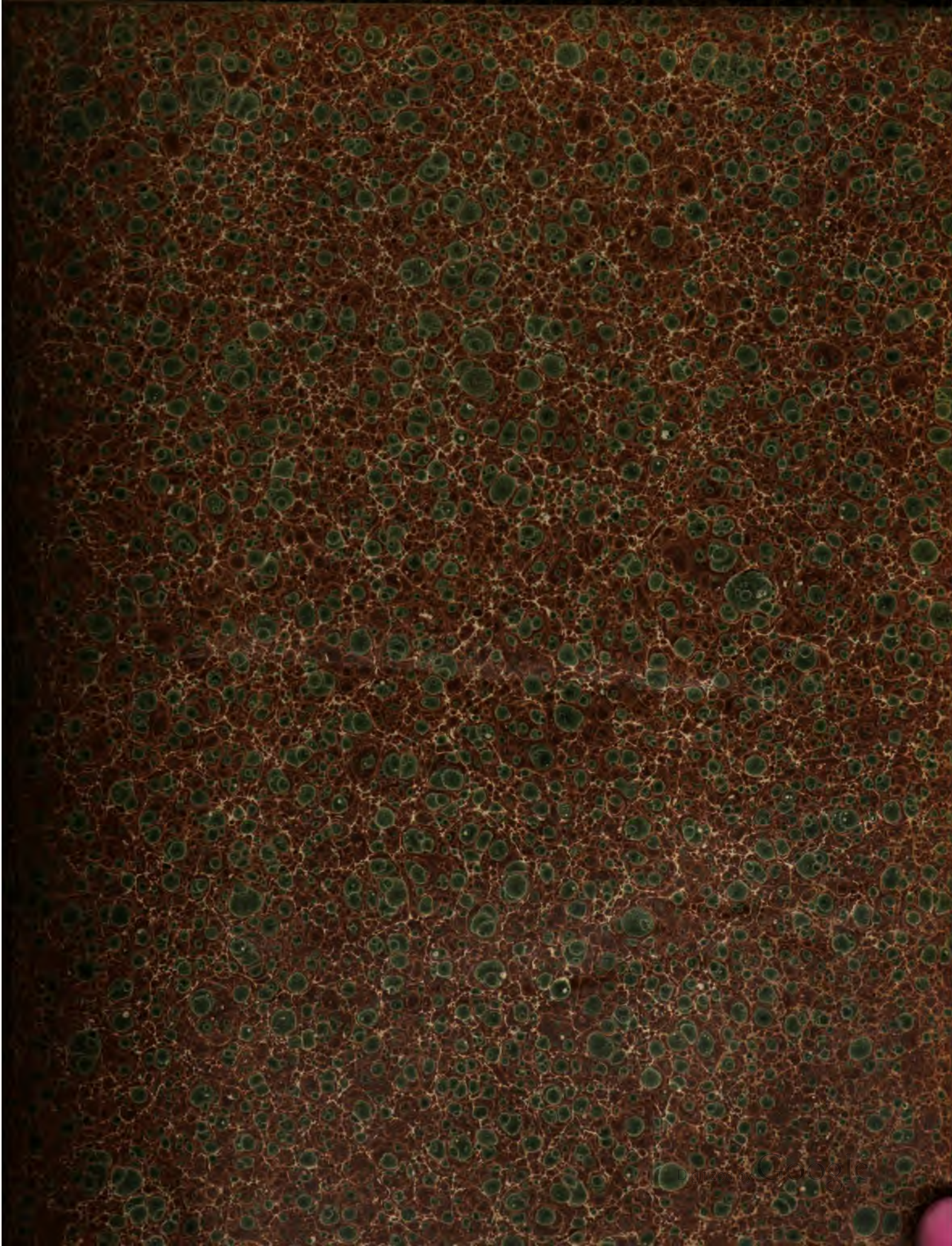

Math. 320.

Math. 320

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DIOPTRICAE

PARS PRIMA

CONTINENS

LIBRVM PRIMVM,

DE

EXPLICATIONE PRINCIPIORVM,

EX QVIBVS

CONSTRVCTIO TAM TELESCOPIORVM

QVAM

MICROSCOPIORVM

EST PETENDA.

AVCTORE

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LIBER PRIMVS

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PRINCIPIORUM

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CAPVT I.

DE DIFFVSIONE IMAGINIS PER VNICAM LENTEM REPRAESENTATAE.

Definitio I.

Imago principalis vocatur ea, quae a radiis axi lentis proximis per lentem refractis repraesentatur. Tab. I. Fig. 1.

Scilicet si lentis PP axis sit EF in eaque tantum spatium minimum ab sit apertum, per quod radiis transitus concedatur, radii a, puncto lucido E emissi in vno puncto F colligentur, quod punctum imago principalis vocatur.

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Coroll. 1.

2. Cum nempe apertura aAa sit minima, omnes radii, qui a puncto quocunque in eam incidunt, ita aequabilem patiuntur refractionem, vt omnes iterum in vnum punctum colligantur, id quod non solum de puncto lucido E in axe lentis sito, sed etiam de quibusuis aliis extra axem positis est intelligendum.

Coroll. 2.

3. Quodsi ergo lentis apertura fuerit minima, singula cuiusuis obiecti puncta post refractionem iterum singulis punctis referentur, sicque imago principalis erit distincta et confusione carebit; siquidem confusio tum demum oritur, quando radii ex vno puncto emissi non iterum in vno puncto colliguntur.

Coroll. 3.

4. Et quamdiu apertura lentis aAa est minima, nihil interest, secundum quamnam figuram facies lentis sint elaboratae; quaecumque enim earum figura fuerit, quoniam tantum portiuncula minima in computum venit, ea semper vt sphaerica spectari poterit.

Coroll. 4.

5. Pendet ergo locus imaginis principalis primum a loco puncti lucidi E, siue id in axe siue extra axem fuerit situm; deinde a sphaericitate vtrius-

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vtriusque faciei aAa et $bB\beta$ refringentis ; tertio ab earum distantia AB seu lentis crassitie, et quarto a ratione refractionis, quam radii in transitu per lentem patiuntur.

Scholion 1.

6. Refractio radiorum per huiusmodi lentium aperturas minimas transmissorum, ideoque determinatio imaginum principalium satis accurate in elementis Dioptricis tradi solet, quod igitur negotium hic fusius non prosequar; sed potius in eam refractionis rationem hic inquirere constitui, quando lentium apertura est modicae quantitatis, in quo imprimis ad vtriusque faciei figuram est spectandum. Hic autem perpetuo lentium figuras sphaericas assumo, propterea quod haec figura vulgo lentibus induci, vel saltem, nisi forte accurate successerit, intendi solet. In praxi certe nulla adhuc alia figura lentibus commode et accurate tribui poterit, atque adeo, a sphaerica figura, etsi ad praxin maxime est accommodata, ab artificibus frequenter aberrari solet. A sollertioribus autem talia vitia iam plerumque satis feliciter euitantur, vnde non adeo erit verendum, ne ea, quae per calculum ex hypothesi sphaericae figurae elicientur, experientiae non consentanea sint futura. Hanc ob rem hic perpetuo postulo, vt ambae facies lentium exactissime secundum sphaericam figuram sint elaboratae.

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Scholion 2.

7. Sphaerica autem figura hoc laborat incommodo, quod statim ac lenti maior apertura tribuatur, non amplius omnes radii, qui quidem ab vno obiecti puncto sunt profecti, post refractionem ad vnum punctum dirigantur radiique EM longius ab axe lentis transmissi non amplius in puncto F concurrant. Vnde confusionem eo maiorem nasci necesse est, quo magis hi radii remotiores ab iis, qui prope axem transeunt, declinauerint, et quoniam talis declinatio a figura sphaerica originem ducit, eo maior euadet, quo maior lenti apertura tribuatur. Quanta igitur quouis casu futura sit haec confusio, hoc capite definire constitui; quae cum ceteris paribus a quantitate aperturae pendeat, hic in perpetuum moneo, me cuiusuis lentis aperturam circularem assumere, per cuius centrum axis lentis transeat, ita vt semidiameter huius circuli simul mensuram aperturae exhibeat. Ita si in superficie lentis PP spatium MAM apertum relinquatur, reliqua parte MP velamine opaco obducta, punctorum extremorum MM distantia diametrum aperturae eiusque semissis semidiametrum aperturae praebebit.

Definitio 2.

8. *Imago extrema est ea, quam radii per extremitatem aperturae transmissi exhibent.*

Iam MM sit apertura lentis, radiique EM, EM a puncto lucido E circa oram aperturae transmissi conueniant in puncto *f*, in hoc ipso puncto *f* erit imago extrema.

Coroll. 1.

9. Si punctum lucidum E est in ipso axe lentis, nullum est dubium, quin radii inde per marginem circularem MM transeuntes iterum in vno axis puncto *f* concurrant, imaginemque distinctam repraesentent, quae imago extrema vocatur.

Coroll. 2.

10. Verum si punctum lucidum non esset in axe lentis, hoc neutiquam eueniet, radiique per marginem illum circularem transmissi non amplius in vno puncto colligentur; vnde hoc casu imago extrema eo magis erit confusa, quo magis punctum lucidum ab axe fuerit remotum.

Scholion.

11. Quemodo se habeat refractio radiorum, quando punctum lucidum extra axem lentis fuerit constitutum, quaestio est non solum difficillima, sed etiam ita prolixis calculis inuoluitur, vt inde vix quicquam concludi possit. Ceterum in vsu, ad quem lentes accommodantur, numquam obiecta ab axe remotiora spectari solent, atque contentos esse nos oportet, dummodo obiecta in ipso axe lentis sita distin-

cta repraesententur, neque etiam confusio, quae obiecta axi proxima afficiuntur, sensibilis esse possit; tum cum imago extrema puncti F in ipso lentis axe siti sit punctum *f*, nulla confusione inquinatum; etiamsi id parumper esset remotum ab axe, vix sensibilis confusio se immiscere poterit. Quam ob causam investigationes sequentes tantum ad obiecta in ipso lentis axe sita adstringam.

Definitio 3.

12. *Spatium diffusionis vocatur interuallum, inter imaginem principalem et extremam interceptum.*

Ita si imago principalis sit in F, extrema vero in *f*, interuallum F*f* appellatur spatium diffusionis.

Coroll. 1.

13. Si ergo apertura lentis MM euanescat, spatium simul diffusionis euanescit, tum enim tantum radii axi proximi transmittuntur, quibus imago distincta in F effingitur. Ex quo intelligi licet, quo maior fuerit apertura lentis, eo maius fore spatium diffusionis F*f*.

Coroll. 2.

14. Cum in F imago a radiis axi proximis, in *f* autem imago a radiis circa marginem circularem MM transmissis formetur; si totam aperturam lentis infinitis circulis concentricis diuisam concipiamus,

radii

radii per ſingulos circulos transmiſſi imagines intermedias exhibebunt, quibus interuallum Ff replebitur.

Coroll. 3.

15. Si enim apertura primum nulla, tum vero continuo increſcens ſtatuatur, imago extrema primum cum principali congruet; tum vero continuo magis ab ea diſcedet, ſicque cum vsque ad MM fuerit aucta, omnes illae imagines etiam nunc ſubſiſtent, ſpatiumque Ff implebunt.

Scholion 1.

16. Spatium hoc diffuſionis cauſam continet confuſionis, qua repraeſentatio imaginis perturbatur; cum enim eiusdem puncti lucidi E infinitae imagines per interuallum Ff diſpoſitae exhibeantur; earum commiſtio confuſionem pariat neceſſe eſt, quae eo maior erit, quo maius fuerit ſpatium diffuſionis Ff. Quemadmodum enim ad repraeſentationem diſtinctam requiritur, vt omnes radii ex eodem obiecti puncto emiſſi iterum in vnico puncto colligantur; ita ſi hi radii in plura puncta coeant, pluresque eiusdem puncti imagines referant, prout hae magis minusue inter ſe diſcrepant, maior inde minorue confuſio naſcetur. Quemadmodum autem hanc confuſionem aeſtimari oporteat, deinceps demum explicare licebit, cum ante ſpatium diffuſionis accurate definire docuerimus: quo circa in hoc capite, cum propoſita fuerit lens quaecunque vitrea faciebus ſphaericis terminata

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pro quauis puncti lucidi E ab ea distantia et quauis apertura spatium diffusionis F*f* inuestigare constitui. Quo facto eandem inuestigationem pro duabus pluribusue lentibus inter se coniunctis suscipi conueniet, vt tandem inde confusionem in quibusuis instrumentis dioptricis assignare valeamus.

Scholion 2.

17. Quaestio igitur principalis huius capitis in hoc versatur, vt proposita lente PP in eiusque axe puncto lucido E, radius quicunque incidens E M considereturr, eiusque per lentem refractio definiatur, vnde punctum *f*, vbi iterum in axem incidat, assignari queat. Cum enim in eodem puncto *f* omnes radii per totam peripheriam circularem M M transmissi concurrant; ab his imago quaepiam puncti E in *f* exprimetur, quae erit extrema, si circulus M M in lente eius aperturam determinet; sin autem capiatur minor, imago quaedam intermedia habebitur. Cum autem hic duplex refractio eueniat, altera in ingressu radii E M in vitrum, altera in egressu eiusdem e vitro, quemadmodum eius directio in vtraque inflectatur, seorsim est inuestigandum; vnde duo nascuntur problemata quasi praeliminaria, ex quorum combinatione deinceps negotium conficietur. Verum quo haec problemata commodius calculo expediri queant, quaedam lemmata ex doctrina angulorum petita praemitti oportet.

Lem-

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Lemma 1.

18. *Si angulus* Φ *triginta gradus non excedat eius ſinus ſatis accurate erit* ſin. $\Phi = \Phi - \frac{1}{6}\Phi^3$, *ſi quidem in circulo, cuius radius eſt* $= 1$, *arcus* Φ, *qui pro illius anguli menſura habetur, in partibus radii exprimatur.*

Demonſtratio.

Quantuscunque fuerit angulus Φ, notum eſt, eius ſinum hac ſerie infinita exprimi:

ſin. $\Phi = \Phi - \frac{1}{6}\Phi^3 + \frac{1}{120}\Phi^5 - \frac{1}{5040}\Phi^7 +$ etc.

ſumtis igitur tantum binis primis terminis, error committitur reliquis neglectis aequalis, ſi ergo ſtatuamus ſin. $\Phi = \Phi - \frac{1}{6}\Phi^3$, hincque pro variis angulis Φ ſinus colligamus, eorum comparatio cum tabulis ſinuum errores manifeſtabit. Ita ſi ſumatur $\Phi = 30^\circ$, quia arcus 180° valet 3, 14159265, erit in partibus radii $\Phi = 0,52359877$, et $\frac{1}{6}\Phi^3 = 0,0239246$, hincque $\Phi - \frac{1}{6}\Phi^3 = 0,4996741$. at eſt reuera

ſin. $\Phi = 0,5000000$ vnde habetur

error $= 0,0003259$, qui ergo ne ad $\frac{1}{5000}$ partem radii quidem aſſurgit. At ſi angulus Φ caperetur duplo minor, ſcilicet $\Phi = 15^\circ$, reperiretur

$\Phi - \frac{1}{6}\Phi^3 = 0,2588088$

cum tamen ſit ſin. $\Phi = 0,2588190$

errore exiſtente $= 0,0000102$

qui tricies bis minor eſt quam caſu praecedente. Cum ergo in praxi error partem adeo tremilleſimam radii adaequans facile tolerari poſſit, multo magis, ſi an-

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gulus Φ fuerit 30° minor, expreſſio $\Phi - \frac{1}{6}\Phi^3$ verum eius ſinum exhibere cenſenda erit.

Lemma 2.

19. *Viciſſim ſi anguli triginta gradibus minoris detur ſinus* $= s$, *ex eo ipſe angulus ita proxime definitur, vt ſit in circulo, cuius radius* $= 1$, *arcus eum metiens* $= s + \frac{1}{6}s^3$.

Demonſtratio.

Si enim Φ deſignet iſtum angulum, cuius ſinus proponitur $= s$; modo vidimus, eſſe ſatis exacte $s = \Phi - \frac{1}{6}\Phi^3$, hinc autem per conuerſionem oritur proxime $\Phi = s + \frac{1}{6}s^3$, quae expreſſio quantum peccet in angulo triginta graduum, vt videamus, ſit $s = \frac{1}{2}$ eritque $s + \frac{1}{6}s^3 = 0,5208333$. Cum autem numerus 3, 14159265 reſpondeat angulo 180°, hic numerus 0, 5208333 praebet angulum 29° 50′ 30″: verus autem angulus eſt 30°, ita vt error ſit 9′ 30″. Sit autem $s = \frac{1}{4}$ cui ſinui reſpondet angulus $= 14°\ 28'\ 39''$, erit $s + \frac{1}{6}s^3 = 0,2526041 = 14°\ 28'\ 23''$, ita vt hoc caſu error non excedat 16″. Patet ergo dum angulus minor ſit 30°, hoc modo ſatis exacte ex dato ſinu reperiri angulum.

Problema 1.

Tab. I. Fig. 2.

20. Si ex puncto lucido E in ſuperficiem ſphaericam reuolutione arcus circularis AM circa axem EC genitam incidat radius EM, definire eius, poſtquam fuerit refractus, concurſum cum axe EC.

Solutio.

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Solutio.

Sit C centrum ſuperficiei ſphaericae, eiusque radius $CA = CM = f$, et diſtantia puncti lucidi E a ſuperficie refringente $EA = a$, tum vero ſit ratio refractionis radiorum ex aere in vitrum $= n : 1$. Vocetur pro radio incidente CM angulus $ACM = \Phi$, eritque huius anguli ſinus $= \Phi - \frac{1}{6}\Phi^3$. Ponatur iam breuitatis gratia diſtantia $EC = a + f = c$, et quia in triangulo ECM dantur latera $CM = f$, $EC = c$ cum angulo $CEM = \Phi$, fiet producta CM in c

$$CM\ (f) : \text{ſin. } CEM = CE\ (c) : \text{ſin. } EMc$$

ideoque

$$\text{ſin. } EMc = \frac{c}{f}\,\text{ſin. }\Phi = \frac{c}{f}\left(\Phi - \tfrac{1}{6}\Phi^3\right);$$

vnde ipſe angulus EMc elicitur, neglectis tertia altioribus poteſtatibus ipſius Φ:

$$EMc = \frac{c}{f}\Phi - \frac{c}{6f}\Phi^3 + \frac{c^3}{6f^3}\Phi^3 = \frac{c}{f}\Phi + \frac{c(cc-ff)}{6f^3}\Phi^3$$

qui cum ſit $= ECM + CEM$, erit

$$ECM = \frac{c-f}{f}\Phi + \frac{c(cc-ff)}{6f^3}\Phi^3$$

Verum angulus EMc eſt angulus incidentiae, ac ſi MO referat radium refractum, erit CMO angulus refractionis, ſicque per hypotheſin

$$\text{ſin. } EMc : \text{ſin. } CMO = n : 1.$$

vnde colligitur

$$\text{ſin. } CMO = \frac{c}{nf}\left(\Phi - \tfrac{1}{6}\Phi^3\right)$$

hincque ipſe angulus

$$CMO = \frac{c}{nf}\Phi + \frac{c(cc-nnff)}{6n^3f^3}\Phi^3$$

quo ablato ab angulo ECM relinquitur angulus

$$COM = \frac{(n-1)c-nf}{nf}\Phi + \frac{c((n^3-1)cc-nn(n-1)ff)}{6n^3f^3}\Phi^3$$

cuius sinus propterea erit

$$\text{fin. } COM = \frac{(n-1)c-nf}{nf}\Phi + \frac{3(n-1)c^3+3(n-1)^2ccf-4n(n-1)cff+nnf^3}{6nnf^3}\Phi^3$$

Iamuero ob

fin. COM : CM (f) = fin. CMO : CO, obtinebitur

$$CO = \frac{\frac{c}{n} - \frac{c}{6n}\Phi\Phi}{\frac{(n-1)c-nf}{nf} + \frac{3(n-1)c^3+3(n-1)^2ccf-4n(n-1)cff+nnf^3}{6nnf^3}\Phi^2}$$

cuius formulae euolutio praebet:

$$CO = \frac{cf}{(n-1)c-nf} - \frac{cf\Phi\Phi}{6((n-1)c-nf)} - \frac{c(3(n-1)c^3+3(n-1)^2ccf-4n(n-1)cff+nnf^3}{6nf((n-1)c-nf)^2}\Phi\Phi$$

quae porro reducitur ad hanc formam:

$$CO = \frac{cf}{(n-1)c-nf} - \frac{(n-1)cc(c-f)(c+nf)}{2nf((n-1)c-nf)^2}\Phi\Phi$$

cui si addamus $CA = f$ prodibit

$$AO = \frac{nf(c-f)}{(n-1)c-nf} - \frac{(n-1)cc(c-f)(c+nf)}{2nf((n-1)c-nf)^2}\Phi\Phi$$

atque hinc positio radii refracti MO primo per interuallum AO modo inuentum definitur, tum vero insuper ex angulo AOM, qui erit:

$$AOM = \frac{(n-1)c-nf}{nf}\Phi + \frac{c((n^3-1)cc-nn(n-1)ff)}{6n^3f^3}\Phi^3$$

siue

$$AOM = \frac{(n-1)c-nf}{nf}\Phi + \frac{(n-1)c((nn+n+1)cc-nnff)}{6n^3f^3}\Phi^3$$

Coroll. 1.

21. Cum posuerimus $c = a+f$ erit

$c-f = a$, et $(n-1)c-nf = (n-1)a-f$,

atque

atque

$(mn+n+1)cc-mff=(nn+n+1)aa+2(m+n+1)af+(n+1)ff$

quibus valoribus substitutis habebimus

$$AO=\frac{n\,a\,f}{(n-1)a-f}-\frac{(n-1)a(a+f)^2(a+(n+1)f)}{2nf((n-1)a-f)^2}\Phi\Phi \text{ et}$$

$$AOM=\frac{(n-1)a-f}{nfa}\Phi+\frac{(n-1)(a+f)(nn+n+1)(a+(n+1)f)+(n+1)ff}{6n^3f^2}\Phi^3$$

Coroll. 2.

22. Si M sit in extremitate aperturae, erit semidiameter aperturae $=f$ sin. ECM $=a\Phi$ satis exacte; neque enim necesse est, aperturam tam exacte nosse. Vnde si semidiameter aperturae ponatur $=x$, erit prope $x=a\Phi$, ideoque $\Phi=\frac{x}{a}$.

Coroll. 3.

23. Si accuratius rem definire velimus, quia est

$$\text{sin. ECM}=\frac{c-e}{f}\Phi+\frac{(c-f)(c-f)}{6ff}\Phi^3=\frac{a}{f}\Phi+\frac{a(a+2f)}{6ff}\Phi^3$$

erit $x=a\Phi+\frac{a(a+2f)}{6f}\Phi^3$

ideoque $\Phi=\frac{x}{a}-\frac{(a+2f)x^3}{6a^3f}$

sed quia in valore ipsius AO non vltra secundam dimensionem ipsius Φ ascendimus, hac expressione non indigemus.

Coroll. 4.

24. Etsi hae expressiones tantum sunt prope verae; tamen in praxi sine errore adhiberi poterunt, dummodo anguli, qui in calculum sunt ingressi, infra

30°

30° subsistant. Non solum ergo necesse est, vt angulus $AEM = \Phi$ sed etiam angulus EMc seu $\frac{c}{f}\Phi = \Phi + \frac{a}{f}\Phi$ minor sit 30 gradibus.

Scholion

25. Summo quidem rigore geometrico distantiam EO definire potuissemus, neque opus fuisset ad approximationes confugere: scilicet si posuissemus angulum $ECM = \omega$, et distantiam $CO = u$; deuenissemus ad hanc determinationem,

$$\frac{cf}{u} = -c\cos.\omega + \sqrt{(nn(c\cos.\omega - f)^2 + (nn-1)cc\sin.\omega^2)}$$

foretque

$$\cos.\omega = \frac{c}{f}\sin.\Phi^2 + \cos.\Phi\sqrt{(1-\frac{cc}{ff}\sin.\Phi^2)} \text{ et}$$

$$\sin.\omega = \frac{c}{f}\sin.\Phi\cos.\Phi - \sin.\Phi\sqrt{(1-\frac{cc}{ff}\sin.\Phi^2)}$$

hisque valoribus substitutis

$$\frac{cf}{u} = \frac{-cc\sin.\Phi^2}{f} - c\cos.\Phi\sqrt{(1-\frac{cc}{ff}\sin.\Phi^2)} + (c\cos.\Phi - \sqrt{(ff-cc\sin.\Phi^2)})\sqrt{(nn-\frac{cc}{ff}\sin.\Phi^2)}$$

vnde ponendo angulo Φ valde paruo per aproximationem superior expressio eliceretur. Verum praecedens analysis magis ad praesens institutum videtur accommodata. Interim si quis propius ad veritatem accedere voluerit, ex formula vera hic exhibita adipiscetur.

$$\frac{cf}{u} = (n-1)c\cos.\Phi - nf + \frac{(n-1)cc}{2nff}((n-1)f + c\cos.\Phi)\sin.\Phi^2 + \frac{c^4}{8n^3f^4}((nn-1)^2f + (n^3-1)c\cos.\Phi)\sin.\Phi^4$$

seu

seu

$$\frac{cf}{u} = (n-1)c - nf + \frac{(n-1)c(c-f)(c+nf)}{2nff}\Phi\Phi$$
$$+ \frac{(n-1)(c-f)(3nn+n+1)c^2 + 2nn(n+1)cf - nnn(n-1)ff}{24n^3f^3}\Phi^3.$$

Problema 2.

26. Si prout in problemate praecedente inuenimus, radius MO in vitrum missus per superficiem sphaericam BN iterum in aerem erumpat, definire punctum V, vbi is cum axe concurret.

Solutio.

Ponatur interuallum AB$=d$; sitque superficiei sphaericae BN centrum in D eiusque radius DB$=$DN$=g$. Quoniam igitur positio radii incidentis MNO datur, ponamus BO$=b$, et angulum BON$=\psi$, et brevitatis ergo interuallum DO$=b+g=e$. Cum nunc in triangulo DON dentur latera DN$=g$, DO$=e$ cum angulo BON$=\psi$, reperitur

sin. DNM$=\frac{e}{g}$ sin. $\psi=\frac{e}{g}\psi-\frac{e}{6g}\psi^3$, hincque

DNM$=\frac{e}{g}\psi+\frac{e(ee-gg)}{6g^3}\psi^3$ et

ODN$=\frac{e-g}{g}\psi+\frac{e(ee-gg)}{6g^3}\psi^3$

Sed hic sin. DNM est sinus incidentiae, cui respondet angulus refractionis VNd, cuius sinus propterea est ad illum vt n : 1, vnde fit

sin. VN$d=\frac{ne}{g}\psi-\frac{ne}{6g}\psi^3$

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hincque

$$VNd = \frac{ne}{g}\psi + \frac{ne(nnee-gg)}{6g^3}\psi^3$$

a quo ablato angulo ODN relinquitur angulus

$$DVN = \frac{(n-1)e+g}{g}\psi + \frac{e((n^3-1)ee-(n-1)gg)}{6g^3}\psi^3$$

ergo

$$\text{fin. } DVN = \frac{(n-1)e+g}{g}\psi + \frac{3n(n-1)e^3-3(n-1)^2eeg-4(n-1)egg-g^3}{6g^3}\psi^3$$

Cum nunc sit

$$\text{fin. } DVN : g = \text{fin. } VNd : DV \text{ erit}$$

$$\frac{g}{DV} = \frac{(n-1)e+g+\frac{1}{6gg}(3n(n-1)e^3-3(n-1)^2eeg-4(n-1)egg-g^3)\psi^2}{ne-\frac{1}{6}ne\psi^2}$$

quae expreſſio reducitur ad hanc formam:

$$\frac{g}{DV} = \frac{(n-1)e+g}{ne} + \frac{(n-1)(e-g)(ne+g)\psi^2}{2ngg}$$

vnde reciproce oritur

$$DV = \frac{neg}{(n-1)e+g} - \frac{n(n-1)ee(e-g)(ne+g)}{2g((n-1)e+g)^2}\psi^2$$

et

$$BV = \frac{g(e-g)}{(n-1)e+g} - \frac{n(n-1)ee(e-g)(ne+g)}{2g((n-1)e+g)^2}\psi^2$$

Puncto autem V inuento notandum eſt eſſe angulum

$$BVN = \frac{(n-1)e+g}{g}\psi + \frac{e((n^3-1)ee-(n-1)gg)}{6g^3}\psi^3$$

Coroll. 1.

27. Cum ſit $e = b+g$ erit $(n-1)e+g = (n-1)b+ng$,

quo

quo valore restituto habebimus

$$BV = \frac{bg}{(n-1)b+ng} - \frac{n(n-1)b(b+g)^2(nb+(n+1)g)}{2g((n-1)b+ng)^2}\psi^2 \text{ et}$$

$$BVN = \frac{(n-1)b+ng}{g}\psi + \frac{(b+g)((n^3-1)(b+g)^2-(n-1)gg)}{6g^3}\psi^3.$$

Coroll. 2.

28. Hae formulae etiam ex praecedentibus erui possunt, si pro litteris n, a, f scribantur $\frac{1}{n}$, $-b$, et $-g$, quoniam hoc modo casus praecedentis problematis ad hunc reducitur. Praeterea autem, qui angulus ibi erat Φ hic est ψ.

Coroll. 3.

29. Quia in praecedente problemate inuenimus tam lineam AO quam angulum AOM erit hoc problemate ad radium illum refractum MO accommodato

$$BO = b = \frac{ng}{(n-1)a-f} - d = \frac{(n-1)n(a+f)^2(a+(n+1)f)}{2n^3((n-1)a-f)^3}\Phi\Phi$$

et

$$\psi = \frac{(n-1)a-f}{nf}\Phi + \frac{(n-1)(a+f)((n+1)(n+2)a+(n+1)(n+2)f)}{6n^3f^3}\Phi^3.$$

Problema 3.

30. Proposita lente vitrea MABN faciebus sphaericis AM et BN terminata, si a puncto quocunque E in eius axe posito incidat in eam radius EM, definire punctum V, in quo is post geminam refractionem iterum cum axe lentis sit concursurus. Tab. I. Fig. 2.

Solutio.

Conſideremus lentem vt vtrinque conuexam, ſitque faciei anterioris AM radius $AC = f$, poſterioris vero BN radius $BD = g$, ipſius lentis autem craſſities $AB = d$. Lens porro ſit vitrea, ita vt ſi in eam radius lucis ex aere incidat, ſit ſinus incidentiae ad ſinum refractionis vt n ad 1. Iam recta iungens centra vtriusque faciei C et D erit axis lentis; in quo reperiatur punctum lucidum E ante lentem in diſtantia $AE = a$, vnde ſub angulo $AEM = \Phi$ in lentem incidat radius EM, qui prima refractione ita inflectatur, vt productus cum axe concurrat in O. Quodſi iam ex iis, quae problemate primo ſunt inuenta, ponamus

$$BO = \frac{naf}{(n-1)a-f} + d - \frac{(n-1)a(a+f)^2(a+(n+1)f}{2nf((n-1)a-f)^3}\Phi\Phi = b$$

$$\text{et ang. } BON = \frac{(n-1)a-f}{nf}\Phi = \psi$$

in valore enim anguli ψ negligere licet terminum Φ^3 inuoluentem, quoniam calculum tantum ad ſecundam poteſtatem ipſius ψ extendimus; his poſitis in problemate ſecundo inuenimus fore

$$BV = \frac{bg}{(n-1)b+ng} - \frac{n(n-1)b(b+g)^2(nb+(n+1)g)}{2g((n-1)b+ng)^3}\psi\psi$$

et

$$BVN = \frac{(n-1)b+ng}{g}\psi.$$

Totum ergo negotium huc redit, vt iſthic pro b et ψ valo-

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valores assignatos substituamus, quod quo facilius fieri possit, statuamus

$$b = P - Q\Phi\Phi \text{ et } \psi = R\Phi \text{ vt sit}$$

$$P = \frac{naf}{(n-1)a-f} - d = \frac{naf-(n-1)ad+df}{(n-1)a-f}$$

$$Q = \frac{(n-1)n(a+f)^2(a+(n+1)f)}{2nf((n-1)a-f)^2}, \text{ et } R = \frac{(n-1)a-f}{nf}$$

Hinc erit:

$$\frac{bg}{(n-1)P+ng} = \frac{Pg-Qg\Phi\Phi}{(n-1)P+ng-(n-1)Q\Phi\Phi} = \frac{Pg}{(n-1)P+ng} - \frac{nQgg\Phi\Phi}{((n-1)P+ng)^2}$$

at in altero membro sufficit pro b scribere P: ex quo obtinebimus

$$BV = \frac{Pg}{(n-1)P+ng} - \frac{nQgg\Phi\Phi}{((n-1)P+ng)^2} - \frac{n(n-1)PRR(P+g)^2(nP+(n+1)g)}{2g((n-1)P+ng)^2}\Phi\Phi$$

$$\text{et } BVN = \frac{(n-1)P+ng}{g}R\Phi$$

Pro his autem substitutionibus notandum est fore:

$$(n-1)P+ng = \frac{n(n-1)a(f+g)-nfg-(n-1)^2ad+(n-1)df}{(n-1)a-f}$$

$$P+g = \frac{naf+(n-1)ag-fg-(n-1)ad+df}{(n-1)a-f}$$

$$nP+(n+1)g = \frac{nnaf+(nn-1)ag-(n+1)fg-n(n-1)ad+ndf}{(n-1)a-f}$$

vnde concluditur:

$$BV = \frac{nafg-(n-1)adg+dfg}{n(n-1)a(f+g)-nfg-(n-1)^2ad+(n-1)df}$$

$$-\frac{(n-1)agg(a+f)^2(a+(n+1)f)}{2f(n(n-1)a(f+g)-nfg-(n-1)^2ad+(n-1)df)^2}\Phi\Phi$$

$$-\frac{(n-1)(naf-(n-1)ad+df)(naf+(n-1)ag-fg-(n-1)ad+df)^2(nnaf+(nn-1)ag-(n+1)fg-n(n-1)ad+ndf)}{2nffg(n(n-1)a(f+g)-nfg-(n-1)^2ad+(n-1)df)^2}\Phi\Phi$$

$$\text{et } BVN = \frac{n(n-1)a(f+g)-nfg+(n-1)df-(n-1)^2ad}{nfg}\Phi.$$

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Coroll. 1.

31. Si angulus Φ prorsus euanescat, punctum V cadet in imaginem principalem, cuius si a lente distantia dicatur $= \alpha$ erit

$$\alpha = \frac{nafg-(n-1)adg+dfg}{n(n-1)a(f+g)-nfg+(n-1)^2ad+(n-1)df}$$

quae si vt data spectetur, hac aequatione relatio inter f et g definitur, vt haec distantia principalis oriatur.

Coroll. 2.

32. Quare si distantia obiecti ante lentem sit $= a$, eiusque imago principalis post lentem ad distantiam $= \alpha$ proiici debeat huic aequationi satisfieri oportet.

$$n(n-1)aa(f+g)-nafg+(n-1)adg-(n-1)^2aad=0$$
$$-n\alpha fg+(n-1)\alpha df \quad -dfg$$

Coroll. 3.

33. Hanc autem imaginis principalis distantiam α in calculum introducendo expressiones nostrae inuentae haud mediocriter contrahentur. Cum enim sit $\alpha = \frac{Pg}{(n-1)P+ng}$, erit hinc $P = \frac{n\alpha g}{g-(n-1)\alpha}$

et porro

$$(n-1)P+ng = \frac{ngg}{g-(n-1)\alpha}; \quad P+g = \frac{fg+\alpha g}{g-(n-1)\alpha} = \frac{g(\alpha+f)}{g-(n-1)\alpha}$$
$$nP+(n+1)g = \frac{g(\alpha+(n+1)f)}{g-(n-1)\alpha}$$

vnde

vnde facta substitutione sequentes determinationes simpliciores assequemur.

$$BV = \alpha - \frac{(n-1)\alpha(\alpha+f)^2(g-(n-1)\alpha)^2(\alpha+(n+1)f)}{2nnffgg((n-1)\alpha-f)^2}\Phi\Phi$$
$$- \frac{(n-1)\alpha(\alpha+g)^2((n-1)\alpha-f)^2(\alpha+(n+1)g)}{2nnffg(g-(n-1)\alpha)^2}\Phi\Phi$$

et

$$BVN = \frac{((n-1)\alpha-f)g}{g-(n-1)\alpha)f}\Phi.$$

Scholion.

34. Non solum hae formulae multo sunt breuiores et concinniores, quam primo inuentae, sed etiam perspicuus ordo in iis obseruatur quo litterae α et g cum litteris α et f ita connexae sunt, vt permutationem admittant. Nullum igitur est dubium, quin si statim distantiam α in calculum introduxissemus, via breuiori ad eas peruenire licuisset. Ceterum quia hae formulae posteriores non amplius crassitiem lentis $AB = d$ involuunt, euidens est eas ex primo inventis nasci, si introducta distantia α crassities d eliminetur, seu eius loco hic valor surrogetur.

$$d = \frac{n(n-1)\alpha\alpha(f+g)-n(\alpha+\alpha)fg}{(n-1)^2\alpha\alpha-(n-1)(\alpha g+\alpha f)+fg} - n\frac{(n-1)\alpha\alpha(f+g)-(\alpha+\alpha)fg}{((n-1)\alpha-f)((n-1)\alpha-g)}$$

qui labor autem ne fuscipi quidem mereretur, nisi iam ante de eximio eius vsu certiores essemus facti. In sequentibus igitur his elegantioribus formulis vtamur, quoad negotium adhuc succinctius expedire dicerimus.

Proble-

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Problema 4.

35. Propoſita lente quacunque faciebus ſphaericis terminata, ſi obiectum in data ab ea diſtantia ſit conſtitutum, pro data lentis apertura ſpatium diffuſionis aſſignare.

Solutio.

Tab. I. Fig. 1. Concipiamus lentem vtrinque conuexam, ſitque faciei anterioris MAM radius $= f$, poſterioris NBN $= g$, lentisque craſſities AB $= d$. Sit porro MM lentis huius apertura, cuius ſemidiameter ſit $= x$; atque in axe lentis expoſitum ſit obiectum vel ſaltem punctum lucidum E, cuius a lente ponatur diſtantia AE $= a$. Iam primo quaeramus eius imaginem principalem, quae cadat in F atque ſupra (31) inuenimus fore

$$BF = \frac{nafg-(n-1)adg+dfg}{n(n-1)a(f+g)-nfg-(n-1)^2ad+(n-1)df}$$

Vocetur ergo haec diſtantia $= \alpha$, et ſi radii EM circa oram aperturae tranſeant, erit angulus AEM $= \Phi = \frac{x}{a}$, quo valore in ſuperioribus formulis ſubſtituto prodibit diſtantia imaginis extremae f a lente ſcilicet:

$$Bf = \alpha - \frac{(n-1)a(a+f)^2(g-(n-1)a)^2(a+(n+1)f)}{2nnfgg((n-1)a-f)^2} \cdot \frac{x}{a}\frac{x}{a}$$
$$- \frac{(n-1)a(a+g)^2((n-1)a-f)^2(a+(n+1)g)}{2nnffg(g-(n-1)a)^2} \cdot \frac{x}{a}\frac{x}{a}$$

ex quo colligitur ſpatium diffuſionis quaeſitum:

$$Ff = +\frac{(n-1)a(a+f)^2(g-(n-1)a)^2(a+(n+1)f)}{2nnfgg((n-1)a-f)^2} \cdot \frac{xx}{aa}$$
$$+ \frac{(n-1)a(a+g)^2((n-1)a-f)^2(a+(n+1)g)}{2nnffg(g-(n-1)a)^2} \cdot \frac{xx}{aa}$$

ac

ac praeterea angulus BfN erit

$$BfN = \frac{((n-1)\alpha - f)g}{(g-(n-1)\alpha)f} \cdot \frac{x}{a}.$$

Coroll. 1.

36. Quoties ergo ſpatium diffuſionis hoc modo expreſſum eſt poſitiuum, imago extrema propius ad lentem cadit quam principalis ſeu eſt Bf < BF. Contra autem ſi iſta expreſſio valorem obtineat negatiuum, imago extrema a lente longius erit remota principali.

Coroll. 2.

37. Patet hinc etiam ſpatium diffuſionis cum apertura ita creſcere, vt ſit quadrato ſemidiametri aperturae proportionale, ſequetur ergo ipſam aperturae rationem.

Coroll. 3.

38. Spatium diffuſionis etiam hoc modo exprimi poteſt

$$Ff = \begin{cases} \dfrac{+(n-1)\alpha\left(1+\frac{\alpha}{f}\right)^2\left(1-\frac{(n-1)\alpha}{g}\right)^2\left(n+1+\frac{\alpha}{f}\right)}{2nn\left(\frac{(n-1)\alpha}{f}-1\right)^2} \cdot \dfrac{xx}{aa} \\ \dfrac{+(n-1)\alpha\left(1+\frac{\alpha}{g}\right)^2\left(\frac{(n-1)\alpha}{f}-1\right)^2\left(n+1+\frac{\alpha}{g}\right)}{2nn\left(1-\frac{(n-1)\alpha}{g}\right)^2} \cdot \dfrac{x\,x}{a\,a} \end{cases}$$

et angulus

$$BfN = \frac{\frac{(n-1)\alpha-1}{f}}{1-\frac{(n-1)\alpha}{g}} \cdot \frac{x}{a}.$$

Coroll. 4.

39. Quia hae formulae introducendis litterarum valoribus reciprocis redditae sunt simpliciores, in hunc modum etiam aequatio (§ 32.) exhibita tractetur quae per $aadfg$ diuisa abit in hanc formam.

$$\alpha(n-1)\tfrac{1}{af}(\tfrac{1}{f}+\tfrac{1}{g})-n\tfrac{1}{af}(\tfrac{1}{\alpha}+\tfrac{1}{a})+(n-1)(\tfrac{1}{af}+\tfrac{1}{ag})-(n-1)^2\tfrac{1}{fg}-\tfrac{1}{aa}=0$$

seu

$$\tfrac{n}{a}((n-1)\tfrac{\alpha}{f}(\tfrac{1}{f}+\tfrac{1}{g})-\tfrac{\alpha}{a}+1)=(\tfrac{(n-1)\alpha}{f}-1)(\tfrac{(n-1)\alpha}{g}-1)$$

quae commodior erit tam ad relationem inter α et a, quam inter f et g definiendam.

Scholion.

40. Scilicet si proposita lente obiectum in variis distantiis exponatur ac pro singulis distantiam imaginis principalis definire velimus, aequatio hoc modo referatur:

$$\tfrac{1}{\alpha\alpha}-(n-1)(\tfrac{1}{\alpha f}+\tfrac{1}{\alpha g})+n(\tfrac{1}{a\alpha}+\tfrac{1}{a\alpha})-n(n-1)(\tfrac{1}{af}+\tfrac{1}{ag})+(n-1)^2\tfrac{1}{fg}=0$$

quae per factores ita adornari poterit

$$(\tfrac{1}{\alpha}-\tfrac{(n-1)}{f}+\tfrac{n}{a})(\tfrac{1}{\alpha}-\tfrac{(n-1)}{g}+\tfrac{n}{a})=\tfrac{nn}{aa}$$

Vnde patet productum ex his duobus factoribus semper esse idem. Cuius resolutio quo facilius instituatur, capiantur numeri μ et ν ita, vt eorum summa sit $=1$, scilicet $\mu+\nu=1$ ac statuatur

$$\tfrac{1}{\alpha}-\tfrac{(n-1)}{f}+\tfrac{n}{a}=\tfrac{\mu n}{\nu a} \text{ erit } \tfrac{1}{\alpha}=\tfrac{n-1}{f}-\tfrac{n}{\nu a}$$

$$\tfrac{1}{\alpha}-\tfrac{(n-1)}{g}+\tfrac{n}{a}=\tfrac{\nu n}{\mu a} \text{ erit } \tfrac{1}{\alpha}=\tfrac{n-1}{g}-\tfrac{n}{\mu a}$$

Quare

Quare si distantia obiecti $AE = a$ ita capiatur, vt sit $\frac{1}{a} = \frac{n-1}{f} - \frac{n}{\nu d}$, distantia imaginis principalis $BF = \alpha$ ita se habebit, vt sit $\frac{1}{\alpha} = \frac{n-1}{g} - \frac{n}{\mu d}$.

Eodem modo si dentur binae distantiae $AE = a$ et $BF = \alpha$ cum crassitie lentis $AB = d$, radii faciei rum f et g ita debent esse comparati, vt sit

$$\frac{1}{f} = \frac{\nu}{(n-1)a} + \frac{n}{\nu(n-1)d} \text{ et } \frac{1}{g} = \frac{1}{(n-1)\alpha} + \frac{n}{\mu(n-1)d}$$

id quod infinitis modis praestari potest, cum numeri μ et ν pro arbitrio accipi queant, dummodo eorum summa $\mu + \nu$ aequetur vnitati. Si hoc modo etiam in formulis pro spatio diffusionis inuentis omnes litterae in denominatores detrudantur, reperietur:

$$F = \frac{(n-1)\alpha a x x}{2nn} \left\{ \frac{\left(\frac{1}{a}+\frac{1}{f}\right)^2\left(\frac{1}{a}+\frac{n+1}{g}\right)^2\left(\frac{n+1}{a}+\frac{1}{f}\right)}{\left(\frac{n-1}{f}-\frac{1}{a}\right)^3} + \frac{\left(\frac{1}{a}+\frac{1}{g}\right)^2\left(\frac{n-1}{f}-\frac{1}{a}\right)^2\left(\frac{n+1}{a}+\frac{1}{g}\right)}{\left(\frac{1}{a}-\frac{(n-1)}{g}\right)^3} \right\}$$

et

$$\text{angulus } BfN = \frac{\frac{n-1}{f}-\frac{1}{a}}{\frac{1}{a}-\frac{(n-1)}{g}} \cdot \frac{x}{\alpha} = \frac{\frac{n}{\nu d}}{\frac{n}{\mu d}} \frac{x}{a} = \frac{\mu}{\nu} \frac{x}{a}$$

Cum autem sit

$$\frac{n-1}{f} - \frac{1}{a} = \frac{n}{\nu d} \text{ et } \frac{1}{\alpha} - \frac{(n-1)}{g} = \frac{-n}{\mu d} \text{ erit}$$

$$F = \frac{(n-1)\alpha a x x}{2nn}\left(\frac{\nu\nu}{\mu\mu}\frac{1}{a}+\frac{1}{f}\right)^2\left(\frac{n+1}{a}+\frac{1}{f}\right)+\frac{\mu\mu}{\nu\nu}\left(\frac{1}{\alpha}+\frac{1}{g}\right)^2\left(\frac{n+1}{\alpha}+\frac{1}{g}\right)$$

et

$$\text{angulus } BfN = -\frac{\mu x}{\nu a} = \frac{\mu x}{(\mu-1)\alpha}.$$

Problema 5.

Tab. I. Fig. 1.

41. Datis distantiis obiecti ante lentem $AE = a$ et imaginis principalis post lentem $BF = \alpha$ vna cum lentis crassitie $AB = d$, definire omnes lentes satisfacientes, simulque pro singulis spatium diffusionis Ff.

Solutio.

Modo vidimus si lentis faciei anterioris AM radius ponatur $= f$, posterioris $BN = g$, lente vt conuexa vtrinque spectata, hos duos radios ita comparatos esse debere, vt sit

$$\frac{n-1}{f} = \frac{1}{a} + \frac{n}{\nu d} \text{ et } \frac{n-1}{g} = \frac{1}{\alpha} + \frac{n}{\mu d}$$

sumtis pro μ et ν numeris quibuscunque, vt sit $\mu + \nu = 1$; vnde infinitae lentes quaesito satisfacientes obtinentur. Deinde si huius lentis semidiameter aperturae ponatur $= x$, spatium diffusionis Ff ita exprimi potest vt sit

$$Ff = \frac{aaxx}{2n(n-1)^2}\left(\frac{\nu\nu}{\mu\mu}\left(\frac{n-1}{a} + \frac{n-1}{f}\right)^2\left(\frac{nn-1}{a} + \frac{n-1}{f}\right) + \frac{\mu\mu}{\nu\nu}\left(\frac{n-1}{a} + \frac{n-1}{g}\right)^2\left(\frac{nn-1}{a} + \frac{n-1}{g}\right)\right)$$

et angulus $BfN = \frac{\mu x}{(\mu - 1)\alpha}$.

Quod si iam hic pro $\frac{n-1}{f}$ et $\frac{n-1}{g}$ valores assignati substituantur spatium diffusionis erit

$$Ff = \frac{n\alpha\alpha xx}{2(n-1)^2}\left(\frac{\nu\nu}{\mu\mu}\left(\frac{1}{a} + \frac{1}{\nu d}\right)^2\left(\frac{n}{a} + \frac{1}{\nu d}\right) + \frac{\mu\mu}{\nu\nu}\left(\frac{1}{\alpha} + \frac{1}{\mu d}\right)^2\left(\frac{n}{\alpha} + \frac{1}{\mu d}\right)\right)$$

seu

$$Ff = \frac{n\alpha\alpha xx}{2(n-1)}\left\{\begin{array}{l} + \frac{\nu\nu}{\mu\mu}\left(\frac{n}{a^3} + \frac{2n+1}{\nu a a d} + \frac{n+2}{\nu\nu a d d} + \frac{1}{\nu^3 d^3}\right) \\ + \frac{\mu\mu}{\nu\nu}\left(\frac{n}{\alpha^3} + \frac{2n+1}{\mu\alpha\alpha d} + \frac{n+2}{\mu\mu\alpha d d} + \frac{1}{\mu^3 d^3}\right) \end{array}\right\}$$

Si

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Si crassities lentis fuerit valde parua, ne confusio fiat enormis, necesse est pro μ et ν sumi numeros vehementer magnos, alterum scilicet positiuum alterum negatiuum. Cum igitur sit $\mu d+\nu d=d$, statuatur $\mu d=\frac{d-k}{2}$ et $\nu d=\frac{d+k}{2}$, vt sit

$$\frac{n-1}{f}=\frac{1}{a}+\frac{2n}{k+d} \text{ et } \frac{n-1}{g}=\frac{1}{\alpha}-\frac{2n}{k-d} \text{ seu}$$

$$f=\frac{(n-1)a(k+d)}{2na+k+d} \text{ et } g=\frac{(n-1)\alpha(k-d)}{k-d-2n\alpha}$$

hincque obtinetur spatium diffusionis

$$Ff=\frac{n\alpha axx}{2(n-1)^2}\left\{\begin{array}{l}+\left(\frac{k+d}{k-d}\right)^2\left(\frac{n}{a}+\frac{2}{k+d}\right)\left(\frac{1}{a}+\frac{2}{k+d}\right)^2\\+\left(\frac{k-d}{k+d}\right)^2\left(\frac{n}{\alpha}-\frac{2}{k-d}\right)\left(\frac{1}{\alpha}-\frac{2}{k-d}\right)^2\end{array}\right\}$$

in quibus formulis paruitas crassitiei lentis $AB=d$ nullum negotium facessit: tum vero est

$$\text{angulus } BfN=\frac{k-d}{k+d}\cdot\frac{x}{\alpha}.$$

Coroll. 1.

42. Propositis ergo binis distantiis $AE=a$ et $BF=\alpha$ vna cum crassitie lentis $AB=d$, infinitis modis lentes idoneae parari possunt cum pro k quantitates pro lubitu siue positiuae siue negatiuae assumi queant.

Coroll. 2.

43. Cum semidiameter aperturae x faciem anteriorem respiciat, et angulus BfN seu BFN sit $=\frac{k-d}{k+d}\cdot\frac{x}{\alpha}$, existente distantia $BF=\alpha$, manifestum est semidiame-

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trum aperturae posterioris faciei esse debere $= \frac{k-d}{k+d} \cdot x$ vel saltem non minorem.

Coroll. 3.

44. Quodsi lentis crassities tam sit parva, vt ea prae k contemni queat, formulae nostrae fient simpliciores

$$f = \frac{(n-1)\alpha k}{k+2n\alpha} \text{ et } g = \frac{(n-1)\alpha k}{k-2n\alpha}$$

et spatium diffusionis

$$F\int = \frac{\alpha\alpha\alpha\beta\beta}{2(n-1)^2}\left(\left(\frac{2}{\alpha}+\frac{2}{k}\right)\left(\frac{1}{\alpha}+\frac{2}{k}\right)^2+\left(\frac{2}{\alpha}-\frac{2}{k}\right)\left(\frac{2}{\alpha}-\frac{2}{k}\right)^2\right)$$

angulusque $BfN = \frac{\alpha}{\beta}$.

Scholion.

45. Sufficiat haec de spatio diffusionis in genere quaecunque fuerit lentis crassities, tradidisse, cum non obstante his satis concinnis transformationibus calculus nimium fieret molestus, si in determinatione spatii diffusionis rationem crassitiei lentis habere vellemus. Licebit autem, vt modo vidimus, crassitiem lentis negligere non solum, quando ipsa est per se valde exigua verum etiam dummodo prae quantitate k fuerit perparua. Atque hinc etiam in sequentibus facile iudicare poterimus, vtrum crassitiem lentis recte in calculo contemserimus, nec ne? quouis enim casu consideretur quantitas pro k assumta, quae si multoties fuerit maior quam d error nullus erit pertimescendus;

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fecedit; contra vero si k non multum superet d multum aberrabitur, quantumuis exigua fuerit crassities ipsa per se.

Vt fiat Ff minimum, definiri potest conueniens valor ipsius k hoc modo. Ponatur $\frac{k-d}{k+d} = z$ et pro minimo peruenitur ad hanc aequationem:

$$0 = 2z^4\left(\frac{n}{a}+\frac{r}{d}\right)\left(\frac{1}{a}+\frac{1}{d}\right)^6$$
$$-\frac{2n}{a}\frac{z^3}{d}\left(\frac{1}{a}+\frac{1}{d}\right)-\frac{z^3}{d}\left(\frac{1}{a}+\frac{2}{d}\right)^2$$
$$+\frac{2n}{a}\frac{z}{d}\left(\frac{1}{a}+\frac{1}{d}\right)+\frac{z}{d}\left(\frac{1}{a}+\frac{2}{d}\right)^2$$
$$-2\left(\frac{n}{a}+\frac{1}{d}\right)\left(\frac{1}{a}+\frac{1}{d}\right)^2$$

vnde valor ipsius z erui debet.

Problema 6.

46. Neglecta lentis crassitie, si detur cum obiecti ante lentem distantia $AE = a$, tum imaginis principalis post lentem distantia $BF = \alpha$, eam definire lentem quae pro data apertura minimam pariat diffusionem.

Solutio.

Positis radiis faciei anterioris $AM = f$ et posterioris $BN = g$ vtraque vt conuexa spectata, vidimus omnes lentes datis distantiis a et α conuenientes his formulis determinari: si scilicet pro k scribamus $2k$

$$\frac{n-1}{f} = \frac{1}{a}+\frac{n}{k} \text{ et } \frac{n-1}{g} = \frac{1}{\alpha}-\frac{n}{k}$$

deno-

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denotante k quantitatem quamcunque. Tum autem ſpatium diffuſionis ita exprimi eſt repertum

$$Ff = \frac{n\alpha\alpha xx}{2(n-1)^2}\left(\left(\frac{n}{\alpha}+\frac{1}{k}\right)\left(\frac{1}{\alpha}+\frac{1}{k}\right)^2+\left(\frac{n}{\alpha}-\frac{1}{k}\right)\left(\frac{1}{\alpha}-\frac{1}{k}\right)^2\right)$$

quae expreſſio ad hanc reducitur formam:

$$Ff = \frac{n\alpha\alpha xx}{2(n-1)^2}\left(\frac{1}{\alpha}+\frac{1}{\alpha}\right)\left(n\left(\frac{1}{\alpha\alpha}-\frac{1}{\alpha\alpha}+\frac{1}{\alpha\alpha}\right)+\frac{2n+1}{k}\left(\frac{1}{\alpha}-\frac{1}{\alpha}\right)+\frac{n+2}{kk}\right)$$

Quaeſtio igitur huc reducitur, vt definiatur quantitas k qua huic expreſſioni valor minimus concilietur; cui quidem requiſito ſatisfacit valor

$$\frac{1}{k} = \frac{-(2n+1)}{2(n+2)}\left(\frac{1}{\alpha}-\frac{1}{\alpha}\right) \text{ ſeu } \frac{1}{k} = \frac{2n+1}{2(n+2)}\left(\frac{1}{\alpha}-\frac{1}{\alpha}\right)$$

quo ſubſtituto habebitur:

$$\frac{n-1}{f} = \frac{4+n-2nn}{2(n+2)\alpha}+\frac{n(2n+1)}{2(n+2)\alpha} \text{ et } \frac{n-1}{g} = \frac{4+n-2nn}{2(n+2)\alpha}+\frac{n(2n+1)}{2(n+2)\alpha}.$$

et ſpatium diffuſionis quod eſt minimum:

$$Ff = \frac{n\alpha\alpha xx}{2(n-1)^2}\left(\frac{1}{\alpha}+\frac{1}{\alpha}\right)\left(n\left(\frac{1}{\alpha\alpha}-\frac{1}{\alpha\alpha}+\frac{1}{\alpha\alpha}\right)-\frac{(2n+1)^2}{4(n+2)}\left(\frac{1}{\alpha}-\frac{1}{\alpha}\right)^2\right)$$

At eſt

$$n\left(\frac{1}{\alpha\alpha}-\frac{1}{\alpha\alpha}+\frac{1}{\alpha\alpha}\right) = n\left(\frac{1}{\alpha}-\frac{1}{\alpha}\right)^2+\frac{n}{\alpha\alpha} \text{ ideoque}$$

$$Ff = \frac{n\alpha\alpha xx}{2(n-1)^2}\left(\frac{1}{\alpha}+\frac{1}{\alpha}\right)\left(\frac{4n-1}{4(n+2)}\left(\frac{1}{\alpha}-\frac{1}{\alpha}\right)^2+\frac{n}{\alpha\alpha}\right)$$

quae expreſſio etiam in hanc formam transfunditur

$$Ff = \frac{n\alpha\alpha xx}{2(n-1)^2}\left(\frac{1}{\alpha}+\frac{1}{\alpha}\right)\left(\frac{4n-1}{4(n+2)}\left(\frac{1}{\alpha}+\frac{1}{\alpha}\right)^2+\frac{(n-1)^2}{(n+2)\alpha\alpha}\right)$$

ſeu

$$Ff = \frac{n(4n-1)\alpha\alpha xx}{8(n-1)^2(n+2)}\left(\frac{1}{\alpha}+\frac{1}{\alpha}\right)\left(\left(\frac{1}{\alpha}+\frac{1}{\alpha}\right)^2+\frac{4(n-1)^2}{(4n-1)\alpha\alpha}\right)$$

Coroll.

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Coroll. 1.

47. Vt igitur lens minimum spatium diffusionis producat, eam ita formari necesse est vt sit

$$f = \frac{2(n-1)(n+2)\alpha\alpha}{n(2n+1)a + (4+n-2nn)\alpha} \text{ et}$$

$$g = \frac{2(n-1)(n+2)\alpha\alpha'}{n(2n+1)a + (4+n-2nn)\alpha}$$

neglecta scilicet lentis crassitie, quae quidem recte negligitur, si modo fuerit vehementer parua prae quantitate $k = \frac{2(n+2)\alpha\alpha}{(2n+1)(a-\alpha)}$.

Coroll. 2.

48. Si igitur sit $a = \alpha$, seu $BF = AE$ crassities lentis, quantacunque fuerit, nihil turbat in spatio diffusionis. Pro hoc autem casu erit $f = g = (n-1)a$ et spatium diffusionis ipsum $Ff = \frac{nnxx}{(n-1)^2 a}$. At quo magis distantiae a et α a se inuicem discrepant, seu minor fuerit quantitas $\frac{a\alpha}{a-\alpha}$, eo magis haec determinatio ob lentis crassitiem fiet erronea.

Coroll. 3.

49. Spatium autem hoc confusionis minimum Ff pluribus modis exhiberi potest, inter quos commodissimum eligi conuenit; sunt autem praecipui:

I. $Ff = \frac{n(4n-1)\alpha\alpha xx}{8(n-1)^2(n+2)}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{4(n-1)^2}{(4n-1)a\alpha}\right)$

II. $Ff = \frac{n(n-1)\alpha\alpha xx}{8(n-1)^2(n+2)}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\left(\frac{1}{a}-\frac{1}{\alpha}\right)^2+\frac{4n(n+2)}{(4n-1)a\alpha}\right)$

III. $Ff = \frac{n(4n-1)\alpha\alpha xx}{8(n-1)^2(n+2)}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\left(\frac{1}{a\alpha}+\frac{1}{\alpha\alpha}\right)+\frac{2(2nn+1)}{(4n-1)a\alpha}\right)$

IV. $Ff = \frac{n(4n-1)\alpha\alpha xx}{8(n-1)^2(n+2)}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\left(\frac{1}{aa}-\frac{1}{a\alpha}+\frac{1}{\alpha\alpha}\right)+\frac{(2n+1)^2}{(4n-1)a\alpha}\right)$

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Coroll. 4.

50. Cum ergo hoc ſpatium diffuſionis ſit minimum, ſi lenti alia quaecunque figura tribuatur, ita tamen, vt obiecti ad diſtantiam $AE = a$ remoti imaginem principalem in diſtantia $BF = \alpha$ exhibeat, ſpatium diffuſionis erit maius, quam hic inuenimus.

Scholion.

51. Quia $n : 1$ denotat rationem refractionis ex aere in vitrum, haec autem pro radiorum natura eſt variabilis, conueniet pro n medium valorem aſſumi, qui eſt $n = \frac{31}{20}$; hinc ergo erit

$$n-1 = \frac{11}{20};\ n+2 = \frac{71}{20};\ 2n+1 = \frac{41}{10};\ 4+n-2nn = \frac{149}{200},$$

hincque

$$\frac{n(2n+1)}{2(n-1)(n+2)} = \frac{8271}{781} = 1,627401$$

$$\frac{4+n-2nn}{2(n-1)(n+2)} = \frac{149}{781} = 0,190781$$

vnde lens minimam confuſionem pariens ita definietur

$$\frac{1}{f} = \frac{149}{781\alpha} + \frac{8271}{781\alpha} = \frac{0,190781}{\alpha} + \frac{1,627401}{\alpha}$$

$$\frac{1}{g} = \frac{149}{781\alpha} + \frac{8271}{781\alpha} = \frac{0,190781}{\alpha} + \frac{1,627401}{\alpha}$$

ſeu

$$f = \frac{781\alpha\alpha}{149\alpha + 8271\alpha} = \frac{\alpha\alpha}{0,190781\alpha + 1,627401\alpha}$$

et

$$g = \frac{781\alpha\alpha}{149\alpha + 8271\alpha} = \frac{\alpha\alpha}{0,190781\alpha + 1,627401\alpha}$$

Pro

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Pro ſpatio autem diffuſionis ipſo definiendo ob $4n-1=\frac{n}{\beta}$,

erit $\frac{n(4n-1)}{3(n-1)^2(n+1)}=\frac{4060}{3591}=0,938191$

et $\frac{4(n-1)^2}{4n-1}=\frac{121}{520}=0,232692$

hincque

$$Ff=0,938191\,\alpha\alpha xx\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{0,232692}{a\alpha}\right)$$

vnde reliquae formulae facile deducuntur. Ceterum hic monendum eſt, quoniam valor $n=\frac{31}{10}$ ex experimentis eſt concluſus, neque ipſe pro omnibus radiorum generibus valet, ſuperfluum fore in praxi hos numeros inuentos nimis ſtudioſe obſeruare; quin etiam ipſa natura minimi aliquam aberrationem permittit. Nihilo vero minus has fractiones decimales ad tot figuras producere viſum eſt, quo facilius quantum ab hac hypotheſi aberretur, dignoſci queat.

Problema 7.

52. Neglecta lentis craſſitie, ſi detur cum obiecti ante lentem diſtantia $AE=a$, tum imaginis principalis poſt lentem diſtantia $BF=\alpha$, eam definire lentem, quae pro data apertura non minimum, ſed datum pariat ſpatium diffuſionis Ff.

Solutio.

Lente vtrinque vt conuexa ſpectata, ſit faciei anterioris radius $=f$, poſterioris $=g$; atque vt ex data

distantia obiecti $AE = a$, data oriatur imaginis principalis distantia $= \alpha$ necesse est sit in genere

$$\frac{n-1}{f} = \frac{1}{a} + \frac{n}{k} \text{ et } \frac{n-1}{g} = \frac{1}{a} - \frac{n}{k}$$

vnde spatium diffusionis fit

$$Ff = \frac{n\alpha\alpha xx}{2(n-1)^2}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(n\left(\frac{1}{aa}-\frac{1}{a\alpha}+\frac{1}{\alpha\alpha}\right)+\frac{2n+1}{k}\left(\frac{1}{\alpha}-\frac{1}{a}\right)+\frac{n+2}{kk}\right)$$

Cum autem spatium diffusionis minimum repertum sit

$$\frac{n(4n-1)\alpha\alpha xx}{8(n-1)^2(n+1)}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{4(n-1)^2}{(4n-1)\alpha\alpha}\right)$$

necesse est vt illud sit maius: statuatur ergo:

$$Ff = \frac{n(4n-1)\alpha\alpha xx}{8(n-1)^2(n+2)}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{4(n-1)^2}{(4n-1)\alpha\alpha}+S\right)$$

et habebitur ista aequatio:

$$4n(n+2)\left(\frac{1}{aa}-\frac{1}{a\alpha}+\frac{1}{\alpha\alpha}\right)+\frac{4(n+2)(2n+1)}{k}\left(\frac{1}{\alpha}-\frac{1}{a}\right)+\frac{4(n+2)^2}{kk} = (4n-1)\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{4(n-1)^2}{a\alpha}+(4n-1)S$$

quae in hanc formam redigitur.

$$(2n+1)^2\left(\frac{1}{a}-\frac{1}{\alpha}\right)^2+\frac{4(n+2)(2n+1)}{k}\left(\frac{1}{\alpha}-\frac{1}{a}\right)+\frac{4(n+2)^2}{kk} = (4n-1)S$$

vnde radice extracta fit

$$(2n+1)\left(\frac{1}{a}-\frac{1}{\alpha}\right)+\frac{2(n+2)}{k} = \sqrt{(4n-1)S}$$

$$\text{et } \frac{1}{k} = \frac{-(2n+1)}{2(n+2)}\left(\frac{1}{a}-\frac{1}{\alpha}\right)+\frac{\sqrt{(4n-1)S}}{2(n+2)}$$

Quare erit

$$\frac{n-1}{f} = \frac{1}{a}-\frac{n(2n+1)}{2(n+2)}\left(\frac{1}{a}-\frac{1}{\alpha}\right)+\frac{n}{2(n+2)}\sqrt{(4n-1)S}$$

$$\frac{n-1}{g} = \frac{1}{\alpha}+\frac{n(2n+1)}{2(n+2)}\left(\frac{1}{a}-\frac{1}{\alpha}\right)-\frac{n}{2(n+2)}\sqrt{(4n-1)S}$$

siue

siue

$$f = \frac{2(n-1)(n+2)\alpha\alpha}{(1+n-2nn)\alpha\alpha+n(2n+1)\alpha\alpha+n\alpha\alpha\sqrt{(4n-1)S}}$$

$$g = \frac{2(n-1)(n+2)\alpha\alpha}{(1+n-2nn)\alpha\alpha+n(2n+1)\alpha\alpha-n\alpha\alpha\sqrt{(4n-1)S}}$$

Oportet ergo pro S sumi quantitatem positiuam, atque vt cum reliqua parte, cui jungitur, sit homogenea, ponamus

$$S = (\lambda - 1)\left(\frac{1}{a} + \frac{1}{\alpha}\right)^2$$

vbi λ denotat numerum vnitate maiorem; atque effici poterit, vt spatium diffusionis ita exprimatur

$$Ff = \frac{n(4n-1)\alpha\alpha xx}{8(n-1)^2(n+1)}\left(\frac{1}{a} + \frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a} + \frac{1}{\alpha}\right)^2 + \frac{4(n-1)^2}{(4n-1)\alpha\alpha}\right)$$

quod cum vt datum spectetur, numerus λ pro dato assumi poterit, ac lens hunc effectum producens ita determinabitur.

$$f = \frac{2(n-1)(n+2)n\alpha}{(4+n-2nn)\alpha+n(2n+1)a+n(a+\alpha)\sqrt{(4n-1)(\lambda-1)}}$$

$$g = \frac{2(n-1)(n+2)n\alpha}{(4+n-2nn)\alpha+n(2n+1)a-n(a+\alpha)\sqrt{(4n-1)(\lambda-1)}}$$

quod ergo cum signum radicale ambiguo illatum inuoluat, duplici modo fieri poterit.

Coroll. 1.

53. Omnes igitur lentes, quae obiecti ad distantiam $AE = a$ remoti, imaginem principalem in distantia $BF = \alpha$ repraesentant, hanc habent proprietatem vt sit:

$$\frac{n-1}{f} + \frac{n-1}{g} = \frac{1}{a} + \frac{1}{\alpha} \text{ seu } \frac{fg}{f+g} = \frac{(n-1)a\alpha}{a+\alpha}.$$

et cum sit

$n = \frac{31}{30}$ erit $\frac{fg}{f+g} = \frac{11aa}{22(a+\alpha)} = \frac{0,55aa}{a+\alpha}$.

Coroll. 2.

54. Si ergo lens sit vtrinque aeque conuexa seu $f = g$, oportet esse $f = g = \frac{2(n-1)a\alpha}{a+\alpha} = \frac{11a'\alpha'}{20(a+\alpha)}$.

sin autem lens desideretur plano-conuexa vt sit $f = \infty$, capi debet

$$g = \frac{(n-1)a\alpha}{a+\alpha} = \frac{11aa}{20(a+\alpha)} = \frac{0,55aa}{a+\alpha}$$

At si lens debeat esse conuexo-plana seu $g = \infty$, capi oportet

$$f = \frac{(n-1)a\alpha}{a+\alpha} = \frac{11aa}{20(a+\alpha)} = \frac{0,55aa}{a+\alpha}$$

Scholion 1.

55. Substituamus pro n valorem ipsi conueni-entem $\frac{31}{30}$, et iam vidimus fore:

$$\frac{n(4n-1)}{3(n-1)^2(n+2)} = \frac{8060}{8591} = 0,938191$$

$$\frac{4(n-1)^2}{4n-1} = \frac{121}{520} = 0,232692$$

$$\frac{4n^2-1-2n}{2(n^2-1)(n+2)} = \frac{149}{781} = 0,190781$$

$$\frac{n(4n^2+1)}{2(n-1)(n+2)} = \frac{1771}{781} = 2,267401$$

nunc vero notari oportet esse:

$$\frac{n\sqrt{(4n-1)}}{2(n-1)(n+2)} = \frac{62\sqrt{130}}{781} = 0,905133$$

Quare

Quare si lens ita construatur vt sit

$$f = \frac{aa}{0,190781aa + 1,627401a + 0,905133(a+x)\sqrt{(\lambda-1)}}$$

$$g = \frac{aa}{0,190781aa + 1,627401a + 0,905133(a+x)\sqrt{(\lambda-1)}}$$

erit pro eius apertura, cuius semidiameter $= x$, spatium diffusionis

$$Ff = 0,938191\,aaxx\left(\frac{1}{a}+\frac{1}{a}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{a}\right)^2 + \frac{0,232602}{aa}\right)$$

Cum autem in posterum hi numeri frequentissime occurrant eorum loco ad abbreuiandum certis characteribus vtamur, ponamus ergo

$$\frac{n(4n-1)}{3(n-1)^2(n+2)} = 0,938191 = \mu$$

$$\frac{4(n-1)^2}{4n-1} = 0,232602 = \nu$$

$$\frac{4+4n-2nn}{2(n-1)(n+2)} = 0,190781 = \varrho$$

$$\frac{n(2n+1)}{4(n-1)(n+2)} = 1,627401 = \sigma$$

$$\frac{n\sqrt{(4n-1)}}{2(n-1)(n+2)} = 0,905133 = \tau$$

seu

$$\mu = \frac{1}{4(n+2)} + \frac{1}{4(n-1)} + \frac{2}{3(n+1)^2}$$

$$\varrho = \frac{1}{2(n-1)} + \frac{1}{n+2} - 1$$

$$\sigma = 1 + \frac{1}{2(n-1)} - \frac{1}{n+2}$$

$$\tau = \frac{1}{2}\left(\frac{1}{2(n-1)} + \frac{1}{n+2}\right)\sqrt{(4n-1)}$$

vnde pro quouis alio medio pellucido hi valores facque supputari possunt

ſicque vt prodeat ſpatium diffuſionis

$$Ff = \mu\alpha\alpha xx\left(\tfrac{1}{a}+\tfrac{1}{\alpha}\right)\left(\lambda\left(\tfrac{1}{a}+\tfrac{1}{\alpha}\right)^2+\tfrac{\nu}{a\alpha}\right)$$

lentis conſtructio erit

$$f = \frac{\alpha\alpha}{\varrho\alpha+\sigma\alpha\pm\tau(a+\alpha)\sqrt{(\lambda-1)}}$$

$$g = \frac{\alpha\alpha}{\varrho\alpha+\sigma\alpha\mp\tau(a+\alpha)\sqrt{(\lambda-1)}}$$

Dummodo igitur λ fuerit numerus poſitiuus vnitate non minor talis lens duplici modo effici poterit. Caſu autem $\lambda = 1$, quo ſpatium diffuſionis eſt minimum, vnica lens propoſito ſatisfaciens conſtrui poteſt.

Coroll. 3.

56. Si igitur lens ſit vtrinque aequaliter conuexa, ideoque $f = g = \frac{2(n-1)a\alpha}{a+\alpha} = \frac{11}{10}\cdot\frac{a\alpha}{a+\alpha}$, in expreſſione noſtra pro ſpatio diffuſionis inuenta valor ipſius λ ex aequalitate inter f et g ſtatuta definietur, vnde fit

$$(\sigma-\varrho)(a-\alpha) = 2\tau(a+\alpha)\sqrt{(\lambda-1)}$$ hincque

$$\lambda = 1+\frac{0{,}629795\alpha\alpha-1{,}259518\alpha\alpha+0{,}629795\alpha\alpha}{(a+\alpha)^2}.$$

Coroll. 4.

57. Si lens capiatur plano conuexa vt ſit:

$$f = \infty \text{ et } g = \frac{(n-1)a\alpha}{a+\alpha} = \frac{11}{20}\cdot\frac{a\alpha}{a+\alpha}$$

pro ſpatio diffuſionis habebitur

$$\varrho\alpha+\sigma\alpha = \mp\tau(a+\alpha)\sqrt{(\lambda-1)}$$

vnde in numeris colligitur

$$\lambda = 1+\frac{0{,}044427\alpha\alpha+0{,}757940\alpha\alpha+3{,}258692\beta\beta}{(a+\alpha)^2}$$

Coroll.

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Corolll. 5.

58. Si denique lens adhibeatur conuexo-plana, vt fit:

$$g = \infty \text{ et } f = \frac{(n-1)aa}{\alpha+a} = \frac{rr}{50} \cdot \frac{pq}{\alpha+a}$$

pro ſpatio diffuſionis inueniendo poni oportet

$$\varrho a + \sigma a = \pm \tau(\alpha \pm a)\sqrt{(\lambda - 1)}$$

vnde in numeris colligimus:

$$\lambda = 1 + \frac{3,232692aa + 0,737940\alpha a + 0,044427\alpha\alpha}{(\alpha+a)^2}$$

Scholion 2.

59. Quod ad aperturam attinet iam initio animaduertimus, in ea maiores arcus comprehendi non debere, quam qui ſint principiis ſtabilitis conformes. Scilicet vt nullus angulus ſupra 30° gradus occurrat, anguli ACM et BDN certe minores 30 gradibus eſſe debent, cum anguli EM*c* et VN*d* ipſis ſint maiores, quorum alter cum quandoque ad duplum aſſurgere poſſit, poterimus hanc regulam ſtatuere, vt aperturae ſemidiameter x neque $\frac{1}{4}f$ neque $\frac{1}{4}g$ ſuperet. Verum quouis caſu ad ipſos angulos EM*c* et VN*d*, qui ſunt maximi, attendi conueniet, atque tanta apertura admitti poterit, vnde neuter horum angulorum 30 gradus ſuperans oriatur, ſi cautius procedere velimus, etiam angulos 20 gradibus minores euitare poterimus, quo pacto apertura magis reſtringetur. Tab. I. Fig. 2.

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Problema 8.

Tab. I. Fig. 3. 60. Non neglecta lentis crassitie AB^8 si pro distantia obiecti $AE=a$ detur distant a imaginis principalis $BF=\alpha$, obiectum autem parumper longius in e remoueatur, definire locum imaginis principalis f.

Solutio.

Posita lentis crassitie $AB=d$, radiisque faciei anterioris $AM=f$ et posterioris $BN=g$, supra vidimus hos radios ita a binis distantiis a et α atque crassitie lentis d pendere debere, vt sit

$$\frac{n-1}{f}=\frac{1}{a}+\frac{2n}{k+d} \text{ et } \frac{n-1}{g}=\frac{1}{\alpha}-\frac{2n}{k-d}$$

denotante k quantitatem quamcunque. Hinc ergo cum sit

$$\frac{k+d}{2n}=\frac{af}{(n-1)a-f} \text{ et } \frac{k-d}{2n}=\frac{\alpha g}{g-(n-1)\alpha}$$

erit eliminando k

$$\frac{d}{n}=\frac{af}{(n-1)a-f}-\frac{\alpha g}{g-(n-1)\alpha}.$$

Ponamus iam distantiam $AE=a$ crescere particula $Ee=da$ ac per differentiationem inueniemus, quantum inde distantia imaginis $BF=\alpha$ immutetur: habebimus scilicet:

$$\frac{-ffda}{((n-1)a-f)^2}-\frac{ggd\alpha}{(g-(n-1)\alpha)^2}=0$$

vnde elicimus

$$d\alpha=\frac{-ff(g-(n-1)\alpha)^2}{gg((n-1)a-f)^2}da=\frac{-\alpha\alpha}{aa}da\left(\frac{k+d}{k-d}\right)^2.$$

Quare

Quare obiecto E per ſpatiolum minimum Ee longius a lente remoto, imago principalis ex F propius ad lentem admouebitur per ſpatiolum minimum Ff, ita vt ſit

$$Ff = \frac{\alpha\alpha'}{aa}\left(\frac{k+d}{k-d}\right)^2 . Ee = \frac{ff(g-(n-1)\alpha)^2}{gg((n-1)a-f)^2} . Ee.$$

Coroll. 1.

61. Quia quantitas $\frac{\alpha\alpha}{aa}\left(\frac{k+d}{k-d}\right)^2$ neceſſario eſt poſitiua, euidens eſt ſi obiectum longius a lente remoueatur, imaginem ſemper propius ad lentem admoueri. Contra ergo etiam ſi obiectum propius ad lentem accedat, imago longius ab ea recedet.

Coroll. 2.

62. Si craſſities lentis d euaneſcat erit $Ff = \frac{\alpha\alpha}{aa} . Ee$, ſin autem ea non euaneſcat fieri poteſt vt ſit vel $Ff > \frac{\alpha\alpha}{aa} Ee$ vel $Ff < \frac{\alpha\alpha}{aa} . Ee$, prius eueniet ſi k ſit quantitas poſitiua, poſterius ſi negatiua. At ſi ſit vel $k = \infty$ vel $k = 0$, vtroque caſu erit $Ff = \frac{\alpha\alpha}{aa} . Ee$ etiamſi craſſities lentis non euaneſcat.

Coroll. 3.

63. Cum in diſtantiis a et α mutatio minima fieri concipiatur, ſpatium diffuſionis nullam inde variationem ſubire cenſendum eſt; ſiue ergo obiectum in E ſiue e reperiatur, ac ſemidiameter aperturae

lentis in facie anteriore fuerit $= x$, erit spatium diffusionis

$$\frac{n\alpha\alpha xx}{2(n-1)}\left\{+\left(\frac{k+d}{k-d}\right)^2\left(\frac{n}{a}+\frac{2}{k+d}\right)\left(\frac{1}{a}+\frac{2}{k+d}\right)^2\right\}$$
$$\left\{+\left(\frac{k-d}{k+d}\right)^2\left(\frac{n}{a}-\frac{2}{k-d}\right)\left(\frac{1}{a}-\frac{2}{k-d}\right)^2\right\}$$

in facie autem posteriori semidiameter aperturae debet esse $= \frac{k-d}{k+d}\, x$.

Problema 9.

Tab. I. Fig. 3.

64. Rationem definire, quam habet magnitudo imaginis ad magnitudinem obiecti non neglecta lentis crassitie.

Solutio.

Sit obiecti ante lentem distantia $AE = a$, imaginis vero $BF = \alpha$ existente lentis crassitie $= d$; vbi quidem tantum imaginem principalem spectamus neglecto spatio diffusionis. Sit iam $E\varepsilon$ obiectum, cui tribuatur magnitudo quam minima $E\varepsilon = x$, normaliter axi lentis insistens; eiusque imago in $F\zeta$ exhibebitur, cuius magnitudo $F\zeta$ quaeritur. Cum igitur punctum ζ a puncto ε oriatur, ita vt radii ab ε emissi in ζ colligantur, consideretur radius quicunque εM, qui productus cum axe in e occurrat: ea perinde est, ac si hic radius ex axis puncto e emanaret. Quare is post refractionem cum axe in f concurret, vt sit $Ef = \frac{aa}{\alpha\alpha}\left(\frac{k+d}{k-d}\right)^2 . E\varepsilon$, indeque ad ζ perget: ex quo magnitudinem $F\zeta$ definire licebit. Statuatur

in

in hunc finem $AM = x$, erit $BN = \frac{k-d}{k+d} x$, hincque colligemus has proportiones:

$$Ee : E\varepsilon = eA : AM = a : x$$

$$Ff : F\zeta = fB : BN = \alpha : \tfrac{k-d}{k+d} x$$

ob spatiola Ee et Ff quam minima: vnde habebimus

$$\tfrac{Ff}{Ee} : \tfrac{F\zeta}{E\varepsilon} = \tfrac{\alpha}{a} : \tfrac{k-d}{k+d} = \tfrac{\alpha a}{aa} \left(\tfrac{k+d}{k-d}\right)^2 : \tfrac{F\zeta}{E\varepsilon}$$

Concluditur ergo $\frac{F\zeta}{E\varepsilon} = \frac{\alpha}{a} \cdot \frac{k+d}{k-d}$, ex quo cum magnitudo obiecti $E\varepsilon$ posita sit $= z$, erit magnitudo imaginis $F\zeta = \frac{\alpha(k+d)}{a(k-d)} z$.

Coroll. 1.

65. Secundum hanc ergo rationem diameter obiecti immutatur. Vbi notandum est, si expressio $\frac{\alpha(k+d)}{a(k-d)}$ habeat positiuum valorem obiecti imaginem situ inuerso repraesentari, contra autem situ erecto, si $\frac{\alpha(k+d)}{a(k-d)}$ negatiuum valorem adipiscatur.

Coroll. 2.

66. Si crassities lentis euanescat, fit $F\zeta = \frac{\alpha}{a} z$: quo ergo casu recta iungens puncta extrema ε et ζ transit per centrum lentis. At si crassities in computum ducatur, recta $\varepsilon\zeta$ modo intra lentem modo extra, axem lentis secare poterit.

Scholion.

67. Sic igitur omnia, quae circa vnam lentem quamcunque nosse oportet, expediuimus, vt etiam

craſſitiei rationem habuerimus. Ac primo quidem ad binas diſtantias lentis quaſi determinatrices ſpectari conuenit, quae ſunt obiecti diſtantia ante lentem $AE=a$ et imaginis principalis poſt lentem diſtantia $BF=\alpha$, quibus addi debet lentis craſſities $AB=d$. His autem conditionibus infinitae lentes ſatisfaciunt; ſi enim radius faciei anterioris AM dicatur $=f$, et poſterioris $BN=g$, vtraque tanquam conuexa conſiderata, conſtructio lentis his continetur formulis:

$$\frac{n-1}{f}=\frac{1}{a}+\frac{2n}{k+d} \text{ et } \frac{n-1}{g}=\frac{1}{\alpha}-\frac{2n}{k-d}$$

ſeu

$$f=\frac{(n-1)a(k+d)}{k+d+2na}, \text{ et } g=\frac{(n-1)\alpha(k-d)}{k-d+2n\alpha}$$

exiſtente $n=\frac{31}{20}$; vbi k eſt quantitas arbitraria, hincque lentes innumerabiles quaeſito ſatisfacientes obtinentur.

Quod ſi iam obiecti diameter ponatur $=z$, erit imaginis principalis repraeſentatae diameter $=\frac{\alpha(k+d)}{a(k-d)}z$, quatenus imago ſitu inuerſo exhibita conſideratur.

Deinde ſi aperturae in facie anterioris lentis ſemidiameter ſit $=x$, ſpatium diffuſionis, quatenus ab imagine principali ad lentem porrigitur, ita exprimetur vt ſit:

$$\frac{n\alpha\alpha xx}{2(n-1)^2}\cdot\left\{\begin{array}{l}+\left(\frac{k+d}{k-d}\right)^2\left(\frac{n}{a}+\frac{2}{k+d}\right)\left(\frac{1}{a}+\frac{2}{k+d}\right)^2\\+\left(\frac{k-d}{k+d}\right)^2\left(\frac{n}{\alpha}-\frac{2}{k-d}\right)\left(\frac{1}{\alpha}-\frac{2}{k-d}\right)^2\end{array}\right\}$$

in facie autem poſteriori aperturae ſemidiameter debet eſſe $=\frac{k-d}{k+d}\cdot x$ vel ſaltem non minor. Quia ſpatium diffu-

diffusionis factorem habet $\alpha\alpha xx$, breuitatis gratia id ita $P\alpha\alpha xx$ indicabimus. Haecque in genere teneantur etiam crassitiei lentis ratione habita. At si crassities lentis euanescat, formulas magis euoluere licuit, scilicet si breuitatis gratia ponatur:

$$\mu = 0,938191 \; ; \; \varrho = 0,190781 \; ; \; \tau = 0,905133$$
$$\nu = 0,232692 \; ; \; \sigma = 1,627401 \; ; \; \lambda > 1 \text{ arbitr:}$$

sumaturque pro formatione lentis:

$$f = \frac{\alpha\alpha}{\varrho a + \sigma a \pm \tau (a+a)\sqrt{(\lambda - 1)}}$$
$$g = \frac{\alpha\alpha}{\varrho a + \sigma a \mp \tau (a+a)\sqrt{(\lambda - 1)}}$$

spatium diffusionis erit pro aperturae semidiametro x

$$\mu\alpha\alpha xx \left(\tfrac{1}{a} + \tfrac{1}{\alpha}\right)\left(\lambda\left(\tfrac{1}{a} + \tfrac{1}{\alpha}\right)^2 + \tfrac{\nu}{a\alpha}\right)$$

et obiecti diametro existente $= z$ imaginis diameter erit $= \frac{\alpha}{a} z$. His ergo pro vna lente determinatis, videamus quomodo in combinatione duarum pluriumve lentium spatium diffusionis definiatur: vt deinceps in omnis generis instrumentis dioptricis siue Telescopiis siue microscopiis confusionem assignare, modumque eam diminuendi inuestigare possimus.

CAPVT II.

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CAPVT II.

DE DIFFVSIONE IMAGINIS PER PLVRES LENTES REPRAESENTATAE.

Problema 1.

68.

Tab. I. Fig. 4. Si loco obiecti adsit imago per spatium Ee diffusa, indeque radii per lentem PP aperturae indefinitae transmittantur, determinare spatium diffusionis Ff per hanc lentem productum.

Solutio.

Loco obiecti veri hic consideramus imaginem iam per aliam lentem repraesentatam, quae sit diffusa per spatium Ee, ita vt in E sit imago principalis per radios axi proximos formata in e autem imago extrema, radiis scilicet per marginem aperturae lentis praecedentis transmissis formata; qui radii cum axe constituant angulum $= \Phi$. Quare a puncto E nonnisi radii axi proximi in lentem PP emittuntur, a puncto e autem eiusmodi tantum radii, qui ad axem eA sub angulo $MeA = \Phi$ sint inclinati. Ponatur iam

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iam distantia $EA = a$, prae qua spatium diffusionis Ee vt valde paruum spectetur, lens autem PP sit eiusmodi vt obiecti in E existentis imaginem principalem referat in distantia $BF = \alpha$, existente lentis crassitie $AB = d$. Hanc ob rem si ponatur faciei anterioris AM radius $= f$, posterioris $BN = g$, lente vt conuexa vtrinque spectata oportet esse:

$$f = \frac{(n-1)a(k+d)}{k+d+2na} \text{ et } g = \frac{(n-1)a(k-d)}{k-d-2na}$$

vbi est $n = \frac{31}{20}$ et k quantitas arbitraria. Hinc autem vti demonstrauimus, pro aperturae semidiametro $= x$ nasceretur spatium diffusionis

$$\frac{\pi a a x x}{2(n-1)^2} \cdot \left\{ \begin{array}{l} +\left(\frac{k+d}{k-d}\right)^2 \left(\frac{n}{a} + \frac{2}{k+d}\right)\left(\frac{1}{a} + \frac{2}{k+d}\right)^2 \\ +\left(\frac{k-d}{k+d}\right)^2 \left(\frac{n}{a} - \frac{2}{k-d}\right)\left(\frac{1}{a} - \frac{3}{k-d}\right)^2 \end{array} \right\}$$

pro quo scribamus breuitatis gratia $P a a x x$. Iam puncti E, quia inde nonnisi radii axi proximi ad lentem emittuntur imago exhibebitur in F, vt sit distantia $BF = \alpha$: videamus ergo quorsum imago puncti e debeat cadere. Ac si radii ex puncto e emissi essent axi proximi, quia id a lente magis est remotum quam E, eius imago propius ad lentem caderet puta in Φ, vt esset, sicut §. 60 definiuimus

$$F\Phi = \frac{\alpha a}{a a}\left(\frac{k+d}{k-d}\right)^2 . Ee.$$

in Φ scilicet caderet imago principalis, si obiectum esset in e. Sed quia ab e tantum radii eM ad axem angulo $AeM = \Phi$ inclinati emittuntur, hi lenti in

punctis M ab A interuallo $AO = eA.\Phi = a\Phi$ remotis occurrent, quandoquidem interuallum Ee praedistantia $Ae = a$ contemnimus, perinde igitur est ac si lenti apertura tribueretur, cuius semidiameter esset $= a\Phi$, et obiecti in e existentis imago extrema definiri deberet, quae cadat in f ita vt Φf sit spatium diffusionis obiecto in e existenti, et aperturae lentis, cuius semidiameter $= a\Phi$, conueniens. Hinc ergo erit interuallum $\Phi f = Paaaa\Phi\Phi$; et quia puncti E imago in F, puncti e autem imago in f exhibetur, erit spatium diffusionis per lentem PP productum

$$Ff = \frac{aa}{aa}\left(\frac{k+d}{k-d}\right)^2 . Ee + Paaaa\Phi\Phi.$$

In imagine autem extrema f radii Nf ita cum axe concurrent, vt sit

$$\text{angulus } BfN = \frac{k-d}{k+d} \cdot \frac{a\Phi}{a}.$$

Coroll. 1.

69. Si ergo imago iam diffusa per spatium Ee locum obiecti teneat respectu lentis PP, per hanc spatium diffusionis nouum producitur Ff, ita vt imago principalis cadat in F extrema vero in f: fierique poterit, vt hoc nouum spatium diffusionis Ff maius sit vel minus proposito Ee.

Coroll. 2.

70. Aperturam lentis PP vt indefinitam assumsi, manifestum autem est sufficere, dum eius semidiameter non sit minor quam $a\Phi$. Si enim esset minor

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minor, radii ex puncto z emissi plane non per lentem transitum inuenirent, neque imago puncti z exprimeretur, sed imaginis Ff punctum extremum responderet imagini cuipiam intermediae spatii Ee.

Coroll. 3.

71. Si diameter obiecti seu potius imaginis in E sitae sit $= z$, tum imaginis inde per lentem PP in F formatae diameter erit $= \frac{\alpha(k+d)}{\alpha(k-d)} z$ quae expressio si sit positiua simul declarat situm imaginis in F esse inuersum respectu eius, quae est in E.

Scholion.

72. Cum in hoc capite plures lentes sim consideraturus, pro cuiuslibet determinatione, spectandae sunt primo binae distantiae determinatrices, altera obiecti seu imaginis, a qua lens radios accipit ante lentem, altera imaginis inde per lentem repraesentatae post lentem: quae distantiae ex imaginibus principalibus aestimentur. Deinde cuiusque lentis crassities in computum est ducenda. Tertio cum his lens nondum prorsus determinetur, insuper pro quaque lente accedit distantia quaedam arbitraria hactenus per litteram k indicata. Quarto vero imprimis ratio haberi deberet, spatii diffusionis, quod cuique lenti pro data apertura conueniat. Quemadmodum autem ex binis distantiis determinatricibus, crassitie lentis et quantitate illa arbitraria k, cum binae lentis facies tum

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etiam ſpatium diffuſionis pro apertura, cuius ſemidiameter ponitur $= x$ definiatur, in praecedente capite fuſius eſt expoſitum. Hic igitur cum plures lentes ſint conſiderandae, pro ſingulis haec elementa ſequentibus litteris indicabo.

Pro Lente	Diſtantiae determinatrices obiecti	imaginis	Craſſities lentis	Quantitas arbitraria	Spatium diffuſionis pro aperturae ſemidiametro x
Prima	a	α	v	k	$P\alpha\alpha xx$
Secunda	b	$\mathfrak{G}$	v'	k'	$Q\mathfrak{G}\mathfrak{G}xx$
Tertia	c	γ	v''	k''	$R\gamma\gamma xx$
Quarta	d	δ	v'''	k'''	$S\delta\delta xx$
Quinta	e	ε	v''''	k''''	$T\varepsilon\varepsilon xx$

Sin autem ratio refractionis in ſingulis lentibus diſcrepet, pro prima lente eam ponamus $= n$; pro ſecunda $= n'$; pro tertia $= n''$ etc.

Prima autem lens hic mihi perpetuo erit ea, quae obiecto eſt proxima indeque recedendo reliquas lentes ordine numero: ex quo ſimul habebuntur diſtantiae lentium, ſcilicet ſecundae a prima $= \alpha + b$, tertiae a ſecunda $= \mathfrak{G} + c$; quartae a tertia $= \gamma + d$; quintae a quarta $= \delta + e$ etc. quae diſtantiae ex ſua natura debent eſſe poſitiuae etiamſi ſingulae diſtantiae determinatrices praeter primam a, quippe quae ad ipſum obiectum neceſſario ante lentem primam conſtituendum refertur, ſint quandoque negatiuae. Craſſitiem lentium

hic

hic littera v designo, quia littera d inter distantias determinatrices, si numerus lentium vltra ternarium assurgat, reperitur. Pro crassitie autem et quantitate arbitraria iisdem litteris vtor, commatibus inscriptis distinguendis ob penuriam litterarum diuersarum. Ceterum obseruandum est me omnes lentes tanquam super communi axe dispositas assumere.

Problema 2.

73. Si radii ab obiecto $E\varepsilon$ emissi per duas lentes PP et QQ transmittantur definire spatium diffusionis Gg, a data apertura primae lentis oriundum; vt et magnitudinem imaginis principalis in G exhibitae. Tab. I. Fig. 5.

Solutio.

Sit obiecti magnitudo $E\varepsilon = x$, eiusque imago principalis per primam lentem PP proiiciatur in $F\zeta$, at per ambas in $G\eta$ erunt distantiae determinatrices pro lente prima PP, obiecti $EA = a$, imaginis $aF = \alpha$ pro lente secunda QQ, obiecti $FB = b$, imaginis $bG = \beta$ Deinde sit

pro lente PP, crassities $Aa = v$, quantitas arbitr: $= k$
pro lente QQ, crassities $Bb = v'$, quantitas arbitr: $= k'$
Denique pro apertura in anteriori facie vtriusque lentis cuius semidiameter sit $= x$

spatium diffusionis primae lentis $PP = P\alpha\alpha xx$

- - - - secundae lentis $QQ = Q\beta\beta xx$.

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His positis pro imagine per primam lentem proiecta $F\zeta$ erit eius magnitudo $E\zeta = \frac{\alpha(k+v)}{a(k-v)}z$ pro inuersa habenda, si haec expressio fuerit positiua. Deinde si lentis primae PP semidiameter aperturae in anteriori facie ponatur $= x$, erit spatium diffusionis $Ff = P\alpha\alpha xx$ et inclinatio radiorum in f ad axem $= \frac{k-v}{k+v} \cdot \frac{x}{\alpha}$. tum vero in facie lentis PP posteriori semidiameter aperturae non minor esse debet quam $\frac{k-v}{k+v}x$. Tota iam haec imago diffusa per spatium Ff respectu alterius lentis QQ tanquam obiectum spectari debet, cuius proinde repraesentatio Gg per propositionem praecedentem determinabitur. Erit autem hic $\Phi = \frac{k-v}{k+v} \cdot \frac{x}{\alpha}$, et spatium ibi expressum $Ee = P\alpha\alpha xx$; tum vero pro a, α, k, d et P hic scribi oportet $b, \mathfrak{b}, k', v'$ et Q, vnde fiet spatium diffusionis quaesitum:

$$Gg = \frac{\mathfrak{b}\mathfrak{b}}{bb}\left(\frac{k'+v'}{k'-v'}\right)^2 P\alpha\alpha xx + \frac{bb\mathfrak{b}\mathfrak{b}}{a\alpha}\left(\frac{k-v}{k+v}\right)^2 Qxx$$

siue

$$Gg = \frac{\mathfrak{b}\mathfrak{b}}{bb}\left(\frac{k'+v'}{k'-v'}\right)^2 P\alpha\alpha xx + \frac{bb}{a\alpha}\left(\frac{k-v}{k+v}\right)^2 Q\mathfrak{b}\mathfrak{b}xx$$

Radiorum porro in g incidentium inclinatio ad axem est $\left(\frac{k-v}{k+v}\right)\left(\frac{k'-v'}{k'+v'}\right)\frac{bx}{a\mathfrak{b}}$. Denique cum sit $F\zeta = \frac{\alpha(k+v)}{a(k-v)}z$ erit magnitudo imaginis in G vt erutae consideratae:

$$G\eta = \frac{\alpha\mathfrak{b}}{ao}\left(\frac{k+v}{k-v}\right)\left(\frac{k'+v'}{k'-v'}\right)z.$$

Coroll. 1.

74. Quod ad aperturam facierum lentis QQ attinet, ea maior esse debet spatio transitus radiorum; hinc ergo erit semidiameter aperturae

faciei

faciei anterioris $> \left(\frac{k-v}{k+v}\right)\frac{bx}{a}$

faciei posterioris $> \left(\frac{k-v}{k+v}\right)\left(\frac{k'-v'}{k'+v'}\right)\frac{bx}{a}$.

Coroll. 2.

75. Si ponatur pro lente prima PP radius faciei anterioris $=f$, et posterioris $=g$ erit posito $n=\frac{11}{10}$

$$f=\frac{(n-1)a(k+v)}{k+v+2na} \text{ et } g=\frac{(n-1)a(k-v)}{k-v-2na}.$$

Similique modo si pro lente altera QQ radius faciei anterioris ponatur $=f'$, et posterioris $=g'$ erit

$$f'=\frac{(n-1)b(k'+v')}{k'+v'+2nb} \text{ et } g'=\frac{(n-1)b(k'-v')}{k'-v'-2nb}$$

omnibus scilicet faciebus vt conuexis consideratis.

Coroll. 3.

76. Pro spatio autem diffusionis ab vtraque lente productae erit quemadmodum inuenimus:

$$P=\frac{n}{2(n-1)^2}\cdot\left\{\begin{matrix}+\left(\frac{k+v}{k-v}\right)^2\left(\frac{n}{a}+\frac{2}{k+v}\right)\left(\frac{1}{a}+\frac{2}{k+v}\right)^2\\+\left(\frac{k-v}{k+v}\right)^2\left(\frac{n}{a}-\frac{2}{k-v}\right)\left(\frac{1}{a}-\frac{2}{k-v}\right)^2\end{matrix}\right\}$$

similique modo

$$Q=\frac{n}{2(n-1)^2}\cdot\left\{\begin{matrix}+\left(\frac{k'+v'}{k'-v'}\right)^2\left(\frac{n}{b}+\frac{2}{k'+v'}\right)\left(\frac{1}{b}+\frac{2}{k'+v'}\right)^2\\+\left(\frac{k'-v'}{k'+v'}\right)^2\left(\frac{n}{b}-\frac{2}{k'-v'}\right)\left(\frac{1}{b}-\frac{2}{k'-v'}\right)^2\end{matrix}\right\}$$

Non opus est vt omnibus lentibus eadem refractionis ratio $n::1$ tribuatur, sed simili modo pro pluribus lentibus poni potest n, n', n'', n''', etc. vt iam supra monuimus vnde nullum aliud discrimen nascitur, nisi

nisi quod in formulis pro f' et g' inuentis loco n scribi debeat n', et Q statui debeat.

$$= \frac{n'}{2(n'-1)^2} \begin{Bmatrix} +(\frac{k'+v'}{k'-v'})^2(\frac{n'}{b}+\frac{2}{k'+v'})(\frac{1}{b}+\frac{2}{k'+v'})^2 \\ +(\frac{k'-v'}{k'+v'})^2(\frac{n'}{\mathfrak{e}}-\frac{2}{k'-v'})(\frac{1}{\mathfrak{e}}-\frac{2}{k'-v'})^2 \end{Bmatrix}$$

Coroll. 4.

77. Sin autem crassities lentium euanescat, et pro quantitate arbitraria k, k' introducamus numerum λ, λ', erit

pro lente PP

$$f = \frac{a\,a}{\varrho a + \sigma a \pm \tau(a+a)\sqrt{(\lambda-1)}}; \quad g = \frac{a\,a}{\varrho a + \sigma a \mp \tau(a+a)\sqrt{(\lambda-1)}}$$

et

$$P = \mu(\tfrac{1}{a}+\tfrac{1}{a})(\lambda(\tfrac{1}{a}+\tfrac{1}{a})^2+\tfrac{\nu}{a\,a})$$

at pro lente QQ

$$f' = \frac{b\mathfrak{e}}{\varrho\mathfrak{e} + \sigma b \pm \tau(b+\mathfrak{e})\sqrt{(\lambda'-1)}}; \quad g' = \frac{b\mathfrak{e}}{\varrho b + \sigma\mathfrak{e} \mp \tau(b+\mathfrak{e})\sqrt{(\lambda'-1)}}$$

$$Q = \mu(\tfrac{1}{b}+\tfrac{1}{\mathfrak{e}})(\lambda'(\tfrac{1}{b}+\tfrac{1}{\mathfrak{e}})^2+\tfrac{\nu}{b\mathfrak{e}}).$$

In expressionibus autem inuentis formulae $(\frac{k-v}{k+v})$ et $(\frac{k'-v'}{k'+v'})$ abeunt in vnitatem.

Numerorum vero $\mu, \nu, \varrho, \sigma, \tau$ indoles in §. 55 exposita est.

Si refractio lentium differat etiam litterae $\mu, \nu, \varrho, \sigma, \tau$ diuersos valores pro singulis lentibus fortientur, quae litte-

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litterae si etiam commatibus a prioribus distinguantur, vt sit

$$\xi' = \frac{1}{2(n'-1)} + \frac{1}{n'+2} - 1$$

$$\sigma' = 1 + \frac{1}{2(n'-1)} - \frac{1}{n'+2}$$

$$\tau' = \tfrac{1}{3}\left(\frac{1}{2(n'-1)} + \frac{1}{n'+2}\right)\sqrt{(4n'-1)}$$

pro secunda lente erit

$$f' = \frac{bc}{\beta' c + \sigma' b \pm \tau(b+c)\sqrt{(\lambda'-1)}}$$

$$g' = \frac{bc}{\beta' b + \sigma' c \mp \tau(b+c)\sqrt{(\lambda'-1)}}$$

et ob eandem rationem erit

$$\mu' = \frac{1}{4(n'+2)} + \frac{1}{4(n'-1)} + \frac{1}{8(n'+1)^2}$$

et $\nu' = \frac{4(n'-1)^2}{4n'-1}$

quod et de sequentibus lentibus est intelligendum, si forte diuersa refractionis lege fuerint praeditae.

Scholion

78. Quo formulas in problemate inuentas magis contrahamus, vt cum ad plures lentes processerimus, succinctiores euadant, ponamus

$$\frac{k-v}{k+v} = i \text{ et } \frac{k'-v'}{k'+v'} = i'$$

ita vt hi numeri i et i' abeant in vnitatem euanescente lentium crassitie v et v'. Tum autem erit spatium diffusionis

$$Gg = \frac{1}{i'i'} \cdot \frac{cc}{bb} \cdot P\alpha axx + ii \cdot \frac{bb}{aa} \cdot Q\gamma\gamma xx$$

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et radiorum in g incidentium inclinatio ad axem $= ii' \cdot \frac{bx}{ae}$: porro magnitudo imaginis $G\eta = \frac{ef}{gh}z$. Ac pro apertura singularum facierum erit vt sequitur

Semidiam. aperturae lentis	faciei anterioris	faciei posterioris
primae PP	x	ix
secundae QQ	$i\frac{bx}{a}$	$ii'\frac{bx}{\alpha}$

aperturae autem hae praeter primam maiores esse debent assignatis: valores enim assignati sufficerent, si obiectum esset merum punctum in axe positum; quando autem habet magnitudinem, radii ab eius terminis per primam faciem ingressi latius diuagantur, et ad sequentes facies maiorem aperturam exigunt.

Problema 3.

79. Si radii ab obiecto $E\varepsilon$ emissi per tres lentes PP, QQ et RR refringantur definire spatium diffusionis $H\beta$ ob datam aperturam lentis primae oriundum, vt et magnitudinem imaginis principalis in H exhibitae. Tab. II. Fig. 6.

Solutio.

Posita obiecti magnitudine $E\varepsilon = z$, cadat eius imago principalis per lentem primam PP in $F\zeta$, dehinc per secundam QQ in $G\eta$, tum vero per tertiam RR in $H\theta$. Erunt ergo distantiae determinatrices

pro lente PP, obiecti $EA = a$, imaginis $aF = \alpha$
pro lente QQ, obiecti $FB = b$, imaginis $bG = \beta$
pro lente RR, obiecti $GC = c$; imaginis $cH = \gamma$

Deinde

Deinde fit

pro lente PP, crassities $A\alpha = v$, quantitas arbitr: $= k$
pro lente QQ, crassities $B\beta = v'$, quantitas arbitr: $= k'$
pro lente RR crassities $C\gamma = v''$, quantitas arbitr: $= k''$
ac ponatur brevitatis gratia:

$$\frac{k-v}{k+v} = i; \quad \frac{k'-v'}{k'+v'} = i'; \quad \frac{k''-v''}{k''+v''} = i''$$

Denique pro quauis lente, si esset singularis, eiusque aperturae semidiameter esset $= x$, sit

spatium diffusionis primae lentis $PP = P\alpha\alpha xx$
- - - - secundae lentis $QQ = Q\beta\beta xx$
- - - - tertiae lentis $RR = R\gamma\gamma xx$

Iam in praecedente problemate inuenimus fore spatium diffusionis per duas lentes priores productum

$$Gg = \frac{1}{(1+ii)^2}\cdot\frac{ee}{\alpha\alpha}P\alpha\alpha xx + ii'\cdot\frac{bb}{\beta\beta}Q\beta\beta xx$$

et radiorum in g concurrentium inclinationem ad axem $= ii'\cdot\frac{bx}{ae}$ imaginisque in G magnitudinem $G\eta = \frac{b}{\beta}\cdot\frac{ee}{dd}\alpha$.

Haec igitur si ad problema 1. transferantur, ibi loco

Ee, $\frac{\alpha\alpha}{aa}$, $\left(\frac{k+d}{k-d}\right)^2$, $P\alpha\alpha\alpha\alpha$ et Φ scribi debebit

Gg; $\frac{\gamma\gamma}{cc}$; $\frac{1}{(1+ii)^2}$, $R\gamma\gamma\gamma\gamma$ et $ii'\cdot\frac{bx}{\beta e}$,

hincque spatium diffusionis per tres lentes productum orietur.

$$Hh = \frac{1}{(1+ii)^2(1+i'i')^2}\cdot\frac{ee\gamma\gamma}{bb\,cc}P\alpha\alpha xx + \frac{ii}{(1+i'i')^2}\cdot\frac{bb\gamma\gamma}{\alpha\alpha cc}Q\beta\beta xx + ii.\,i'i'.\,\frac{bb\,cc}{\alpha\alpha ee}R\gamma\gamma xx$$

at radiorum in b concurrentium inclinatio ad axem

erit $= ii'i''.\frac{bcx}{\alpha\beta\gamma}$. Denique imaginis in H formatae magnitudo erit $H\theta = \frac{1}{ii'i''}.\frac{\alpha\beta\gamma}{abc}x$ ad situm inuersum relata.

Coroll. 1.

80. Quod ad aperturas singularum facierum attinet, eas sequenti modo comparatas esse conuenit.

Semid. aperturae Lentis	faciei anterioris	faciei posterioris
Primae P P	x	$i\,x$
Secundae Q Q	$i.\frac{bx}{a}$	$ii'.\frac{bx}{a}$
Tertiae R R	$ii'.\frac{bcx}{a\beta}$	$ii'i''.\frac{bcx}{a\beta\gamma}$

his scilicet valoribus non debent esse minores.

Coroll. 2.

81. Si inclinatio radiorum in b concurrentium ad axem vocetur $= \Phi$. Cum sit $\Phi = ii'i''.\frac{bcx}{\alpha\beta\gamma}$ et $H\theta = \frac{1}{ii'i''}.\frac{\alpha\beta\gamma}{abc}x$ erit $\Phi . H\theta = \frac{xx}{a}$: quae proprietas pro lentium numero quantumuis magno locum habet.

Coroll. 3.

82. Quemadmodum radii singularum facierum determinandi sint, ex praecedentibus facile liquet:

Erit nempe pro	Radius faciei anterioris	Radius faciei posterioris
Lente prima P P	$\frac{(n-1)\alpha(k+v)}{k+v+2n\alpha}$	$\frac{(n-1)\alpha(k-v)}{k-v-2n\alpha}$
Lente secunda Q Q	$\frac{(n-1)\beta(k'+v')}{k'+v'+2n\beta}$	$\frac{(n-1)\beta(k'-v')}{k'-v'-2n\beta}$
Lente tertia R R	$\frac{(n-1)\gamma(k''+v'')}{k''+v''+2n\gamma}$	$\frac{(n-1)\gamma(k''-v'')}{k''-v''-2n\gamma}$

existente pro vitro $n = \frac{31}{20}$.

Coroll.

Coroll. 4.

83. Tum vero valores literarum P, Q, R ita se habebunt

$$P = \frac{\pi}{2(n-1)^2}\left(\frac{1}{ii}\left(\frac{\pi}{a}+\frac{\varepsilon}{k+v}\right)\left(\frac{1}{a}+\frac{1}{k+v}\right)^2+\frac{1}{i'i'}\left(\frac{\pi}{a}-\frac{\varepsilon}{k-v}\right)\left(\frac{1}{a}-\frac{1}{k-v}\right)^2\right)$$

$$Q = \frac{\pi}{2(n-1)^2}\left(\frac{1}{i'i'}\left(\frac{\pi}{b}+\frac{\varepsilon}{k'+v'}\right)\left(\frac{1}{b}+\frac{1}{k'+v'}\right)^2+\frac{1}{i''i''}\left(\frac{\pi}{b}-\frac{\varepsilon}{k'-v'}\right)\left(\frac{1}{b}-\frac{1}{k'-v'}\right)^2\right)$$

$$R = \frac{\pi}{2(n-1)^2}\left(\frac{1}{i''i''}\left(\frac{\pi}{c}+\frac{\varepsilon}{k''+v''}\right)\left(\frac{1}{c}+\frac{1}{k''+v''}\right)^2+\frac{1}{i'''i'''}\left(\frac{\pi}{c}-\frac{\varepsilon}{k''-v''}\right)\left(\frac{1}{c}-\frac{1}{k''-v''}\right)^2\right)$$

quibus valoribus spatia diffusionis definiuntur.

Si refractio differat:

litteris	tribuatur refr.
P	π
Q	π^b
R	π^{bb}

Coroll. 5.

84. Iuuabit etiam spatia diffusionis, prout per vnam, duas ac tres lentes producuntur, inter se comparasse, quod ita commodissime fieri videtur.

$$Ff = aaxx.P$$

$$Gg = \mathfrak{G}\mathfrak{G}xx\left(\frac{1}{\mathfrak{v}\mathfrak{v}}.\frac{aa}{bb}P+ii.\frac{bb}{aa}Q\right)$$

$$Hh = \mathfrak{y}\mathfrak{y}xx\left(\frac{1}{\mathfrak{w}\mathfrak{w}}.\frac{aa\mathfrak{e}\mathfrak{e}}{bbcc}P+\frac{ii}{\mathfrak{v}\mathfrak{v}}.\frac{bb\mathfrak{e}\mathfrak{e}}{aacc}Q+ii.i'i'.\frac{bbcc}{aa\mathfrak{e}\mathfrak{e}}R\right)$$

Schol. 1.

85. Hinc facile perspicitur, quomodo determinatio spatii diffusionis, ad plures lentes extendi debeat;

vnde problema generale statim ad numerum lentium quemcunque accommodabo, idque pro quacunque lentium crassitie.

Problema 4.

86. Si radii ab obiecto $E\varepsilon$ emissi per lentes quotcunque PP, QQ, RR, SS etc. super communi axe dispositas refringantur, definire spatium diffusionis a data apertura lentis primae oriundum et magnitudinem imaginis repraesentatae.

Solutio.

Posita obiecti magnitudine $E\varepsilon = z$ imagines eius principales contemplemur: cadat igitur eius imago principalis per lentem primam PP in $F\zeta$ deinde per secundam QQ in $G\eta$, tum per tertiam RR in $H\vartheta$, porro per quartam SS in $I\iota$, per quintam TT in $K\varkappa$ etc. Hinc pro singulis lentibus habebimus distantias determinatrices quas ita litteris indicemus.

crassitiem

Pro lente PP obiecti $EA = a$, imaginis $\alpha F = \alpha$; $A\alpha = v$

Pro lente QQ obiecti $FB = b$; imaginis $\beta G = \mathfrak{b}$; $B\beta = v'$

Pro lente RR obiecti $GC = c$; imaginis $\gamma H = \gamma$; $C\gamma = v''$

Pro lente SS obiecti $HD = d$; imaginis $\delta I = \delta$; $D\delta = v'''$

Pro lente TT obiecti $IE = e$; imaginis $\varepsilon K = \mathfrak{e}$; $E\varepsilon = v''''$

etc.

Deinde

Deinde cum determinatio cuiusuis lentis non ſolum has diſtantias determinatrices cum craſſitie cuiusque ſed etiam quantitatem quandam arbitrariam inuoluat, a qua ſpatium diffuſionis cuiusque pendet; poniamus ſi quaelibet lens eſſet ſolitaria eiusque aperturae ſemidiameter $= x$.

Pro lente	quant: arbitr:	Spatium Diffuſionis
Prima PP	k	$P\alpha\alpha xx$
Secunda QQ	k'	$Q\beta\beta xx$
Tertia RR	k''	$R\gamma\gamma xx$
Quarta SS	k'''	$S\delta\delta xx$
Quinta TT	k''''	$T\varepsilon\varepsilon xx$
	etc.	

Hinc ergo conſtructio cuiusque lentis ita ſe habebit poſito $n = \frac{31}{30}$: vel ſi refractio differat, cuique lenti ſuus tribuatur valor, primae n, ſecundae n', tertiae n'' etc.

Erit nempe pro	Radius faciei anterioris	Radius faciei posterioris
Lente prima PP	$\frac{(n-1)\alpha(k+v)}{k+v+2n\alpha}$	$\frac{(n-1)\alpha(k-v)}{k-v-2n\alpha}$
Lente ſecunda QQ	$\frac{(n-1)\beta(k'+v')}{k'+v'+2n\beta}$	$\frac{(n-1)\beta(k'-v')}{k'-v'-2n\beta}$
Lente tertia RR	$\frac{(n-1)\gamma(k''+v'')}{k''+v''+2n\gamma}$	$\frac{(n-1)\gamma(k''-v'')}{k''-v''-2n\gamma}$
Lente quarta SS	$\frac{(n-1)\delta(k'''+v''')}{k'''+v'''+2n\delta}$	$\frac{(n-1)\delta(k'''-v''')}{k'''-v'''-2n\delta}$
Lente quinta TT	$\frac{(n-1)\varepsilon(k''''+v'''')}{k''''+v''''+2n\varepsilon}$	$\frac{(n-1)\varepsilon(k''''-v'''')}{k''''-v''''-2n\varepsilon}$
	etc.	

at

at si ponamus breuitatis gratia:

$\frac{k-v}{k+v} = i$; $\frac{k'-v'}{k'+v'} = i'$; $\frac{k''-v''}{k''+v''} = i''$; $\frac{k'''-v'''}{k'''+v'''} = i'''$; $\frac{k''''-v''''}{k''''+v''''} = i''''$ etc.

pro spatiis diffusionis habebimus hos valores:

$$P = \frac{n}{2(n-1)^2}\left(\frac{1}{ii}\left(\frac{n}{\alpha}+\frac{2}{k+v}\right)\left(\frac{1}{a}+\frac{2}{k+v}\right)^2 + ii\left(\frac{n}{\alpha}-\frac{2}{k-v}\right)\left(\frac{1}{a}-\frac{2}{k-v}\right)^2\right)$$

$$Q = \frac{n}{2(n-1)^2}\left(\frac{1}{i'i'}\left(\frac{n}{\beta}+\frac{2}{k'+v'}\right)\left(\frac{1}{b}+\frac{2}{k'+v'}\right)^2 + i'i'\left(\frac{n}{\beta}-\frac{2}{k'-v'}\right)\left(\frac{1}{b}-\frac{2}{k'-v'}\right)^2\right)$$

$$R = \frac{n}{2(n-1)^2}\left(\frac{1}{i''i''}\left(\frac{n}{\gamma}+\frac{2}{k''+v''}\right)\left(\frac{1}{c}+\frac{2}{k''+v''}\right)^2 + i''i''\left(\frac{n}{\gamma}-\frac{2}{k''-v''}\right)\left(\frac{1}{c}-\frac{2}{k''-v''}\right)^2\right)$$

$$S = \frac{n}{2(n-1)^2}\left(\frac{1}{i'''i'''}\left(\frac{n}{\delta}+\frac{2}{k'''+v'''}\right)\left(\frac{1}{d}+\frac{2}{k'''+v'''}\right)^2 + i'''i'''\left(\frac{n}{\delta}-\frac{2}{k'''+v'''}\right)\left(\frac{1}{d}-\frac{2}{k'''-v'''}\right)^2\right)$$

$$T = \frac{n}{2(n-1)^2}\left(\frac{1}{i''''i''''}\left(\frac{n}{\varepsilon}+\frac{2}{k''''+v''''}\right)\left(\frac{1}{e}+\frac{2}{k''''+v''''}\right)^2 + i''''i''''\left(\frac{n}{\varepsilon}-\frac{2}{k''''-v''''}\right)\left(\frac{1}{e}-\frac{2}{k''''-v''''}\right)^2\right)$$

etc.

His constitutis pro magnitudine singularum imaginum habebimus:

ad situm

pro vna lente imaginem $F\zeta = \frac{1}{i}\cdot\frac{\alpha}{a}z$ inuersum

pro duabus lentibus .... $G\eta = \frac{1}{ii'}\cdot\frac{\alpha\beta}{ab}z$ erectum

pro tribus lentibus .... $H\vartheta = \frac{1}{ii'i''}\cdot\frac{\alpha\beta\gamma}{abc}z$ inuersum

pro quatuor lentibus .... $I\iota = \frac{1}{ii'i''i'''}\cdot\frac{\alpha\beta\gamma\delta}{abcd}z$ erectum

pro quinque lentibus .... $K\varkappa = \frac{1}{ii'i''i'''i''''}\cdot\frac{\alpha\beta\gamma\delta\varepsilon}{abcde}z$ inuersum

etc.

Deni-

Denique dum aperturae lentium non sint minores, quam sequentes formulae exhibent:

Semid. aperturae Lentis	faciei anterioris	faciei posterioris
primae P P	x	$i\,x$
secundae Q Q	$i.\frac{bx}{a}$	$ii.\frac{bx}{a}$
tertiae R R	$ii^{l}.\frac{bcx}{a\mathfrak{b}}$	$ii^{l}i^{ll}.\frac{bcx}{a\mathfrak{b}}$
quartae S S	$ii^{l}i^{ll}.\frac{bcdx}{a\mathfrak{b}\gamma}$	$ii^{l}i^{ll}i^{lll}.\frac{bcdx}{a\mathfrak{b}\gamma}$
quintae T T	$ii^{l}i^{ll}i^{lll}.\frac{bcdex}{a\mathfrak{b}\gamma\delta}$	$ii^{l}i^{ll}i^{lll}i^{llll}.\frac{bcdex}{a\mathfrak{b}\gamma\delta}$

etc.

erit vt sequitur pro quolibet lentium numero:

I. Pro vna lente

Spatium diffusionis $Ff = aaxx.\ P$

inclinatio radiorum in f concurrentium ad axem $= i.\frac{x}{a}$

II. Pro duabus lentibus

Spatium diffusionis

$$Gg = \mathfrak{b}\mathfrak{b}xx\left(\frac{1}{i^{l}i^{l}}.\frac{aa}{bb}P + ii.\frac{bb}{aa}Q\right)$$

et radiorum in g concurrentium inclinatio ad axem $= ii^{l}.\frac{bx}{a\mathfrak{b}}$

III. Pro tribus lentibus

Spatium diffusionis:

$$Hh = \gamma\gamma xx\left(\frac{1}{i^{l}i^{ll}i^{lll}}.\frac{aa\mathfrak{b}\mathfrak{b}}{bbcc}P + \frac{ii}{i^{l}i^{ll}}.\frac{bb\mathfrak{c}\mathfrak{c}}{aacc}.Q + ii.i^{l}i^{ll}.\frac{bbcc}{aa\mathfrak{b}\mathfrak{b}}R\right)$$

et radiorum in h concurrentium inclinatio ad axem $= ii^{l}i^{ll}.\frac{bcx}{a\mathfrak{b}\gamma}$

IV. Pro quatuor lentibus.

Spatium diffusionis:

$$Ei = \delta\delta xx \left\{ \frac{1}{i'i''i'''i^{iv}} \cdot \frac{\alpha\alpha\varepsilon\varepsilon\gamma\gamma}{bbccdd} P + \frac{i'i''}{i''i'''i^{iv}i^{v}} \cdot \frac{bb\varepsilon\varepsilon\gamma\gamma}{\alpha\alpha cddd} Q \right.$$
$$\left. + \frac{ii.i'i''}{i''i^{iv}} \cdot \frac{bb\varepsilon\varepsilon\gamma\gamma}{\alpha\alpha\varepsilon ddd} R + ii\,i'i''i'''i^{iv}i^{v}i^{vi} \cdot \frac{bbccdd}{\alpha\alpha\varepsilon\varepsilon\gamma\gamma} S \right\}$$

et radiorum in i concurrentium

inclinatio ad axem $= ii\,i'i''i'''i^{iv} \cdot \frac{bc\varepsilon\gamma xx}{a\varepsilon\gamma\delta}$

V. Pro quinque lentibus

Spatium diffusionis:

$$Ek = \varepsilon\varepsilon xx \left\{ \begin{array}{l} + \frac{1}{i'i''i'''i^{iv}i^{v}i^{vi}i^{vii}} \cdot \frac{\alpha\alpha\varepsilon\varepsilon\gamma\gamma\delta\delta}{bbccddee} P \\ + \frac{i.i}{i''i'''i^{iv}i^{v}i^{vi}i^{vii}} \cdot \frac{bb\varepsilon\varepsilon\gamma\gamma\delta\delta}{\alpha\alpha ccddee} Q \\ + \frac{ii.i'i''}{i'''i^{iv}i^{v}i^{vi}i^{vii}} \cdot \frac{bbc\alpha\gamma\gamma\delta\delta}{\alpha\alpha\varepsilon ddee} R \\ + \frac{ii.i'i''i'''i^{iv}}{i^{iv}i^{v}i^{vi}i^{vii}} \cdot \frac{bb\varepsilon\varepsilon dd\delta\delta}{\alpha\alpha\varepsilon\gamma\gamma ee} S \\ + ii.i'i''i'''i^{iv}i^{v}i^{vi}i^{vii} \cdot \frac{bbccddee}{\alpha\alpha\varepsilon\varepsilon\gamma\gamma\delta\delta} T \end{array} \right\}$$

et radiorum in k concurrentium

inclinatio ad axem $= ii\,i'i''i'''i^{iv}i^{v} \cdot \frac{bcde xx}{a\varepsilon\gamma\delta\varepsilon}$

Vnde progressio harum formularum ad plures adhuc lentes satis est manifesta. Si in lentibus ratio refractionis sit diuersa atque ad singulas lentes ordine his litteris indicetur n, n', n'', n''', n^{iv}, etc.; haec diuersitas facile ad formulas hic inuentas accommodabitur. Primum enim haec correctio occurrit in formulis pro radiis lentium,

lentium, ita, vt quemadmodum formulae f et g pro prima lente numerum n involunt, ita pro secunda lente numerus n', pro tertia n'' et ita porro introducatur. Similem correctionem etiam requirunt valores litterarum P, Q, R, S etc. et loco litterae n, quae in valore P occurrit, in valoribus Q, R, S etc. scribi oportet n', n'', n''' etc.

Coroll. 1.

87. Si obiectum sit tantum punctum in axe positum, sufficit vt lentium aperturae sint tantae, quantas assignauimus sin autem obiectum habeat quandam magnitudinem, tum aperturae praeter primam eo magis mensuras assignatas superare debent, quo maior fuerit obiecti magnitudo z.

Coroll. 2.

88. In expressione spatii diffusionis quadratum semidiametri aperturae primae faciei xx primo multiplicatur per quadratum distantiae postremae imaginis ab vltima lentium: quae ergo si fuerit infinita, etiam spatium diffusionis fit infinitum.

Coroll. 3.

89. Ceteris ergo paribus, quotcunque fuerint lentes, spatium diffusionis semper est proportionale quadrato diametri aperturae primae faciei, hoc est ipsi huic aperturae. Vnde diametro aperturae primae

ciei ad semissem redacto spatium diffusionis quadruplo fiet minus.

Scholion.

90. Considerauimus hic statim loca singularum imaginum principalium tanquam data ex iisque structuram cuiusque lentis quantitatem arbitrariam introducendo determinauimus. Quod si vero ipsae lentes fuerint datae ita vt tam radii ambarum facierum cuiusque quam crassities, vna cum earum interuallis cognoscantur tum ope formularum exhibitarum, vicissim distantiae determinatrices innotescunt. Sint scilicet radii facierum anterioris et posterioris primae lentis PP, f, g, secundae lentis QQ, f', g'; tertiae lentis RR, f'', g'' etc. crassitie earum existente v, v' v'' etc. tum verodentur distantiae aB$=$F; bC$=$G; cD$=$H etc. Praeterea autem distantia obiecti ante lentem primam sit AE$=a$, ac sequenti modo omnia elementa ad problema superius necessaria elicientur;

$$\begin{cases} 1.\ \frac{n-1}{f} = \frac{1}{a} + \frac{2n}{k+v}, \text{ hinc reperitur } k \\ 2.\ \frac{n-1}{g} = \frac{1}{a} - \frac{2n}{k-v}, \text{ hinc vero } \alpha \\ 3.\ \mathrm{F} = a + b \text{ vnde } b = \mathrm{F} - a \end{cases}$$

$$\begin{cases} 4.\ \frac{n-1}{f''} = \frac{1}{b} + \frac{2n}{k'+v'}, \text{ hinc reperitur } k' \\ 5.\ \frac{n-1}{g''} = \frac{1}{b} - \frac{2n}{k'-v'}, \text{ hinc vero porro } \mathfrak{b} \\ 6.\ \mathrm{G} = \mathfrak{b} + c \text{ vnde } c = \mathrm{G} - \mathfrak{b} \end{cases}$$

(7.

$$\begin{cases} 7.\ \frac{n-1}{f''} = \frac{1}{c} + \frac{2\pi}{k''+v''}, \text{ hinc reperitur } k'' \\ 8.\ \frac{n-1}{g''} = \frac{1}{\gamma} - \frac{2\pi}{k''-v''}, \text{ hinc vero } \gamma \\ 9.\ H = \gamma + d \text{ vnde } d = H - \gamma \end{cases}$$

etc.

Vtcunque ergo lentes datae fuerint dispositae super axe communi, si ante eas constituatur obiectum in data distantia $AE = a$, inde singulae distantiae determinatrices $\alpha, b, \mathfrak{C}, c, \gamma$ etc. cum arbitrariis k, k', k'', k''' etc. facile definiuntur ex iisque porro spatium diffusionis cum reliquis phaenomenis in solutione problematis commemoratis assignabitur. Operae pretium autem erit casum, quo crassities lentium vt euanescens spectatur, accuratius euoluisse.

Problema 5.

91. Si crassities lentium euanescat, et quotcunque huiusmodi lentes super communi axe fuerint dispositae ante quas existat obiectum $E\varepsilon$, definire spatium diffusionis, per quod imago erit dissipata, vt et magnitudinem imaginis.

Solutio.

Si obiecti magnitudo $E\varepsilon = \varepsilon$, cuius imagines principales successive cadant in $F\zeta, G\eta, H\theta, I\iota, K\varkappa$ etc. hincque pro singulis lentibus sequentes habebimus distantias determinatrices, cum imago per quamuis lentem repraesentata respectu lentis sequentis vicem obiecti gerat.

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distantia — distantia

Pro lente PP obiecti $EA = a$; imaginis $aF = \alpha$
Pro lente QQ obiecti $FB = b$; imaginis $bG = \beta$
Pro lente RR obiecti $GC = c$; imaginis $cH = \gamma$
Pro lente SS obiecti $HD = d$; imaginis $dI = \delta$
Pro lente TT obiecti $IE = e$; imaginis $eK = \varepsilon$
etc.

Porro autem sint numeri arbitrarii vnitate maiores cuiusque lentis figuram determinantes, λ pro lente PP, λ' pro QQ, λ'' pro RR, λ''' pro SS, λ^{IV} pro TT, etc. ita vt ponendo breuitatis gratia

$\varrho = 0,190781$, $\sigma = 1,627401$, $\tau = 0,905133$

Pro lente PP radius faciei
$$\text{anter.} = \frac{a\alpha}{\varrho\alpha + \sigma\alpha \pm \tau(a+\alpha)\sqrt{(\lambda-1)}}$$
$$\text{poster.} = \frac{a\alpha}{\varrho\alpha + \sigma\alpha \mp \tau(a+\alpha)\sqrt{(\lambda-1)}}$$

Pro lente QQ radius faciei
$$\text{anter.} = \frac{b\beta}{\varrho\beta + \sigma b \pm \tau(b+\beta)\sqrt{(\lambda'-1)}}$$
$$\text{poster.} = \frac{b\beta}{\varrho\beta + \sigma b \mp \tau(b+\beta)\sqrt{(\lambda'-1)}}$$

Pro lente RR radius faciei
$$\text{anter.} = \frac{c\gamma}{\varrho\gamma + \sigma c \pm \tau(c+\gamma)\sqrt{(\lambda''-1)}}$$
$$\text{poster.} = \frac{c\gamma}{\varrho\gamma + \sigma c \mp \tau(c+\gamma)\sqrt{(\lambda''-1)}}$$

Pro lente SS radius faciei
$$\text{anter.} = \frac{d\delta}{\varrho\delta + \sigma d \pm \tau(d+\delta)\sqrt{(\lambda'''-1)}}$$
$$\text{poster.} = \frac{d\delta}{\varrho\delta + \sigma d \mp \tau(d+\delta)\sqrt{(\lambda'''-1)}}$$

Pro lente TT radius faciei
$$\text{anter.} = \frac{e\varepsilon}{\varrho\varepsilon + \sigma e \pm \tau(e+\varepsilon)\sqrt{(\lambda^{IV}-1)}}$$
$$\text{poster.} = \frac{e\varepsilon}{\varrho\varepsilon + \sigma e \mp \tau(e+\varepsilon)\sqrt{(\lambda^{IV}-1)}}$$
etc.

Deinde

Deinde si quaelibet lens cum binis suis distantiis determinatricibus seorsim consideretur, eiusque aperturae semidiameter foret $= x$, posito $\mu = 0,938192$ et $\nu = 0,232692$ esset spatium diffusionis

Lentis PP $= \mu\alpha\alpha xx\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{a\alpha}\right)$

Lentis QQ $= \mu\beta\beta xx\left(\frac{1}{b}+\frac{1}{\beta}\right)\left(\lambda'\left(\frac{1}{b}+\frac{1}{\beta}\right)^2+\frac{\nu}{b\beta}\right)$

Lentis RR $= \mu\gamma\gamma xx\left(\frac{1}{c}+\frac{1}{\gamma}\right)\left(\lambda''\left(\frac{1}{c}+\frac{1}{\gamma}\right)^2+\frac{\nu}{c\gamma}\right)$

Lentis SS $= \mu\delta\delta xx\left(\frac{1}{d}+\frac{1}{\delta}\right)\left(\lambda'''\left(\frac{1}{d}+\frac{1}{\delta}\right)^2+\frac{\nu}{d\delta}\right)$

Lentis TT $= \mu\varepsilon\varepsilon xx\left(\frac{1}{e}+\frac{1}{\varepsilon}\right)\left(\lambda''''\left(\frac{1}{e}+\frac{1}{\varepsilon}\right)^2+\frac{\nu}{e\varepsilon}\right)$

etc.

His constitutis pro magnitudine singularum imaginum habebimus

Pro vna lente $F\zeta = \frac{a}{\alpha}z$ situ inuerso

Pro duabus lentibus $G\eta = \frac{a\beta}{ab}z$ situ erecto

Pro tribus lentibus $H\theta = \frac{a\beta\gamma}{ab\alpha}z$ situ inuerso

Pro quatuor lentibus $I\iota = \frac{a\beta\gamma\delta}{abc\alpha}z$ situ erecto

Pro quinque lentibus $K\varkappa = \frac{a\beta\gamma\delta\varepsilon}{abcd\varepsilon}z$ situ inuerso

etc.

At si semidiameter aperturae primae lentis PP ponatur $= x$, necesse est, vt reliquarum lentium aperturae sequentes valores

Semi-

Semidiameter aperturae
Lentis ſecundae $QQ > \frac{b}{a}x$
Lentis tertiae $RR > \frac{bc}{a\mathfrak{e}}x$
Lentis quartae $SS > \frac{bcd}{a\mathfrak{e}\gamma}x$
Lentis quintae $TT > \frac{bcde}{a\mathfrak{e}\gamma\mathfrak{z}}x$
etc.

Hinc ſpatium diffuſionis pro quolibet lentium numero ita ſe habebit.

I. Pro vna lente

ſpatium diffuſionis

$$Ff = \mu\alpha\alpha xx\left(\tfrac{1}{a}+\tfrac{1}{a}\right)\left(\lambda\left(\tfrac{1}{a}+\tfrac{1}{a}\right)^2+\tfrac{\nu}{a\alpha}\right)$$

radiorum in f concurrentium
inclinatio ad axem $= \frac{x}{a}$

II. Pro duabus lentibus

ſpatium diffuſionis

$$Gg = \mu\mathfrak{e}\mathfrak{e}xx\begin{cases}+\frac{\alpha\alpha}{bb}\left(\frac{1}{a}+\frac{1}{a}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{a}\right)^2+\frac{\nu}{a\alpha}\right)\\ +\frac{bb}{a\alpha}\left(\frac{1}{b}+\frac{1}{\mathfrak{e}}\right)\left(\lambda'\left(\frac{1}{b}+\frac{1}{\mathfrak{e}}\right)^2+\frac{\nu}{b\mathfrak{e}}\right)\end{cases}$$

et radiorum in g concurrentium
inclinatio ad axem $= \frac{bx}{a\mathfrak{e}}$

III. Pro tribus lentibus

ſpatium diffuſionis

$$Hh = \mu\gamma\gamma xx\begin{cases}+\frac{\alpha\alpha\mathfrak{e}\mathfrak{e}}{bbcc}\left(\frac{1}{a}+\frac{1}{a}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{a}\right)^2+\frac{\nu}{a\alpha}\right)\\ +\frac{bb\mathfrak{e}\mathfrak{e}}{\alpha\alpha cc}\left(\frac{1}{b}+\frac{1}{\mathfrak{e}}\right)\left(\lambda'\left(\frac{1}{b}+\frac{1}{\mathfrak{e}}\right)^2+\frac{\nu}{b\mathfrak{e}}\right)\\ +\frac{bbcc}{\alpha\alpha\mathfrak{e}\mathfrak{e}}\left(\frac{1}{c}+\frac{1}{\gamma}\right)\left(\lambda''\left(\frac{1}{c}+\frac{1}{\gamma}\right)^2+\frac{\nu}{c\gamma}\right)\end{cases}$$

et

et radiorum in b concurrentium inclinatio ad axem $\frac{bcx}{\alpha\varepsilon\gamma}$

IV. Pro quatuor lentibus ſpatium diffuſionis

$$I\,i = \mu\,\delta\,\delta\,x\,x \begin{cases} +\frac{\alpha\alpha\varepsilon\varepsilon\gamma\gamma}{bb\,cc\,dd}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{a\alpha}\right) \\ +\frac{bb\varepsilon\varepsilon\gamma\gamma}{\alpha\alpha\,cc\,dd}\left(\frac{1}{b}+\frac{1}{\varepsilon}\right)\left(\lambda'\left(\frac{1}{b}+\frac{1}{\varepsilon}\right)^2+\frac{\nu}{b\varepsilon}\right) \\ +\frac{bbcc\gamma\gamma}{\alpha\alpha\varepsilon\varepsilon\,dd}\left(\frac{1}{c}+\frac{1}{\gamma}\right)\left(\lambda''\left(\frac{1}{c}+\frac{1}{\gamma}\right)^2+\frac{\nu}{c\gamma}\right) \\ +\frac{bbccdd}{\alpha\alpha\varepsilon\varepsilon\gamma\gamma}\left(\frac{1}{d}+\frac{1}{\delta}\right)\left(\lambda'''\left(\frac{1}{d}+\frac{1}{\delta}\right)^2+\frac{\nu}{d\delta}\right) \end{cases}$$

et radiorum in i concurrentium inclinatio ad axem $= \frac{bc\,dx}{\alpha\varepsilon\gamma\delta}$

V. Pro quinque lentibus ſpatium diffuſionis

$$K\,k = \mu\,\varepsilon\,\varepsilon\,x\,x \begin{cases} +\frac{\alpha\alpha\varepsilon\varepsilon\gamma\gamma\delta\delta}{bbccddee}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{a\alpha}\right) \\ +\frac{bb\varepsilon\varepsilon\gamma\gamma\delta\delta}{\alpha\alpha ccddee}\left(\frac{1}{b}+\frac{1}{\varepsilon}\right)\left(\lambda'\left(\frac{1}{b}+\frac{1}{\varepsilon}\right)^2+\frac{\nu}{b\varepsilon}\right) \\ +\frac{bbcc\gamma\gamma\delta\delta}{\alpha\alpha\varepsilon\varepsilon ddee}\left(\frac{1}{c}+\frac{1}{\gamma}\right)\left(\lambda''\left(\frac{1}{c}+\frac{1}{\gamma}\right)^2+\frac{\nu}{c\gamma}\right) \\ +\frac{bbccdd\delta\delta}{\alpha\alpha\varepsilon\varepsilon\gamma\gamma ee}\left(\frac{1}{d}+\frac{1}{\delta}\right)\left(\lambda'''\left(\frac{1}{d}+\frac{1}{\delta}\right)^2+\frac{\nu}{d\delta}\right) \\ +\frac{bbccddee}{\alpha\alpha\varepsilon\varepsilon\gamma\gamma\delta\delta}\left(\frac{1}{e}+\frac{1}{\varepsilon}\right)\left(\lambda''''\left(\frac{1}{e}+\frac{1}{\varepsilon}\right)^2+\frac{\nu}{e\varepsilon}\right) \end{cases}$$

et radiorum in k concurrentium inclinatio ad axem $= \frac{bcdex}{\alpha\varepsilon\gamma\delta\varepsilon}$

neque ergo caſus, quibus plures occurrunt lentes, vlla amplius laborant difficultate.

Si lentes ratione refractionis diſcrepent, ad easque referendae ſint litterae n, n', n'', n''', etc. formu-

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lae in hoc problemate inuentae sequenti modo facile ad hunc casum latius patentem adcommodabuntur. Primo scilicet in formulis pro radiis facierum inuentis litterae ϱ, σ, et τ tantum ad primam lentem pertinent, earumque loco pro secunda lente scribi oportet ϱ', σ', et τ'; pro tertia autem ϱ'', σ'' et τ'' et ita porro. Praeterea vero spatia diffusionis hinc aliquam mutationem requirunt, eritque spatium diffusionis.

I. Pro vna lente

$$\alpha\alpha xx\left(\tfrac{1}{a}+\tfrac{1}{\alpha}\right)\mu\left(\lambda\left(\tfrac{1}{a}+\tfrac{1}{\alpha}\right)^2+\tfrac{\nu}{a\alpha}\right).$$

II. Pro duabus lentibus

$$\varepsilon\varepsilon xx\begin{cases}\frac{\mu\alpha\alpha}{bb}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{N}{a\alpha}\right)\\ +\frac{\mu'bb}{\alpha\alpha}\left(\frac{1}{b}+\frac{1}{\varepsilon}\right)\left(\lambda'\left(\frac{1}{b}+\frac{1}{\varepsilon}\right)^2+\frac{\nu'}{b\varepsilon}\right).\end{cases}$$

III. Pro tribus lentibus

$$\gamma\gamma xx\begin{cases}+\frac{\mu\alpha\alpha\varepsilon\varepsilon}{bbcc}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{a\alpha}\right)\\ +\frac{\mu'bb\varepsilon\varepsilon}{\alpha\alpha cc}\left(\frac{1}{b}+\frac{1}{\varepsilon}\right)\left(\lambda'\left(\frac{1}{b}+\frac{1}{\varepsilon}\right)^2+\frac{\nu'}{b\varepsilon}\right)\\ +\frac{\mu''bbcc}{\alpha\alpha\varepsilon\varepsilon}\left(\frac{1}{c}+\frac{1}{\gamma}\right)\left(\lambda''\left(\frac{1}{a}+\frac{1}{\gamma}\right)^2+\frac{\nu''}{c\gamma}\right).\end{cases}$$

IV. Pro quatuor lentibus

$$\delta\delta xx\begin{cases}+\frac{\mu\alpha\alpha\varepsilon\varepsilon\gamma\gamma}{bbccdd}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{a\alpha}\right)\\ +\frac{\mu'bb\varepsilon\varepsilon\gamma\gamma}{\alpha\alpha ccdd}\left(\frac{1}{b}+\frac{1}{\varepsilon}\right)\left(\lambda'\left(\frac{1}{b}+\frac{1}{\varepsilon}\right)^2+\frac{\nu'}{b\varepsilon}\right)\\ +\frac{\mu''bbcc\gamma\gamma}{\alpha\alpha\varepsilon\varepsilon dd}\left(\frac{1}{c}+\frac{1}{\gamma}\right)\left(\lambda''\left(\frac{1}{c}+\frac{1}{\gamma}\right)^2+\frac{\nu''}{c\gamma}\right)\\ +\frac{\mu'''bbccdd}{\alpha\alpha\varepsilon\varepsilon\gamma\gamma}\left(\frac{1}{d}+\frac{1}{\delta}\right)\left(\lambda'''\left(\frac{1}{d}+\frac{1}{\delta}\right)^2+\frac{\nu'''}{d\delta}\right).\end{cases}$$

valores autem harum litterarum μ', ν', μ'', ν'' etc. iam supra definiuimus §. 77.

Coroll.

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Coroll. 1.

92. Si lentis primae PP ponatur radius faciei anterioris $=f$ et posterioris $=g$ erit

$$\tfrac{1}{f}=\tfrac{\varrho}{a}+\tfrac{\sigma}{\alpha}\pm\tau\left(\tfrac{1}{a}+\tfrac{1}{\alpha}\right)\sqrt{(\lambda-1)}$$

$$\tfrac{1}{g}=\tfrac{\varrho}{\alpha}+\tfrac{\sigma}{a}\mp\tau\left(\tfrac{1}{a}+\tfrac{1}{\alpha}\right)\sqrt{(\lambda-1)}$$

vnde si detur distantia obiecti $EA=a$, primo inuenitur α ex hac aequatione

$$\tfrac{1}{f}+\tfrac{1}{g}=(\varrho+\sigma)\left(\tfrac{1}{a}+\tfrac{1}{\alpha}\right)=1,\ 81818182\left(\tfrac{1}{a}+\tfrac{1}{\alpha}\right)=\tfrac{20}{11}\left(\tfrac{1}{a}+\tfrac{1}{\alpha}\right)$$

Inuenta autem distantia α numerus λ reperitur ex hac aequatione.

$$\tfrac{1}{f}-\tfrac{1}{g}+(\sigma-\varrho)\left(\tfrac{1}{a}-\tfrac{1}{\alpha}\right)=2\tau\left(\tfrac{1}{a}+\tfrac{1}{\alpha}\right)\sqrt{(\lambda-1)}.$$

Coroll. 2.

93. Deinde si distantia secundae lentis a prima sit $=F$, ob $F=\alpha+b$ habetur $b=F-\alpha$; qua distantia b cognita, si pro lente secunda datus sit radius faciei anterioris $=f'$, et posterioris $=g'$ habebuntur iterum duae aequationes

$$\tfrac{1}{f'}=\tfrac{\varrho}{b}+\tfrac{\sigma}{\beta}\pm\tau\left(\tfrac{1}{b}+\tfrac{1}{\beta}\right)\sqrt{(\lambda'-1)}$$

$$\tfrac{1}{g'}=\tfrac{\varrho}{\beta}+\tfrac{\sigma}{b}\mp\tau\left(\tfrac{1}{b}+\tfrac{1}{\beta}\right)\sqrt{(\lambda'-1)}$$

ex quibus cum distantiam β tum numerum λ' definire licet. Similique modo ex forma sequentium lentium earumque distantia reliqua elementa innotescent.

Coroll. 3.

94. Si singulae lentes ad minimum spatium diffusionis fuerint accommodatae, erit $\lambda=1, \lambda'=1, \lambda''=1, \lambda'''=1$ etc. sin autem hae lentes alia forma fuerint praeditae, isti numeri erunt vnitate maiores.

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Scholion.

95. Quo plures fuerint lentes eo pluribus constabit membris spatium diffusionis ab iis productum; neque tamen propterea aucto lentium numero spatium diffusionis necessario augetur. Cum enim quantitates $a, b, \mathfrak{C}, c, \gamma, d, \delta$, etc. valores quoque negatiuos recipere queant, dummodo binarum summae $a+b$; $\mathfrak{C}+c$; $\gamma+d$; $\delta+e$ etc. vtpote lentium distantiae maneant positiuae, fieri potest, vt vnum vel aliquot membra fiant negatiua, hincque spatium diffusionis diminuatur, quin etiam interdum prorsus euanescat, quo casu repraesentatio sine dubio erit perfectissima. Verum in instrumentis dioptricis ad visionem instructis veluti Telescopiis ac Microscopiis non tam hoc, quod definiuimus, spatium diffusionis quam confusio in ipsa visione orta spectari debet; quae autem etsi a spatio diffusionis plurimum differt, tamen ex eo definiri potest vti mox explicabimus. Ante autem conueniet lentes compositas seu multiplicatas considerare, cuiusmodi oriuntur, si duae pluresue lentes, quarum craffities tam est parua vt negligi queat, immediate iungantur quo quidem pacto instar lentium simplicium spectari possunt; verum tali coniunctione effici potest, vt spatium diffusionis multo fiat minus quam si lens simplex adhiberetur, atque adeo euanescat valorque numeri λ istiusmodi lenti compositae conueniens vnitate minor sit proditurus vnde maxima commoda ad confusionem diminuendam obtinebuntur.

CAPVT III.

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CAPVT III.

DE LENTIBVS COMPOSITIS SEV MVLTIPLICATIS.

Definitio 1.

96.

Lens duplicata oritur, si duae lentes super communi axe sibi immediate iungantur.

Crassitiem hic vtriusque lentis tanquam nullam assumo et quia distantia inter lentes nulla ponitur, crassities etiam lentis duplicatae pro nulla haberi poterit.

Coroll. 1.

97. Binae ergo lentes PP et QQ sibi ad contactum fere coniunctae lentem duplicatam constituunt; de qua tamen notandum est, eius crassitiem minus tuto negligi posse quam vtriusque lentis simplicis seorsim sumtae. Si enim lentes immediate se in puncto contingerent; phaenomena colorum a *Newtono* obseruata essent metuenda tum vero etiam ostendemus, quomodo ratio distantiae inter binas lentes haberi possit. Tab. III. Fig. 7.

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Coroll. 2.

98. Si lentis anterioris PP distantiae determinatrices sint a et α, lentis posterioris vero QQ, b et $\mathfrak{E}$, necesse est vt sit $a+b=0$ seu $a=-b$. Tum vero obiecti ante lentem ad distantiam $AE=a$ positi imago principalis repraesentabitur post lentem ad distantiam $G=\mathfrak{E}$.

Coroll. 3.

99. Erunt ergo a et $\mathfrak{E}$ quasi distantiae determinatrices lentis duplicatae; ac sumendo α vel b ad libitum infinita paria lentium pro his distantiis exhiberi possunt. Cum deinde vtraque lens praeterea numerum indefinitum λ recipiat insuper infinita varietas locum habet.

Coroll. 4.

100. Quia crassities pro nihilo reputatur, aperturae in singulis faciebus eadem est ratio; scilicet si in prima facie semidiameter aperturae sit $=x$, in reliquis quoque faciebus apertura eadem vel saltem non minor esse debet.

Scholion.

101. Non opus est, vt lentes plane ad contactum coniungantur quoniam forte refractionis lex turbari posset: hoc autem vel minima interposita distantia euitabitur id quod ad institutum nostrum sufficit, cum etiam crassities vtriusque non omnino sit nulla.

Proble-

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Problema 1.

102. Omnes lentes duplicatas describere, quibus obiectum Ee in data ante lentem distantia AE propositum post lentem in data distantia bG repraesentetur, simulque diffusionem imaginis Gg pro data lentis apertura definire.

Solutio.

Sit distantia obiecti $AE = a$, imaginis principalis $bG = \mathfrak{E}$, tum vero lentis primae PP distantiae determinatrices α, et α; lentis posterioris vero QQ, β et $\mathfrak{E}$; iam quia distantia lentium est nulla, erit $\alpha + \beta = 0$, seu $\alpha = -\beta$; Hinc si obiecti magnitudo sit $Ee = z$, erit imaginis principalis magnitudo $G\eta = \frac{\mathfrak{E}}{a} z$ pro situ inuerso. Cum porro vtraque lens infinitis modis formari possit, sit λ numerus arbitrarius pro prima PP et λ' pro secunda QQ, quarum ergo constructio ita se habebit:

Pro Lente

PP radius faciei $\begin{cases} \text{anterioris} = \frac{\alpha\alpha}{\varrho\alpha + \sigma\alpha \pm \tau(\alpha+\alpha)\sqrt{(\lambda-1)}} \\ \text{posterioris} = \frac{\alpha\alpha}{\varrho\alpha + \sigma\alpha \mp \tau(\alpha+\alpha)\sqrt{(\lambda-1)}} \end{cases}$

QQ radius faciei $\begin{cases} \text{anterioris} = \frac{\beta\mathfrak{E}}{\varrho\mathfrak{E} + \sigma\beta \pm \tau(\beta+\mathfrak{E})\sqrt{(\lambda'-1)}} \\ \text{posterioris} = \frac{\beta\mathfrak{E}}{\varrho\beta + \sigma\mathfrak{E} \mp \tau(\beta+\mathfrak{E})\sqrt{(\lambda'-1)}} \end{cases}$

Si ratio refractionis sit diuersa; pro secunda lente scribi debet ϱ', σ' et τ', loco ϱ, σ et τ.

Pro

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Pro ſpatio autem diffuſionis Gg inueniendo, ſit ſemidiameter aperturae lentis duplicatae $= x$, eritque

$$Gg = \mu \mathfrak{G}\mathfrak{E} x x \left\{ \begin{array}{l} +\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{a\alpha}\right) \\ +\left(\frac{1}{b}+\frac{1}{\mathfrak{E}}\right)\left(\lambda'\left(\frac{1}{b}+\frac{1}{\mathfrak{E}}\right)^2+\frac{\nu}{b\mathfrak{E}}\right) \end{array} \right.$$

et radiorum in g concurrentium inclinatio ad axem $= \frac{x}{\mathfrak{E}}$. Si ratio refractionis diſcrepet; in parte ex lente ſecunda orta ſcribatur μ', ν', loco μ, ν.

Coroll. 1.

103. Huius ergo lentis duplicatae diſtantiae determinatrices ſunt a et $\mathfrak{E}$, praeterea vero duo numeri arbitrarii λ et λ', vna cum diſtantia α vel b eius perfectam determinationem conſtituunt, vnde in huiusmodi lentibus multo maior varietas locum habet, quam in lentibus ſimplicibus.

Coroll. 2.

104. Si ex eisdem diſtantiis determinatricibus a et $\mathfrak{E}$ adiungendo numero arbitrario λ^0 lens ſimplex conſtruatur, ea imaginem eadem magnitudine $G\eta = \frac{\mathfrak{E}}{a}x$ referet; ſed pro eadem apertura, cuius ſemidiameter $= x$ habebitur ſpatium diffuſionis

$$Gg = \mu \mathfrak{G}\mathfrak{E} x x \left(\tfrac{1}{a}+\tfrac{1}{\mathfrak{E}}\right)\left(\lambda^0\left(\tfrac{1}{a}+\tfrac{1}{\mathfrak{E}}\right)^2+\tfrac{\nu}{a\mathfrak{E}}\right)$$

Coroll. 3.

105. Fieri ergo poterit vt lens duplicata modo maiorem modo minorem diffuſionem gignat. Lens autem

autem ſimplex minimam parit diffuſionem, ſi $\lambda^\circ = 1$; ergo tum demum lentes duplicatae ſimplicibus erunt praeferendae, cum adhuc minorem diffuſionem producent.

Scholion.

106. Concipi quidem ſemper poterit lens ſimplex eandem diffuſionem gignens ac lens duplicata, ſi pro numero λ° omnes valores admittamus; quomodocunque enim lens duplicata fuerit comparata, ſi ſpatium diffuſionis inde productum huic ex lente ſimplici nato aequale ſtatuatur, determinatus valor pro numero λ° elicitur: qui ſi fuerit poſitiuus et vnitate maior, realis lens ſimplex aequiualens exhiberi poterit, ſin autem prodeat vnitate minor, vel adeo negatiuus, lens ſimplex inter imaginaria erit referenda. Quando autem fit $\lambda^\circ > 1$, euidens eſt lentem ſimplicem eundem plane effectum eſſe edituram ac duplicatam ideoque ſemper expediet lente ſimplici potius vti quam duplicata; quodſi vero prodierit $\lambda^\circ < 1$, quo caſu lens ſimplex fit imaginaria, tum lentes duplicatae effectum praeſtabunt a ſimplicibus non expectandum, qui adeo cum inſigni hoc commodo, quod ſpatium diffuſionis futurum ſit minus, erit coniunctus. Talibus ergo lentibus duplicatis maximo cum ſucceſſu vti poterimus, eoque magis eae ſimplicibus erunt anteponendae, quo minor fuerit valor numeri λ° iis reſpondens.

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Problema 2.

107. Data lente duplicata ad binas distantias determinatrices $AE=\alpha$ et $bG=\varepsilon$ relata pro iisdem distantiis definire lentem simplicem, quae pro eadem apertura, eandem diffusionem imaginis producat.

Solutio.

Totum ergo negotium huc redit, vt spatium diffusionis Gg a lente simplici productum (104) aequale ponatur spatio diffusionis a lente duplicata orto, cuius expressio in problemate praecedente (101) est inuenta, indeque valor numeri λ° pro constructione lentis simplicis eliciatur. Quae inuestigatio quo commodius institui possit, ponamus:

$$\tfrac{1}{a}=\tfrac{f-1}{\alpha}+\tfrac{f}{\varepsilon} \text{ eritque } \tfrac{1}{b}=\tfrac{-f+1}{\alpha}-\tfrac{f}{\varepsilon}$$

ita vt loco quantitatis α vel b numerum f introduamus eritque

$$\tfrac{1}{a}+\tfrac{1}{\alpha}=f\left(\tfrac{1}{\alpha}+\tfrac{1}{\varepsilon}\right) \text{ et } \tfrac{1}{b}+\tfrac{1}{b}=(1-f)\left(\tfrac{1}{\alpha}+\tfrac{1}{\varepsilon}\right)$$

vnde spatium diffusionis a lente duplicata ortum prodit:

$$Gg=\mu\varepsilon\varepsilon xx\begin{cases}+f\left(\frac{1}{\alpha}+\frac{1}{\varepsilon}\right)\left(\lambda ff\left(\frac{1}{\alpha}+\frac{1}{\varepsilon}\right)^2+\nu\left(\frac{f-1}{\alpha\alpha}+\frac{f}{\alpha\varepsilon}\right)\right)\\ +(1-f)\left(\frac{1}{\alpha}+\frac{1}{\varepsilon}\right)\left(\lambda'(1-f)^2\left(\frac{1}{\alpha}+\frac{1}{\varepsilon}\right)^2+\nu\left(\frac{1-f}{\alpha\varepsilon}-\frac{f}{\varepsilon\varepsilon}\right)\right)\end{cases}$$

quae expressio reducitur ad hanc formam:

$$Gg=\mu\varepsilon\varepsilon xx\left(\tfrac{1}{\alpha}+\tfrac{1}{\varepsilon}\right)\left(\left(\lambda f^3+\lambda'(1-f)^3\right)\left(\tfrac{1}{\alpha}+\tfrac{1}{\varepsilon}\right)^2+\nu\left(\tfrac{f(f-1)}{\alpha\alpha}+\tfrac{1-2f+2ff}{\alpha\varepsilon}+\tfrac{f(f-1)}{\varepsilon\varepsilon}\right)\right)$$

Verum postremum membrum $\frac{f(f-1)}{\alpha\alpha}+\frac{1-2f+2ff}{\alpha\varepsilon}+\frac{f(f-1)}{\varepsilon\varepsilon}$ mutatur

mutatur in $f(f-1)(\frac{1}{a}+\frac{1}{b})^2+\frac{1}{ab}$, ſicque habebimus pro lente duplicata:

$$Gg=\mu\mathfrak{G}\mathfrak{G}xx(\tfrac{1}{a}+\tfrac{1}{b})\left((\lambda f^3+\lambda'(1-f)^3-\nu f(1-f))(\tfrac{1}{a}+\tfrac{1}{b})^2+\tfrac{\nu}{ab}\right)$$

quae forma iam facillime cum ſpatio diffuſionis lentis ſimplicis comparatur indeque manifeſto colligitur:

$$\lambda^0=\lambda f^3+\lambda'(1-f)^3-\nu f(1-f)$$

Cum autem loco quantitatum a et b numerum f introduxerimus conſtructio lentis duplicatae ita ſe habebit

Pro lente — radius faciei

prima PP { anterioris $=\frac{ab}{(\varrho-\sigma(1-f))b+\sigma fa\pm\tau f(a+b)\sqrt{(\lambda-1)}}$; posterioris $=\frac{ab}{(\sigma-\varrho(1-f))b+\varrho fa\pm\tau f(a+b)\sqrt{(\lambda-1)}}$

secunda QQ { anterioris $=\frac{ab}{(\sigma-\varrho f)a+\varrho(1-f)b\pm\tau(1-f)(a+b)\sqrt{(\lambda-1)}}$; posterioris $=\frac{ab}{(\varrho-\sigma f)a+\sigma(1-f)b\pm\tau f(a+b)\sqrt{(\lambda-1)}}$

Tum vero inuento numero λ^0 conſtructio lentis ſimplicis aequiualentis erit

Radius faciei { anterioris $=\frac{ab}{\varrho b+\sigma a\pm\tau(a+b)\sqrt{(\lambda-1)}}$; posterioris $=\frac{ab}{\varrho a+\sigma b\pm\tau(a+b)\sqrt{(\lambda-1)}}$

Coroll. 1.

108. Quoties ergo fuerit

$$\lambda f^3+\lambda'(1-f)^3-\nu f(1-f)>1$$

ſemper lens ſimplex parari poteſt duplicatae aequiua-

lens iisque ergo casibus praestabit lente simplici vti potius quam lente duplicata.

Coroll. 2.

109. Verum si fuerit

$$\lambda f^3+\lambda'(1-f)^3-\nu f(1-f)<1$$

ob $\lambda^\circ<1$, constructio lentis simplicis fit impossibilis, ac lens duplicata minorem pariet diffusionem quam per vllam lentem simplicem obtineri potest.

Coroll. 3

110. Si esset $f=0$, prodiret $\lambda^\circ=\lambda'$, et in lente duplicata anteriori nullam refractionem produceret ob facies parallelas, resque eodem rediret, ac si posterior sola adesset. Sin autem sumatur $f=1$, fit $\lambda^\circ=\lambda$, et lens posterior superflua; vtroque ergo casu nullum lucrum impetratur.

Coroll. 4.

111. Cum autem numeri λ et λ' vnitate nequeant esse minores, sitque $\nu=0,232692$, patet pro f nullum valorem inter limites 0 et 1 assumi posse, vnde fiat $\lambda^\circ=0$. At si f extra hos limites capiatur, vtique pro λ et λ' eiusmodi numeri vnitate maiores assignari poterunt, vt fiat $\lambda^\circ=0$.

Scholion 1.

112. Ratione ergo numeri f tres casus lentium duplicatarum considerari conuenlet, prout vel f intra limi-

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limites 0 et 1 continetur, vel fuerit $f > 1$ vel $f < 0$. Primo casu euenire non potest, vt fiat $\lambda^\circ = 0$; sed plurimum intererit eam determinasse lentem duplicatam, pro qua λ° minimum obtineat valorem qui quo magis infra vnitatem cadat, eo perfectior lens erit censenda, maioreque iure simplicibus anteferenda. Binis reliquis vero casibus $f > 1$ et $f < 0$ eiusmodi adeo lentes duplicatae parari poterunt, quae praebeant $\lambda^\circ = 0$, quae ergo pro perfectissimis essent habendae. Verum hic quoque ad praxin est respiciendum, quae cum semper a praeceptis Theoriae aberrare soleat, euenire potest, vt leui errore commisso numerus λ° non solum non euanescat, sed adeo vnitatem excedat quo casu vtique expediret lente vti simplici.

Scholion 2.

113. Cum tanti sit momenti rationem aberrationis a qua praxis vix liberari potest habere, eam etiam in lentibus simplicibus perpendi conueniet. Postulauimus autem pro datis distantiis determinatricibus lentem construi posse in qua numerus λ datum obtineat valorem dum ne sit vnitate minor; hic igitur obseruari oportet, quo maior fuerit λ, eo difficilius fore errorem euitare; si enim in constructione leuissimus error committatur alius eo magis diuersus valor pro λ orietur, quo maior fuerit λ. Verum e contrario cum vnitas sit minimus valor, quem λ recipere potest ex natura minimi liquet etiamsi in praxi a praescripta regula notabiliter recedatur tamen inde vix

ſenſibile diſcrimen in valorem λ eſſe redundaturum. Ex quo concludimus feliciſſimo cum ſucceſſu eiusmodi lentes ſimplices parari poſſe, pro quibus futurum ſit $\lambda = 1$, neque hic errores praxeos, niſi fuerint enormes, admodum eſſe pertimeſcendos. Deinde quo minus numerus λ vnitatem ſuperare debeat, eo certiores eſſe poterimus de ſucceſſu, ſed non eo gradu, quo caſu, $\lambda = 1$; at ſi opus ſit eiusmodi lente, pro qua valor ipſius λ debeat eſſe numerus ſatis magnus, difficillime per praxin ſatisfiet ac fortaſſe ingentem lentium numerum parare oportebit, antequam vna obtineatur ſcopo ſatisfaciens. Quamobrem ſi praxi conſulere velimus, vix alias lentes exigere debemus, niſi pro quibus numerus λ vel ſit vnitas ipſa, vel parumper maior. Sin autem ad inſigne aliquod commodum aliae lentes requirantur, labori non erit parcendum, cum fortaſſe non niſi poſt plurimos conatus irritos voti tandem compotes reddi queamus.

Problema 3.

114. Definire eam lentem duplicatam, pro qua, ſi numerus f intra limites 0 et 1 accipiatur numerus λ° minimum adipiscatur valorem.

Solutio.

Poſitis iisdem, quae in praecedentibus problematibus ſunt conſtituta, inuenimus eſſe

$$\lambda^\circ = \lambda f^3 + \lambda'(1-f)^3 - \nu f(1-f)$$

vbi cum f intra limites 0 et 1 aſſumi debeat, ambo termini

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termini λf^3 et $\lambda'(1-f)^3$ erunt positiui. Quare vt λ° omnium minimum valorem nanciscatur, necesse est vtrique numero λ et λ' minimum valorem cuius est capax tribui.

Sit ergo $\lambda = 1$ et $\lambda' = 1$, ac habebimus

$$\lambda^\circ = 1 - 3f + 3ff - \nu f + \nu ff = n - (3+\nu) f(1-f)$$

quae expressio vt minima reddatur, oportet fieri $f(1-f)$ maximum, id quod fit sumendo $f = \frac{1}{2}$; hincque oritur

$$\lambda^\circ = 1 - \tfrac{1}{4}(3+\nu) = \tfrac{1-\nu}{4} = 0,191827$$

Quare constructio huius lentis duplicatae ita se habebit

Pro lente P P radius faciei $\begin{cases} \text{anterioris} = \frac{2\alpha\varepsilon}{(2\varrho-\sigma)\varepsilon+\sigma a} \\ \text{posterioris} = \frac{2\alpha\varepsilon}{(2\sigma-\varrho)\varepsilon+\varrho a} \end{cases}$

Pro lente QQ radius faciei $\begin{cases} \text{anterioris} = \frac{2\alpha\varepsilon}{(2\sigma-\varrho)a+\varrho\varepsilon} \\ \text{posterioris} = \frac{2\alpha\varepsilon}{(2\varrho-\sigma)a+\sigma\varepsilon} \end{cases}$

et si aperturae semidiameter sit $= x$, erit spatium diffusionis

$$Gg = \mu\varepsilon\varepsilon xx\left(\tfrac{1}{a}+\tfrac{1}{\varepsilon}\right)(0,191827\ \left(\tfrac{1}{a}+\tfrac{1}{\varepsilon}\right)^2 + \tfrac{\nu}{a\varepsilon})$$

Si eiusmodi vitro vtamur, pro quo est $n = 1,60 = \frac{8}{5}$ tum ob $\nu = \frac{4}{15}$, prodiret $\lambda^\circ = 0,183333$ ideoque haec vitri species adhuc minorem confusionem pareret.

Coroll.

Coroll. 1.

115. Si pro iisdem distantiis determinatricibus a et $\mathfrak{b}$ lens simplex minimam diffusionem pariens construatur quod fit sumendo

radium faciei $\begin{cases} \text{anterioris} = \frac{a\mathfrak{b}}{\varrho\mathfrak{b}+\sigma a} \\ \text{posterioris} = \frac{a\mathfrak{b}}{\varrho a+\sigma\mathfrak{b}} \end{cases}$

spatium diffusionis foret

$$\mu\,\mathfrak{b}\mathfrak{b}\,xx\left(\tfrac{1}{a}+\tfrac{1}{\mathfrak{b}}\right)\left(\left(\tfrac{1}{a}+\tfrac{1}{\mathfrak{b}}\right)^2+\tfrac{\nu}{a\mathfrak{b}}\right).$$

Coroll. 2.

116. Apparet ergo a lente duplicata descripta multo minorem oriri diffusionem, quam a lente simplici, etiamsi haec iam ad minimam diffusionem sit instructa: Cum enim ceterae partes sint pares coefficiens membri $\left(\frac{1}{a}+\frac{1}{\mathfrak{b}}\right)^2$ plus quam quintuplo minor est in duplicata quam in simplici.

Coroll. 3.

117. Si ponamus $\lambda = 1$ et $\lambda' = 1$, vel saltem $\lambda' = \lambda$, minor valor pro λ° obtineri nequit, quam inuenimus, etiamsi pro f alios valores admittere velimus. Vnde si vtraque lens per se iam minimam diffusionem pariat, pro lente duplicata valor ipsius λ° minor quam $0,191827$ fieri nequit.

Scholion.

118. Huiusmodi ergo lentes duplicatae maxime sunt notatu dignae cum loco simplicium adhibitae multo

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multo minorem diffusionem pariant, ex quo in constructione Telescopiorum et Microscopiorum earum amplissimus erit vsus. Neque vero hac insigni proprietate sunt praeditae, sed etiam earum constructio in praxi minimis difficultatibus est obnoxia: propterea quod etsi a praescriptis regulis parumper aberretur, effectus tamen inde vix vllam mutationem patiatur. Siue enim in constructione vtriusque seorsim leuis error committatur, valores numerorum λ et λ' vnitatem haud sensibiliter excedent, siue in quantitate f valor iustus $f = \frac{1}{2}$ non exacte obseruetur, error vix sentietur, quoniam hi numeri ex natura minimi sunt eruti. Quomodocunque autem a regulis praescriptis aberretur, valor ipsius λ° inde pauliſper maior prodibit. Veluti si eiusmodi errores committantur, vt sit $\lambda = 1 + \frac{1}{10}$; $\lambda' = 1 + \frac{1}{10}$ et $f = \frac{1}{2} + \frac{1}{20}$; prodibit $\lambda^\circ = 0,191827 + 0,02558$ seu $\lambda^\circ = 0,2174$, ita vt discrimen partem tantum quadragesimam vnitatis conficiat. Facile autem intelligitur, dummodo λ° prodeat minus quam $\frac{1}{4}$, quod nunquam non, facile praestari posse videtur, ab his lentibus duplicatis insignem vtilitatem expectandam esse. Etsi ergo eiusmodi lentes duplicatae confici possunt, pro quibus numerus λ° plane euanescat, ob harum lubricam constructionem illae istis anteferendae videntur, vt mox clarius patebit.

Problema 4.

119. Pro datis distantiis determinatricibus $AE = a$ et $bG = c$ eas lentes duplicatas inuenire, in quibus sit $\lambda^\circ = 0$.

Solutio.

Cum fieri nequeat $\lambda^0 = 0$, nisi numerus f extra limites 0 et 1 accipiatur, simulque numeri λ et λ' fuerint inaequales, ita vt alterutra saltem lens non debeat seorsim minimam diffusionem parere; ponamus esse vel $f > 1$ vel $f < 0$. Sit ergo primo $f = 1 + \xi$, et cum sit

$$\lambda^0 = \lambda(1+\xi)^3 - \lambda'\xi^3 + \nu\xi(1+\xi)$$

vt fiat $\lambda^0 = 0$ oportet esse:

$$\lambda' = \lambda(1+\tfrac{1}{\xi})^3 + \tfrac{\nu(1+\xi)}{\xi^2}$$

vnde λ' necessario vnitatem superabit, cuius valor ne prodeat nimis magnus, sumi conueniet $\lambda = 1$, ita vt sit

$$\lambda' = (1+\tfrac{1}{\xi})^3 + \tfrac{0,252602(1+\xi)}{\xi^2} \text{ et } \lambda = 1.$$

Quicunque ergo valor ipsi ξ tribuatur, lens duplicata habetur, pro qua sit $\lambda^0 = 0$, ac propterea spatium diffusionis $= \mu\,\mathfrak{G}\mathfrak{G}xx(\tfrac{1}{a}+\tfrac{1}{\mathfrak{G}})\tfrac{\nu}{a\mathfrak{G}}$. Videamus nonnullos casus speciales.

$f = \frac{1}{2}$; $\xi = \frac{1}{2}$; $\lambda = 1$ et $\lambda' = 28,396152$

$f = 2$; $\xi = 1$; $\lambda = 1$ et $\lambda' = 8,465384$

$f = 3$; $\xi = 2$; $\lambda = 1$ et $\lambda' = 3,549519$

$f = 4$; $\xi = 3$; $\lambda = 1$ et $\lambda' = 2,473789$

$f = 5$; $\xi = 4$; $\lambda = 1$ et $\lambda' = 2,025841$

$f = 6$; $\xi = 5$; $\lambda = 1$ et $\lambda' = 1,783846$

etc.

Pro

Pro altero casu sit $f = -\xi$, ideoque

$$\lambda^6 = -\lambda\xi^6 + \lambda'(1+\xi)^3 + \nu\xi(1+\xi)$$

vnde facto $\lambda^\circ = 0$ prodit

$$\lambda = \lambda'(1+\tfrac{1}{\xi})^3 + \tfrac{\nu(1+\xi)}{\xi^5}.$$

Statui ergo conueniet $\lambda' = 1$, ac pro λ notentur casus sequentes.

$f = -\frac{1}{2}$; $\xi = \frac{1}{2}$; $\lambda = 28,396152$ et $\lambda' = 1$

$f = -1$; $\xi = 1$; $\lambda = 8,465384$ et $\lambda' = 1$

$f = -2$; $\xi = 2$; $\lambda = 3,549519$ et $\lambda' = 1$

$f = -3$; $\xi = 3$; $\lambda = 2,473789$ et $\lambda' = 1$

$f = -4$; $\xi = 4$; $\lambda = 2,025841$ et $\lambda' = 1$

$f = -5$; $\xi = 5$; $\lambda = 1,783846$ et $\lambda' = 1$

ficque patet infinitis modis huiusmodi lentes duplicatus parari posse, pro quibus sit $\lambda^\circ = 0$, et spatium diffusionis

$$Gg = \mu\mathfrak{G}\mathfrak{G}xx\left(\tfrac{1}{a}+\tfrac{1}{\mathfrak{a}}\right)\tfrac{\nu}{a\mathfrak{a}}.$$

Scholion 1.

128. Si huiusmodi lentes accuratissime parari possent, nullum est dubium, quin praecedentibus essent anteferendae propterea quod diffusio iis adhuc magis diminuitur. Verum dolendum est, quod minimus error in earum constructione commissus omnem

fere vsum destruat. Quo hoc facilius diiudicare queamus, examinemus eum casum, quo est

$$f=5,\ \lambda=1 \text{ et } \lambda'=2,025841$$

hincque

$$\lambda^\circ=\lambda f^2-\lambda'(f-1)^2+\nu f(f-1)=0.$$

Ponamus autem in constructione errorem esse commissum vt reuera non sit $f=5$ sed $f=5\frac{1}{10}$, dum numeri λ et λ' suos iustos valores obtineant: ob hunc autem vix vitandum errorem non fit $\lambda^\circ=0$, sed adeo $\lambda^\circ=-2,011$; sicque haec lens duplicata simplicibus longe est postponenda, simili modo si f esset $=5$, sed vel λ vel λ' tantillum a praescripto valore aberraret, enorme statim discrimen in valorem ipsius λ° redundaret. Minus quidem error metuendus videtur in ea specie, qua $\lambda=1$, $\lambda'=28,396152$ et $f=\frac{3}{2}$; sed praeterquam, quod lens simplex posterior difficillime parari queat, pro qua λ' praecise valorem assignatum consequatur, talis lens ad vsum dioptricum plane est inepta, ob ingentem alterius faciei curuaturam. Quae cum ita sint, quia tam exiguus error in paratione huiusmodi lentium commissus facit, vt λ° adeo supra vnitatem excrescat, vix sperare poterimus, vt vnquam talis lens duplicata perficiatur, pro qua λ° vsque ad $\frac{1}{2}$ diminuatur. In lentibus autem praecedentis generis successus vix fallere poterit, nisi in praxi enormiter a praescripta regula aberretur: ex quo his solis lentibus duplicatis cum fructu vti licebit, dum

contra

contra eae, quas in praesente problemate descripsimus, penitus profligandae videntur.

Scholion 2.

121. Pro datis ergo binis distantiis determinatricibus a et $\mathfrak{b}$ semper eiusmodi lens duplicata parari potest, ex qua nascatur spatium diffusionis

$$Gg = \mu\xi\mathfrak{b}xx\left(\tfrac{1}{a}+\tfrac{1}{\mathfrak{b}}\right)\left(\lambda^{0}\left(\tfrac{1}{a}+\tfrac{1}{\mathfrak{b}}\right)^{2}+\tfrac{\nu}{a\mathfrak{b}}\right)$$

ita vt λ^{0} numerum quemcunque denotare possit. Ad hoc enim satisfieri oportet huic aequationi:

$$\lambda^{0} = \lambda f^{2}+\lambda'(1-f)^{2}-\nu f(1-f)$$

id quod semper fieri potest, cum f plane ab arbitrio nostro pendeat, et numeri λ et λ' tantum non vnitate minores accipi debeant. Definitis autem his tribus numeris λ, λ' et f ita, vt λ^{0} datum valorem obtineat, binae lentes simplices, ex quibus duplicata est componenda, secundum formulas §. 107 datas confici debent. Vbi quidem tenenda sunt ea, quae modo obseruauimus, in praxi eas lentes facillime obtineri, quando numeri λ et λ' vnitatem parum superant, f vero propemodum $\frac{1}{2}$ denotat, cum contra quo magis hi numeri ab istis terminis recedant, eo malus sit periculum ne effectus enormiter fallat. Ceterum cum diffusionem a) lente duplicata oriundam ad eandem formam reduxerimus, qua diffusio lentis simplicis exprimitur, inde id commodi consequimur; vt simili modo diffusionem a lentibus magis multiplicatis ortum definire valeamus.

Supplementum I.

De lentibus duplicatis.

Si pro lente anteriori ratio refractionis sit $n : 1$, pro lente posteriori vero alia ratio $n' : 1$ locum habeat, problemata hic tractata sequenti modo resolui poterunt

Pro Problemate I.

Si pro numeris ϱ, σ, τ, qui ex n oriuntur, quaerantur simili modo ex n' valores ϱ', σ', et τ' erunt

Pro Lente

P P radius faciei $\begin{cases} \text{anter.} = \frac{aa}{\varrho a + \sigma a \pm \tau(a+a)\sqrt{(\lambda-1)}} \\ \text{poster.} = \frac{aa}{\varrho a + \sigma a \mp \tau(a+a)\sqrt{(\lambda-1)}} \end{cases}$

Q Q radius faciei $\begin{cases} \text{anter.} = \frac{bb}{\varrho' b + \sigma' b \pm \tau'(b+b)\sqrt{(\lambda'-1)}} \\ \text{poster.} = \frac{bb}{\varrho' b + \sigma' b \mp \tau'(b+b)\sqrt{(\lambda'-1)}} \end{cases}$

et definitis simili modo valoribus μ', ν' ex ratione $n' : 1$ reperietur spatium diffusionis

$$\mathfrak{b}\,\mathfrak{b}\,x\,x \begin{cases} +\mu\left(\frac{1}{a}+\frac{1}{c}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{c}\right)^2+\frac{\nu}{ac}\right) \\ +\mu'\left(\frac{1}{b}+\frac{1}{\mathfrak{c}}\right)\left(\lambda'\left(\frac{1}{b}+\frac{1}{\mathfrak{c}}\right)^2+\frac{\nu'}{b\mathfrak{c}}\right) \end{cases}$$

reliqua manent vt in problemate.

Pro Problemate II.

Quoniam hic duae vitri species occurrunt, ponamus lentem simplicem quaesitam ex alio quocunque vitri genere parari, cuius ratio refractionis sit $n^\circ : 1$, vnde prodeant numeri μ° et ν°; superfluum autem foret, numeros ϱ°, σ° et τ° computari, quoniam radii

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dii facierum fiunt imaginarii, ita, vt eos exprimere non sit opus, et quoniam pro hac lente simplici aequiualente numerus λ° est introductus, erit huius lentis spatium diffusionis

$$\mathfrak{GG}xx.\mu^\circ\left(\tfrac{1}{a}+\tfrac{1}{\mathfrak{G}}\right)\left(\lambda^\circ\left(\tfrac{1}{a}+\tfrac{1}{\mathfrak{G}}\right)^2+\tfrac{\nu^\circ}{a\mathfrak{G}}\right)$$

quod vt cum spatio diffusionis lentis duplicatae aequale fiat; ponatur, vti in problemate est factum,

$$\tfrac{1}{\alpha}=\tfrac{f-1}{a}+\tfrac{f}{\mathfrak{G}} \text{ et } \tfrac{1}{\beta}=\tfrac{1-f}{a}-\tfrac{f}{\mathfrak{G}}$$

vt prodeat

$$\tfrac{1}{\alpha}+\tfrac{1}{\alpha}=f\left(\tfrac{1}{a}+\tfrac{1}{\mathfrak{G}}\right) \text{ et } \tfrac{1}{\beta}+\tfrac{1}{\mathfrak{G}}=(1-f)\left(\tfrac{1}{a}+\tfrac{1}{\mathfrak{G}}\right)$$

vnde peruenietur ad hanc aequationem

$$\mu f\left(\lambda f^2\left(\tfrac{1}{a}+\tfrac{1}{\mathfrak{G}}\right)^2+\nu\left(\tfrac{f-1}{a^2}+\tfrac{f}{a\mathfrak{G}}\right)\right)$$
$$+\mu'(1-f)\left(\lambda'.(1-f)^2\left(\tfrac{1}{a}+\tfrac{1}{\mathfrak{G}}\right)^2+\nu'\left(\tfrac{1-f}{a\mathfrak{G}}-\tfrac{f}{\mathfrak{G}^2}\right)\right)$$
$$=\mu^\circ\left(\lambda^\circ\left(\tfrac{1}{a}+\tfrac{1}{\mathfrak{G}}\right)^2+\tfrac{\nu^\circ}{a\mathfrak{G}}\right)$$

quae ita repraesentari potest

$$\left(\tfrac{1}{a}+\tfrac{1}{\mathfrak{G}}\right)^2\left(\mu\lambda f^3+\mu'\lambda'(1-f)^3\right)$$
$$+\left(\tfrac{f-1}{a}+\tfrac{f}{\mathfrak{G}}\right)\left(\tfrac{\mu\nu f}{a}-\tfrac{\mu'\nu'(1-f)}{\mathfrak{G}}\right)$$
$$=\mu^\circ\lambda^\circ\left(\tfrac{1}{a}+\tfrac{1}{\mathfrak{G}}\right)^2+\tfrac{\nu^\circ\mu^\circ}{a\mathfrak{G}}$$

Vnde

$$\lambda^\circ=\tfrac{\mu\lambda f^3+\mu'\lambda'(1-f)^3}{\mu^\circ}+$$
$$\tfrac{aa\mathfrak{G}\mathfrak{G}}{a\mathfrak{G}+\mathfrak{G}\mathfrak{G}}\left(\left(\tfrac{f-1}{a}+\tfrac{f}{\mathfrak{G}}\right)\left(\tfrac{\mu\nu f}{a}-\tfrac{\mu'\nu'(1-f)}{\mathfrak{G}}\right)-\tfrac{\nu^\circ\mu^\circ}{a\mathfrak{G}}\right)$$

Denique

Denique si ex ratione refractionis $n' : 1$ computentur numeri ϱ', σ' et τ', radii facierum lentis duplicatae erunt

Pro Lente

P P radius facici { anterioris $= \frac{\alpha\varepsilon}{(\varrho-\sigma(1-f))\varepsilon+\sigma f\alpha+\tau f(\alpha+\varepsilon)\sqrt{(\lambda-1)}}$

posterioris $= \frac{\alpha\varepsilon}{(\sigma-\varrho(1-f))\varepsilon+\varrho f\alpha+\tau f(\alpha+\varepsilon)\sqrt{(\lambda-1)}}$

Q Q radius faciei { anter. $= \frac{\alpha\varepsilon}{(\sigma'-\varrho' f)\alpha+\varrho'(1-f)\varepsilon+\tau'(1-f)(\alpha+\varepsilon)\sqrt{(\lambda-1)}}$

poster. $= \frac{\alpha\varepsilon}{(\varrho'-\sigma' f)\alpha+\sigma'(1-f)\varepsilon+\tau'(1-f)(\alpha+\varepsilon)\sqrt{\lambda'-1}}$

Ad Problema III.

Hoc problema non solum pro diuersa refractione n et n' hic generalius pertractabo, sed etiam rationem distantiae inter binas lentes habebo. Primum igitur vtramque lentem ad distantias determinatrices lentis duplicatae, a et ε, reuocabo, ponendo

$$\tfrac{1}{a}+\tfrac{1}{\alpha}=f(\tfrac{1}{a}+\tfrac{1}{\varepsilon}) \text{ et } \tfrac{1}{b}+\tfrac{1}{\varepsilon}=g(\tfrac{1}{a}+\tfrac{1}{\varepsilon})$$

vt sit

$$\tfrac{1}{\alpha}=\tfrac{f-1}{a}+\tfrac{f}{\varepsilon} \text{ et } \tfrac{1}{b}=\tfrac{g}{a}+\tfrac{g-1}{\varepsilon}$$

hincque

$$\alpha=\frac{a\varepsilon}{fa+(f-1)\varepsilon} \text{ et } b=\frac{a\varepsilon}{(g-1)a+g\varepsilon}$$

ideoque distantia lentium

$$\alpha+b=\frac{a\varepsilon(f+g-1)(a+\varepsilon)}{(fa+(f-1)\varepsilon)((g-1)a+g\varepsilon)}$$

quae si deberet esse $=0$, capi oporteret $g=1-f$ sed si distantiam aliquam inter lentes admittamus, sta-

tuamus

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tuamus $f+g-1=\omega$, denotante ω fractionem quandam minimam, siue positiuam siue negatiuam vt distantia lentium prodeat positiua et valde parua. Cum hinc igitur sit $g=1+\omega-f$; erit lentium distantia

$$\alpha+b=\frac{\alpha\varepsilon(\alpha+\varepsilon)\omega}{(f\alpha+(f-1)\varepsilon)((\omega-f)\alpha+(1+\omega-f)\varepsilon)}$$

Spatium autem diffusionis nunc ita exprimetur:

$$\varepsilon\varepsilon xx.\left(\tfrac{1}{\alpha}+\tfrac{1}{\varepsilon}\right)\begin{cases}+\lambda\mu f^3(\frac{1}{\alpha}+\frac{1}{\varepsilon})^2+\lambda'\mu' g^3(\frac{1}{\alpha}+\frac{1}{\varepsilon})^2\\+\frac{\mu\nu f}{\alpha}(\frac{f-1}{\alpha}+\frac{f}{\varepsilon})\\+\frac{\mu'\nu' g}{\varepsilon}(\frac{g}{\alpha}+\frac{g-1}{\varepsilon})\end{cases}$$

quae cum in hoc problemate ita tractari debeat, vt tam f, quam g positiue sumantur, et haec formula minima reddatur; euidens est, litteris λ et λ' minimos valores tribui debere, scilicet $\lambda=1$ et $\lambda'=1$; deinde pro hoc casu minimi f et g conuenienter definiantur, vbi quia $f+g-1=\omega$ ideoque constans in differentiatione, habebimus $dg=-df$; vndeobtinebimus hanc aequationem

$$(3\mu f^2-3\mu' g^2)(\tfrac{1}{\alpha}+\tfrac{1}{\varepsilon})^2+\frac{\mu\nu}{\alpha}(\frac{2f-1}{\alpha}+\frac{2f}{\varepsilon})$$
$$-\frac{\mu'\nu'}{\varepsilon}(\frac{2g}{\alpha}+\frac{2g-1}{\varepsilon})=0$$

cui proxime satisfit, ponendo $f=g=\frac{1+\omega}{2}$ quibus valoribus substitutis spatium diffusionis ipsum minimum, erit proxime

$$\frac{\varepsilon^2 x^2(1+\omega)}{4}(\tfrac{1}{\alpha}+\tfrac{1}{\varepsilon})\begin{cases}(\frac{1}{\alpha}+\frac{1}{\varepsilon})^2(1+\omega)^2(\mu+\mu')\\+\frac{2\mu\nu}{\alpha}(\frac{\omega-1}{\alpha}+\frac{1+\omega}{\varepsilon})\\+\frac{2\mu'\nu'}{\varepsilon}(\frac{1+\omega}{\alpha}+\frac{\omega-1}{\varepsilon})\end{cases}$$

Tum vero distantia lentium erit

$$\alpha+b=\frac{4\alpha\beta(\alpha+\beta)\omega}{((1+\omega)\alpha+(\omega-1)\beta)((\omega-1)\alpha+(1+\omega)\beta)}$$
$$=\frac{4\alpha\beta(\alpha+\beta)\omega}{-(1-\omega)(\alpha+\beta)+2(1+\omega)\alpha\beta}$$

cuius denominator cum sit negatiuus ob ω minimum; necesse est, fractionem ω sumi debere negatiuam; hincque adeo spatium diffusionis minus reddetur.

Coroll.

Si ergo distantia obiecti α fuerit infinita seu $\alpha=\infty$, habebitur primo distantia lentium $=\frac{4\beta\omega}{1-\omega^2}$; et secundo spatium diffusionis

$$\frac{\varkappa^2(1+\omega)}{4\beta}\left\{\begin{array}{l}(1+\omega)^2(\mu+\mu')\\-2\mu'\nu'(1-\omega)\end{array}\right.$$

quod cum ω debeat esse negatiuum, non mediocriter minus erit, quam si distantia lentium esset nulla.

Ad Problema IV.

In hoc problemate etiam distantiam lentium non negligamus; factaque reductione, vt ante, statuamus spatium diffusionis plane euanescens; id quod fieri nequit, nisi altera litterarum f et g sit negatiua, quod cùm etiam fiat, quando confusio a diuersa radiorum refrangibilitate oriunda ad nihilum redigi debet, vt infra videbimus; hic casus multo magis euolutionem meretur. Ponatur igitur $g=-\zeta f$, vbi ζ ex illa

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illa conditione determinatur, vt primo pro distantia lentium sit

$$\alpha+b=\frac{-\alpha\beta(\alpha+\beta)(f-\zeta f-1)}{(f\alpha+(f-1)\beta)((\zeta f+1)\alpha+\zeta f\beta)}$$

et posito $f+g-1=\omega$, prodeat $f-\zeta f=1+\omega$ ideoque

$$f=\frac{1+\omega}{1-\zeta} \text{ et } g=\frac{-\zeta(1+\omega)}{1-\zeta}$$ sicque

$$\alpha+b=\frac{-\alpha\beta(\alpha+\beta)\omega(1-\zeta)^2}{((1+\omega)\alpha+(\omega+\zeta)\beta)((1+\omega\zeta)\alpha+\zeta(1+\omega)\beta)}$$

ac si in denominatore ω reiiciatur, erit

$$\alpha+b=\frac{-\alpha\beta(\alpha+\beta)(1-\zeta)^2\omega}{(\alpha+\zeta\beta)}$$

sicque patet ω negatiue capi debere.

Posito autem $g=-\zeta f$, fit spatium diffusionis

$$\beta^2 x^2\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)\begin{cases}(\lambda\mu f^3-\lambda'\mu'\zeta^3 f^3)\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)^2\\ +\frac{\mu\nu f}{\alpha}\left(\frac{f-1}{\alpha}+\frac{f}{\beta}\right)\\ +\frac{\mu'\nu\zeta f}{\beta}\left(\frac{-\zeta f}{\alpha}-\frac{(\zeta f+1)}{\beta}\right)\end{cases}$$

ad nihilum redigendum; vnde sequitur fore

$$\lambda'=\frac{\lambda\mu}{\mu'\zeta^3}+\frac{\mu\nu.\alpha.\beta^2}{\mu'\zeta^3 f^2(\alpha+\beta)^2}\left(\frac{f-1}{\alpha}+\frac{f}{\beta}\right)$$
$$+\frac{\nu.\alpha^2\beta}{\zeta^2 f^2.(\alpha+\beta)^2}\left(+\frac{\zeta f}{\alpha}+\frac{\zeta f+1}{\beta}\right)$$

qui valor cum debeat esse maior vnitate, si forte eueniat, vt minor prodeat, tunc non λ' sed λ definiri conueniet, vbi notandum est esse $f=\frac{1+\omega}{1-\zeta}$. Hincque per formulas ante datas facile eruuntur radii facierum vtriusque lentis.

Etsi formula superius data pro spatio diffusionis iam ad casum, quo distantia lentium est nulla, est

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adcommodata; tamen quia hic distantiam minimam assumimus, nullus inde error est metuendus.

Definitio 2.

122. *Lens triplicata est, quae consistit ex tribus lentibus simplicibus sibi immediate iunctis ad communem axem.*

Hic quidem etiam crassitiem negligo, etiamsi necessario maior sit quam in lentibus duplicatis. In suplemento autem ostendetur, quomodo etiam distantiarum inter lentes ratio sit habenda.

Corollarium.

123. Potest ergo lens triplicata considerari, quasi composita ex lente duplicata et lente simplici, hocque duplici modo, prout vel binae anteriores, vel binae posteriores lentem duplicatam constituere concipiuntur.

Scholion.

124. Lentibus triplicatis tum demum vsum in praxi concedi conueniet, cum eadem commoda per lentes simplices vel duplicatas consequi non licet. Haec autem commoda in paruitate numeri λ consistunt, qui quamdiu vnitate fuerit maior, semper lente simplici vti praestat, cum vero circumstantiae in expressione diffusionis minorem numerum λ requirunt, ad lentes multiplicatas erit confugiendum. Quoniam igitur eiusmodi lentes duplicatas conficere docuimus, in

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in quibus valor ipsius λ non solum ad nihilum vsque, sed adeo ad negatiua diminui queat, vsus lentium triplicatarum superfluus videtur. Verum iam obseruauimus in praxi aegre eiusmodi duplicatas lentes parari posse, pro quibus valor ipsius λ minor sit quam 0,191827, propterea quod si leuissimus error committatur, omnis labor irritus reddatur. His igitur casibus imprimis, quando minori valore numeri λ opus est, lentes triplicatae in vsum erunt vocandae: et quia respectu ad praxin habito non omnes aequo successu construere licet, errores ineuitabiles hic quoque imprimis spectari oportet, vt pateat quousque numerus λ cum successu diminui queat, ac si adhuc minore valore opus fuerit lentes adeo quadruplicate erunt inducendae.

Problema 5.

125. Datis binis distantiis determinatricibus omnes lentes triplicatas definire, simulque spatium diffusionis ab iis ortum. Tab. II. Fig. 8.

Solutio.

Sit Ee obiectum, cuius magnitudo $= z$, et distantia a lente $AE = a$: tum primae lentis PP distantiae determinatrices sint α et α, secundae lentis QQ β et $\mathfrak{G}$; ac tertiae RR, γ et γ. His positis quia lentes immediate iunctae sumuntur, erit $\alpha + \beta = 0$, $\mathfrak{G} + \gamma = 0$, et α et γ distantiae determinatrices lentis triplicatae, ita vt α et $\mathfrak{G}$ arbitrio nostro relinquantur.

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Si igitur lens prima sola PP adesset, imago repraesentaretur in $F\zeta$, vt esset $AF = a$ et $F\zeta = \frac{\alpha}{a} z$: si binae priores PP et QQ solae adessent imago exhiberetur in $G\eta$, vt esset $AG = \mathfrak{b}$ et $G\eta = \frac{\mathfrak{c}}{a} z$; at per lentem triplicatam referetur in $H\theta$ vt sit $cH = \gamma$ et $H\theta = \frac{\gamma}{a} z$, pro situ inuerso ob $\frac{\alpha}{b}\frac{\mathfrak{c}}{c} = 1$. Hactenus scilicet res perinde se habet, ac si in A haberetur lens simplex ad distantias determinatrices a et γ accommodata.

At si ad spatium diffusionis Hb respiciamus figuram singularum lentium in computum ducere debemus, quatenus praeter distantias determinatrices numeri arbitrarii $\lambda, \lambda', \lambda''$ inuoluuntur; ex quibus facies singularum lentium supra §. 91 sunt definitae. Indidem autem colligitur pro apertura cuius semidiameter $= x$, spatium diffusionis ob $\frac{\alpha}{b} = -1$ et $\frac{\mathfrak{c}}{c} = -1$ fore:

$$Hb = \mu\gamma\gamma xx \begin{cases} + (\frac{1}{a}+\frac{1}{\alpha})(\lambda(\frac{1}{a}+\frac{1}{\alpha})^2 + \frac{\nu}{\alpha a}) \\ + (\frac{1}{b}+\frac{1}{\mathfrak{c}})(\lambda'(\frac{1}{b}+\frac{1}{\mathfrak{c}})^2 + \frac{\nu}{b\mathfrak{c}}) \\ + (\frac{1}{c}+\frac{1}{\gamma})(\lambda''(\frac{1}{c}+\frac{1}{\gamma})^2 + \frac{\nu}{c\gamma}) \end{cases}$$

quae expressio, vt ad formam vni lenti respondentem reducatur statuamus:

$$\frac{1}{a}+\frac{1}{\alpha} = f(\frac{1}{a}+\frac{1}{\gamma})$$
$$\frac{1}{b}+\frac{1}{\mathfrak{c}} = g(\frac{1}{a}+\frac{1}{\gamma})$$
$$\frac{1}{c}+\frac{1}{\gamma} = h(\frac{1}{a}+\frac{1}{\gamma})$$

et

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et quia $\frac{1}{\alpha}+\frac{1}{\beta}=0$, et $\frac{1}{\beta}+\frac{1}{\gamma}=0$, his aequationibus addendis adipiſcimur: $f+g+h=1$. Porro vero erit

$$\frac{1}{\alpha}=f\left(\frac{1}{a}+\frac{1}{\gamma}\right)-\frac{1}{a};\quad \frac{1}{b}=\frac{1}{a}-f\left(\frac{1}{a}+\frac{1}{\gamma}\right)$$

$$\frac{1}{\beta}=(f+g)\left(\frac{1}{a}+\frac{1}{\gamma}\right)-\frac{1}{a};\quad \frac{1}{c}=\frac{1}{a}-(f+g)\left(\frac{1}{a}+\frac{1}{\gamma}\right)$$

ſiue ob $1=f+g+h$

$$\frac{1}{\alpha}=\frac{f+g+h}{a};\quad \frac{1}{\alpha}=\frac{-g-h}{a}+\frac{f}{\gamma}$$

$$\frac{1}{b}=\frac{g+h}{a}-\frac{f}{\gamma};\quad \frac{1}{\beta}=-\frac{h}{a}+\frac{f+g}{\gamma}$$

$$\frac{1}{c}=\frac{h}{a}-\frac{f-g}{\gamma};\quad \frac{1}{\gamma}=\frac{f+g+h}{\gamma}$$

Vnde cum ſpatium diffuſionis fiat.

$$Hb=\mu\gamma\gamma xx\left(\frac{1}{a}+\frac{1}{\gamma}\right)\left(\lambda f^3+\lambda' g^3+\lambda'' h^3\right)\left(\frac{1}{a}+\frac{1}{\gamma}\right)^2+\nu\left(\frac{f}{a\alpha}+\frac{g}{b\beta}+\frac{h}{c\gamma}\right)$$

reducetur id ad hanc formam

$$Hb=\mu\gamma\gamma xx\left(\frac{1}{a}+\frac{1}{\gamma}\right)\left(\lambda f^3+\lambda' g^3+\lambda'' h^3-\nu(1-f)(1-g)(1-h)\right)\left(\frac{1}{a}+\frac{1}{\gamma}\right)^2+\frac{\nu}{a\gamma}\Big)$$

Radiorum autem in b concurrentium inclinatio ad axem erit $=\frac{x}{\gamma}$.

Coroll. 1.

126. Haec igitur lens triplicata idem producit ſpatium diffuſionis quod produceret lens ſimplex ad easdem diſtantias determinatrices inſtructa; numero eius arbitrario (per litteram λ indicato) exiſtente

$$\lambda f^3+\lambda' g^3+\lambda'' h^3-\nu(1-f)(1-g)(1-h)$$

vbi quidem eſt $f+g+h=1$.

Coroll.

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Coroll. 2.

127. Quatenus ergo haec quantitas reddi potest minor non solum vnitate; sed etiam fractione 0,191827 ita scilicet, vt praxis non enormi aberrationi sit exposita, eatenus lentibus triplicatis vsus erit concedendus.

Coroll. 3.

128. Si sit vel $f=0$, vel $g=0$, vel $h=0$, vna lentium habebit facies parallelas, et lens triplicata aequivalebit duplicatae ac si duae litterarum f,g,h simul euanescant, tertia in vnitatem abeunte, casus habebitur lentis simplicis.

Coroll. 4.

129. Si sit $f=1$, ideoque $h=-g$, valor numeri λ pro lente triplicata erit $\lambda+(\lambda'-\lambda'')g^3$, ideoque si $\lambda''=\lambda'$ lens triplicata simplici aequivalebit, quod idem euenit si fuerit vel $g=1$ vel $h=1$.

Coroll. 5.

130. Sumtis autem pro f,g,h numeris idoneis ob $f+g+h=1$, constructio lentis triplicatae ex formulis §. 91 exhibitis est petenda sumendo.

$$\frac{1}{\alpha}=\frac{-1+f}{g}+\frac{f}{\gamma};\quad \frac{1}{b}=\frac{1-f}{a}-\frac{f}{\gamma}$$

$$\frac{1}{\beta}=\frac{-b}{a}+\frac{1-b}{\gamma};\quad \frac{1}{c}=\frac{b}{a}-\frac{1+b}{\gamma}$$

Scholion

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Scholion 1

131. Quemadmodum hinc expressio inuenta pro casu, notandum est fore $\frac{f}{aa}+\frac{g}{b\mathfrak{b}}+\frac{h}{c\gamma}=$

$$+\tfrac{1}{aa}(-f(1-f)-gh(1-f))$$
$$+\tfrac{1}{a\gamma}(+ff+g(1-f)(1-h)+fgh+hh)$$
$$+\tfrac{1}{\gamma\gamma}(-fg(1-h)-h(1-h))$$

sed $-f(1-f)-gh(1-f)=-(1-f)(f+gh)=-(1-f)(1-g)(1-h)$ ob $f=1-g-h$ ideoque $f+gh=(1-g)(1-h)$. Simili modo pro $\frac{1}{\gamma\gamma}$ est $-fg(1-h)-h(1-h)=-(1-h)(h+fg)=-(1-f)(1-g)(1-h)$ ob $h=1-f-g$. Denique pro $\frac{1}{a\gamma}$, quia est $ff+hh=(f+h)^2-2fh=(1-g)^2-2fh=1-2g+gg-2fh$, hoc valore substituto coefficiens ipsius $\frac{1}{a\gamma}$ erit

$$1-2g-2fh+gg+fgh+g(1-f)(1-h)=$$
$$1+(g+fh)(g-2)-g(1-f)(1-h)=1-2(1-f)(1-g)(1-h)$$

ob $g+fh=(1-f)(1-h)$. Consequenter colligitur

$$\tfrac{f}{aa}+\tfrac{g}{b\mathfrak{b}}+\tfrac{h}{c\gamma}=-(1-f)(1-g)(1-h)(\tfrac{2}{a}+\tfrac{1}{\gamma})^2+\tfrac{4}{a\gamma}.$$

Scholion 2

132. Hoc problema etiam ope praecedentium facilius sequenti modo resolui potest. Considerentur scilicet binae lentes PP et QQ iunctim sumtae tanquam lens duplicata ad distantias determinatrices a et $\mathfrak{a}$ per numeros arbitrarios λ, λ' et f instructa, ac posito $\lambda f^4+\lambda'(1-f)^2-\nu f(1-f)=\lambda^{(2)}$, spatium diffusionis ex ea sola ortum erit

$$\mu\mathfrak{b}\mathfrak{b}xx(\tfrac{1}{a}+\tfrac{1}{\mathfrak{a}})(\lambda^{(2)}(\tfrac{1}{a}+\tfrac{1}{\mathfrak{a}})^2+\tfrac{\nu}{a\mathfrak{a}})$$

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quae lens si iam in compositione cum tertia RR tanquam simplex tractetur exinde elicitur spatium diffusionis perinde atque ex coniunctione duarum simplicium

$$Hb = \mu\gamma\gamma xx \begin{cases} +(\frac{1}{a}+\frac{1}{\varepsilon})(\lambda^{(2)}(\frac{1}{a}+\frac{1}{\varepsilon})^2+\frac{\nu}{c\varepsilon}) \\ +(\frac{1}{c}+\frac{1}{\gamma})(\lambda''(\frac{1}{c}+\frac{1}{\gamma})^2+\frac{\nu}{c\gamma}) \end{cases}$$

vbi notandum est esse $\varepsilon+c=0$, et constructio lentis duplicatae ex §. 107 erit petenda lentis vero simplicis RR ex distantiis determinatricibus $c=-\varepsilon$ et γ vna cum numero arbitrario λ''. Ponatur iam $\frac{1}{\varepsilon}=\frac{\xi-1}{a}+\frac{\xi}{\gamma}$ vt sit $\frac{1}{c}=\frac{1-\xi}{a}-\frac{\xi}{\gamma}$, eritque spatium diffusionis huius lentis triplicatae,

$$Hb=\mu\gamma\gamma xx(\tfrac{1}{a}+\tfrac{1}{\gamma})((\lambda^{(2)}g^2+\lambda''(1-g)^2-\nu g(1-g))(\tfrac{1}{a}+\tfrac{1}{\gamma})^2+\tfrac{\nu}{a\gamma})$$

Quare si pro lente triplicata ponatur

$$\lambda^{(2)}g^2+\lambda''(1-g)^2-\nu g(1-g)=\lambda^{(3)}$$

ita vt iam numerus g insuper arbitrio nostro relinquatur, habebitur spatium diffusionis more hactenus recepto expressum

$$Hb=\mu\gamma\gamma xx(\tfrac{1}{a}+\tfrac{1}{\gamma})(\lambda^{(3)}(\tfrac{1}{a}+\tfrac{1}{\gamma})^2+\tfrac{\nu}{a\gamma})$$

Hinc iam id intelligitur, quod ex praecedente solutione minus patet; si numerus $\lambda^{(2)}$ fuerit vnitate maior loco binarum priorum lentium PP et QQ commodius vnicam simplicem adhiberi; ex quo eatenus tantum lentes triplicatae resultare censendae sunt, quatenus

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numerus $\lambda^{(2)}$ vnitate est minor. Vidimus autem successum tuto sperari non posse, nisi $\lambda^{(2)}$ aequalis sit fractioni 0,191827, vel ea non multo maior; vnde si praxi consulere velimus, ipsi $\lambda^{(2)}$ minorem valorem tribui non conuenit atque ob eandem rationem numerus g intra terminos 0 et 1 accipi debebit; cuius valor imprimis ad praxin erit accommodatus si reddat numerum $\lambda^{(3)}$ minimum, quia tum leues errores negotium minime turbant. Quodsi vero pro $\lambda^{(2)}$ valorem assignatum substituamus, obtinebimus

$$\lambda^{(3)} = \lambda f^3 g^3 + \lambda' g^3 (1-f)^3 + \lambda''(1-g)^3 - \nu g^3 f(1-f) - \nu g(1-g)$$

Conducet autem vtramque expressionem pro numero $\lambda^{(3)}$, quo spatium diffusionis a lente triplicata ortum definitur, hic exposuisse cum aliae conclusiones ex altera facilius deducantur. Etsi autem hinc omnes valores pro $\lambda^{(3)}$ obtineri possunt tamen eos tantum, qui prope minimum subsistunt ad praxin adhiberi conueniet.

Problema 6.

133. Datis distantiis determinatricibus $AE = a$ et $cH = \gamma$ definire eam lentem triplicatam, quae minimum spatium diffusionis producat.

Solutio 1.

Duplici modo hoc problema solui potest, prout spatium diffusionis vel ita exprimitur, vti in solutione problematis praecedentis, vel in Scholio 2. Pri-

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ori modo numeros f, g, h ita determinari oportet, vt minima reddatur haec expreſſio:

$$\lambda f^3+\lambda' g^3+\lambda'' h^3-\nu(1-f)(1-g)(1-h)$$

vbi notandum eſt eſſe $f+g+h=1$. Eius ergo differentiali nihilo aequali poſito habebimus:

$$3\lambda ff df+3\lambda' gg dg+3\lambda'' hh dh$$
$$+\nu df(1-g)(1-h)+\nu dg(1-f)(1-h)+\nu dh(1-f)(1-g)=0.$$

Cum autem ſit $dh=-df-dg$ erit

$$\left.\begin{aligned}&+3\lambda ff df-3\lambda'' hh df+\nu df(1-g)(f-h)\\&+3\lambda' gg dg-3\lambda'' hh dg+\nu dg(1-f)(g-h)\end{aligned}\right\}=0$$

quia vero bina differentialia df et dg a ſe inuicem non pendent ambo membra huius aequationis ſeorſim euaneſcere debent, vnde ob $1-g=f+h$ et $1-f=g+h$ has duas nanciſcimur aequationes:

$$3\lambda ff-3\lambda'' hh+\nu ff-\nu hh=0$$
$$3\lambda' gg-3\lambda'' hh+\nu gg-\nu hh=0$$

ex quibus elicimus:

$$f=h\sqrt{\tfrac{3\lambda''+\nu}{3\lambda+\nu}} \text{ et } g=h\sqrt{\tfrac{3\lambda''+\nu}{3\lambda'+\nu}}.$$

Cum autem ſit

$$f+g+h=1 \text{ ſeu } \tfrac{1}{h}=1+\tfrac{f}{h}+\tfrac{g}{h} \text{ erit}$$

$$\tfrac{1}{h}=1+\sqrt{\tfrac{3\lambda''+\nu}{3\lambda+\nu}}+\sqrt{\tfrac{3\lambda''+\nu}{3\lambda'+\nu}}$$
$$\tfrac{1}{g}=1+\sqrt{\tfrac{3\lambda'+\nu}{3\lambda+\nu}}+\sqrt{\tfrac{3\lambda'+\nu}{3\lambda''+\nu}}$$
$$\tfrac{1}{f}=1+\sqrt{\tfrac{3\lambda+\nu}{3\lambda'+\nu}}+\sqrt{\tfrac{3\lambda+\nu}{3\lambda''+\nu}}$$

Quicun-

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Quicunque ergo numeri λ, λ', λ'' in constructione singularum lentium fuerint vsurpati, hinc numeri f, g, et b determinantur, ex quibus spatium diffusionis minimum resultet. Pro lentium autem constructione hinc distantiae α, b, $\mathfrak{C}$, c ita definiuntur, vt sit

$$\frac{1}{a}=\frac{-1+f}{a}+\frac{f}{\gamma}; \quad \frac{1}{b}=\frac{1-f}{a}-\frac{f}{\gamma}$$

$$\frac{1}{\mathfrak{C}}=\frac{-b}{a}+\frac{1-b}{\gamma}; \quad \frac{1}{c}=\frac{b}{a}-\frac{1+b}{\gamma}$$

est vero:

$$f=\frac{\sqrt{(3\lambda'+\nu)(3\lambda''+\nu)}}{\sqrt{(3\lambda+\nu)(3\lambda'+\nu)}+\sqrt{(3\lambda+\nu)(3\lambda''+\nu)}+\sqrt{(3\lambda'+\nu)(3\lambda''+\nu)}}$$

$$g=\frac{\sqrt{(3\lambda+\nu)(3\lambda''+\nu)}}{\sqrt{(3\lambda+\nu)(3\lambda'+\nu)}+\sqrt{(3\lambda+\nu)(3\lambda''+\nu)}+\sqrt{(3\lambda'+\nu)(3\lambda''+\nu)}}$$

$$b=\frac{\sqrt{(3\lambda+\nu)(3\lambda'+\nu)}}{\sqrt{(3\lambda+\nu)(3\lambda'+\nu)}+\sqrt{(3\lambda+\nu)(3\lambda''+\nu)}+\sqrt{(3\lambda'+\nu)(3\lambda''+\nu)}}$$

Ex distantiis autem a, α, b, $\mathfrak{C}$, c, γ cum numeris λ, λ', λ'' lentes ipsae per formulas §. 91 exhibitas construuntur.

Coroll. 1.

134. Si pro hac lente triplicata ponatur

$$\lambda f^3+\lambda' g^3+\lambda'' b^3-\nu(1-f)(1-g)(1-b)=\lambda^{(3)}$$

substituendis his valoribus pro f, g, et b reperietur

$$\lambda^{(3)}=\frac{1}{3\left(\frac{1}{\sqrt{(3\lambda+\nu)}}+\frac{1}{\sqrt{(3\lambda'+\nu)}}+\frac{1}{\sqrt{(3\lambda''+\nu)}}\right)^2}-\frac{1}{3}\nu$$

vnde spatium diffusionis fit

$$\mathfrak{H}b=\mu\gamma\gamma\pi\pi\left(\frac{1}{a}+\frac{1}{\gamma}\right)\left(\lambda^{(3)}\left(\frac{1}{a}+\frac{1}{\gamma}\right)^2+\frac{\nu}{a\gamma}\right)$$

Coroll. 2.

135. Hoc autem ſpatium diffuſionis omnium fiet minimum ſi numeris λ, λ', λ'' minimi valores, quos accipere poſſunt, tribuantur. Sit ergo $\lambda = 1$, $\lambda' = 1$, $\lambda'' = 1$, eritque

$$f = \tfrac{1}{3}; \quad g = \tfrac{1}{3}, \quad b = \tfrac{1}{3}$$

$$\text{et } \lambda^{(3)} = \tfrac{\nu+3}{27} - \tfrac{\nu}{3} = \tfrac{2-3\nu}{27} = 0,042165$$

ob $0,232692$; qui ergo valor multo eſt minor, quam caſu lentium duplicatarum.

Coroll. 3.

136. Hoc porro caſu erit $\frac{1}{\alpha} = -\frac{2}{3a} + \frac{1}{3\gamma}$; $\frac{1}{\beta} = \frac{2}{3a} - \frac{1}{3\gamma}$; $\frac{1}{\delta} = \frac{2}{3a} + \frac{2}{3\gamma}$ et $\frac{1}{\epsilon} = \frac{1}{3a} - \frac{2}{3\gamma}$; vnde conſtructio lentium ternarum ſimplicium ita ſe habebit.

Pro Lente

Prima radius faciei { anterioris $= \frac{3a\gamma}{(3\varrho - 2\sigma)\gamma + \sigma a}$; poſterioris $= \frac{3a\gamma}{(3\sigma - 2\varrho)\gamma + \varrho a}$

Secunda radius faciei { anterioris $= \frac{3a\gamma}{(2\varrho - \sigma)\gamma + (2\sigma - \varrho)a}$; poſterioris $= \frac{3a\gamma}{(2\sigma - \varrho)\gamma + (2\varrho - \sigma)a}$

Tertia radius faciei { anterioris $= \frac{3a\gamma}{\varrho\gamma + (3\sigma - 2\varrho)a}$; poſterioris $= \frac{3a\gamma}{\sigma\gamma + (3\varrho - 2\sigma)a}$

Solutio altera Problematis.

137. Conſideremus binas lentes priores PP et QQ vt lentem duplicatam ad diſtantias determinatrices α et ϵ ita

ita instructam vt posito $\lambda f^2 + \lambda'(1-f)^2 - \nu f(1-f) = \lambda^{(2)}$ spatium diffusionis, inde oriundum sit

$$= \mu\,\mathfrak{G}\,\alpha\alpha\left(\tfrac{1}{a}+\tfrac{1}{\beta}\right)\left(\lambda^{(2)}\left(\tfrac{1}{a}+\tfrac{1}{\beta}\right)^2+\tfrac{\nu}{a\beta}\right).$$

sed pro constructione binarum lentium simplicium, ex quibus haec lens est composita, recordandum est esse

$$\tfrac{1}{a} = \tfrac{1+f}{a} + \tfrac{f}{\alpha} \text{ et } \tfrac{1}{\beta} = \tfrac{1-f}{a} - \tfrac{f}{\alpha}.$$

Adiuncta iam tertia lente RR ad distantias determinatrices $c = -\mathfrak{G}$ et γ per numerum arbitrarium λ'' instructa si ponamus $\tfrac{1}{\alpha} = \tfrac{1+g}{a} + \tfrac{g}{\gamma} = -\tfrac{1}{\mathfrak{G}}$, et

$$\lambda^{(3)} g^2 + \lambda''(1-g)^2 - \nu g(1-g) = \lambda^{(3)}$$

erit spatium diffusionis

$$\mathfrak{H}\,b = \mu\,\gamma\gamma\,\alpha\alpha\left(\tfrac{1}{a}+\tfrac{1}{\gamma}\right)\left(\lambda^{(3)}\left(\tfrac{1}{a}+\tfrac{1}{\gamma}\right)^2+\tfrac{\nu}{a\gamma}\right)$$

quod vt fiat minimum, valor ipsius $\lambda^{(3)}$ minimus reddi debet, quaeratur ergo primo g dum $\lambda^{(3)}$ vt numerus datus spectatur et habebimus

$$3\lambda^{(2)} g g - 3\lambda''(1-g)^2 - \nu + 2\nu g = 0$$

vnde elicitur

$$g = \frac{-3\lambda'' - \nu + \sqrt{(3\lambda^{(2)}+\nu)(3\lambda''+\nu)}}{3\lambda^{(2)} - 3\lambda''} \quad \text{siue}$$

$$g = \frac{\sqrt{(3\lambda''+\nu)}}{\sqrt{(3\lambda^{(2)}+\nu)} + \sqrt{(3\lambda''+\nu)}}$$

et hinc valor ipsius $\lambda^{(3)}$ erit

$$\lambda^{(3)} = \frac{1}{3\left(\frac{1}{\sqrt{(3\lambda^{(2)}+\nu)}} + \frac{1}{\sqrt{(3\lambda''+\nu)}}\right)^2} - \frac{1}{3}\nu$$

vel

vel

$$\frac{1}{3\lambda^{(3)}+\nu}=\left(\frac{1}{\sqrt{(3\lambda^{(2)}+\nu)}}+\frac{1}{\sqrt{(3\lambda''+\nu)}}\right)^2$$

Sicque fit:

$$\frac{1}{\sqrt{(3\lambda^{(3)}+\nu)}}=\frac{1}{\sqrt{(3\lambda^{(2)}+\nu)}}+\frac{1}{\sqrt{(3\lambda''+\nu)}}$$

Simili modo si f ita definiatur, vt $\lambda^{(2)}$ fiat minimum reperietur:

$$f=\frac{\sqrt{(3\lambda'+\nu)}}{\sqrt{(3\lambda+\nu)}+\sqrt{(3\lambda'+\nu)}}$$

hocque valore substituto

$$\frac{1}{\sqrt{(3\lambda^{(2)}+\nu)}}=\frac{1}{\sqrt{(3\lambda+\nu)}}+\frac{1}{\sqrt{(3\lambda'+\nu)}}$$

Quare si numeris $\lambda, \lambda', \lambda''$, arbitrio nostro relictis bini numeri f et g ita definiantur, vt $\lambda^{(3)}$ consequatur valorem minimum erit

$$\frac{1}{\sqrt{(3\lambda^{(3)}+\nu)}}=\frac{1}{\sqrt{(3\lambda+\nu)}}+\frac{1}{\sqrt{(3\lambda'+\nu)}}+\frac{1}{\sqrt{(3\lambda''+\nu)}}$$

vnde idem valor pro $\lambda^{(3)}$ reperitur, quem ante inuenimus.

Coroll. 1.

138. Ex hac ergo solutione numeri f et g ita definiuntur vt sit

$$\frac{1}{f\sqrt{(3\lambda+\nu)}}=\frac{1}{\sqrt{(3\lambda+\nu)}}+\frac{1}{\sqrt{(3\lambda'+\nu)}} \quad \text{et}$$

$$\frac{1}{g\sqrt{(3\lambda^{(2)}+\nu)}}=\frac{1}{\sqrt{(3\lambda^{(3)}+\nu)}}+\frac{1}{\sqrt{(3\lambda''+\nu)}}$$

siue

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sive hoc modo

$$\left(\frac{1}{f}-1\right)\frac{1}{\sqrt{(3\lambda+\nu)}}=\frac{1}{\sqrt{(3\lambda'+\nu)}} \text{ et}$$

$$\left(\frac{1}{g}-1\right)\left(\frac{1}{\sqrt{(3\lambda+\nu)}}+\frac{1}{\sqrt{(3\lambda'+\nu)}}\right)=\frac{1}{\sqrt{(3\lambda''+\nu)}}$$

Coroll. 2.

139. Ex inuentis minimis valoribus numerorum $\lambda^{(2)}$ et $\lambda^{(3)}$ numeri f et g etiam ita definiuntur vt sit

$$\frac{1}{f\sqrt{(3\lambda+\nu)}}=\frac{1}{\sqrt{(3\lambda^{(2)}+\nu)}} \text{ seu } f=\sqrt{\left(\frac{3\lambda^{(2)}+\nu}{3\lambda+\nu}\right)} \text{ et}$$

$$\frac{1}{g\sqrt{(3\lambda^{(2)}+\nu)}}=\frac{1}{\sqrt{(3\lambda^{(3)}+\nu)}} \text{ seu } g=\sqrt{\left(\frac{3\lambda^{(3)}+\nu}{3\lambda^{(2)}+\nu}\right)}$$

Coroll. 3

140. Distantiae autem determinatrices singularum lentium ita per a et γ prodibunt expressae:

$$\frac{1}{a}=-\frac{1}{b}=\frac{=1+fg}{a}+\frac{fg}{\gamma}$$

$$\frac{1}{e}=-\frac{1}{c}=\frac{=1+g}{a}+\frac{g}{\gamma}$$

vbi cum sit

$$g=\sqrt{\frac{3\lambda^{(3)}+\nu}{3\lambda^{(2)}+\nu}} \text{ notandum est esse } fg=\sqrt{\frac{3\lambda^{(3)}+\nu}{3\lambda+\nu}}.$$

Coroll. 4.

141. Eliminando autem numero $\lambda^{(2)}$ erit

$$\frac{1}{a}=-\frac{1}{b}=-\frac{1}{a}\sqrt{\frac{3\lambda^{(3)}+\nu}{3\lambda'+\nu}}-\frac{1}{a}\sqrt{\frac{3\lambda^{(3)}+\nu}{3\lambda''+\nu}}+\frac{1}{\gamma}\sqrt{\frac{3\lambda^{(3)}+\nu}{3\lambda+\nu}}$$

$$\frac{1}{e}=-\frac{1}{c}=-\frac{1}{a}\sqrt{\frac{3\lambda^{(3)}+\nu}{3\lambda''+\nu}}+\frac{1}{\gamma}\sqrt{\frac{3\lambda^{(3)}+\nu}{3\lambda+\nu}}+\frac{1}{\gamma}\sqrt{\frac{3\lambda^{(3)}+\nu}{3\lambda'+\nu}}$$

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ex quibus formulis, inuento iam valore minimo $\lambda^{(s)}$ singulae lentes commodissime determinantur.

Coroll. 5.

142. Si praeterea singulae lentes ita fuerint comparatae vt per se minimam confusionem pariant quod fit si $\lambda = \lambda' = \lambda'' = 1$, erit

$$\frac{1}{\sqrt{(3\lambda^{(2)}+\nu)}} = \frac{2}{\sqrt{(3+\nu)}} \text{ hincque } 3\lambda^{(2)}+\nu = \frac{3+\nu}{4}$$

vnde fit $\lambda^{(2)} = \frac{1-\nu}{4}$. Deinde vero habebitur:

$$\frac{1}{\sqrt{(3\lambda^{(3)}+\nu)}} = \frac{3}{\sqrt{(3+\nu)}} \text{ hincque } 3\lambda^{(3)}+\nu = \frac{3+\nu}{9}$$

ac propterea

$\lambda^{(3)} = \frac{3-4\nu}{27}$. Sin autem fuerit tantum $\lambda = \lambda' = \lambda''$ reperietur:

$$\lambda^{(2)} = \frac{2\lambda-3\nu}{12} \text{ et } \lambda^{(3)} = \frac{3\lambda-4\nu}{27}.$$

Coroll. 6.

143. Eodem autem casu, quo $\lambda = \lambda' = \lambda''$, ob $\sqrt{(3\lambda^{(3)}+\nu)} = \frac{1}{3}\sqrt{(3\lambda+\nu)}$, distantiae determinatrices pro lentibus simplicibus erunt:

$$\frac{1}{a} = \frac{-2}{3a} + \frac{8}{3\gamma}; \quad \frac{1}{b} = \frac{2}{3a} - \frac{1}{3\gamma}$$

$$\frac{1}{c} = \frac{-1}{3a} + \frac{2}{3\gamma}; \quad \frac{1}{c} = \frac{1}{3a} - \frac{2}{3\gamma}$$

vnde eaedem formulae pro earum constructione nascuntur quae supra (136) sunt allatae nisi quod iam in denominatoribus membra $\pm\tau(a+\gamma)\sqrt{(\lambda-1)}$ adiungi debeant. Scholi-

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Scholion.

144. Ternarum ergo lentium ſimplicium idonea coniunctione effici poteſt, vt in expreſſione ſpatii diffuſionis $H\,b = \mu\,\gamma\,\gamma\,x\,x\,(\frac{1}{a}+\frac{1}{\gamma})\,(\lambda^{(2)}\,(\frac{1}{a}+\frac{1}{\gamma})^2+\frac{\nu}{a\gamma})$ numerus $\lambda^{(3)}$ fiat $=0,042165$. In lentibus autem duplicatis vidimus minimum valorem numeri $\lambda^{(2)}$ eſſe $=0,191827$; ſicque in triplicatis hic numerus fere quinquies minor reddi poteſt at is fere vicies quater eſt minor, quam per lentes ſimplices obtineri poteſt. Loquor hic autem de lentibus ex principio minimi petitis, quippe quae ad praxin maxime ſunt accommodatae, dum conſtructio leuibus erroribus non admodum turbatur. Quanquam enim lentes triplicatae perinde ac duplicatae parari poſſent, pro quibus numerus conueniens λ non ſolum nihilo aequalis, ſed etiam negatiuus reſultaret tamen earum conſtructio tam eſt lubrica, vt minimus error totum laborem irritum reddat. Interim tamen periculum in triplicatis non tantum eſt quam duplicatis, vnde ſequens problema ſoluiſſe operae erit pretium.

Problema 7.

145. Pro datis diſtantiis determinatricibus a et γ eas definire lentes triplicatas, pro quibus valor ipſius $\lambda^{(1)}$ prorſus in nihilum abeat.

Solutio.

Conſideretur numerus $\lambda^{(2)}$ ex duabus prioribus

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lentibus natus vt datus, et cum sit secundum solutionem posteriorem praecedentis problematis

$$\lambda^{(3)} = \lambda^{(2)} g^3 + \lambda^{II}(1-g)^2 - \nu g(1-g)$$

definiri debet g ita vt ista quantitas euanescat. Verum cum $\lambda^{(2)}$ commode nequeat minor effici quam $\frac{1-\nu}{4}$, statuamus $\lambda^{(2)} = \frac{1-\nu}{4}$, fierique oportet:

$$0 = \tfrac{1}{4} g^3 + \lambda^{II}(1-g)^2 - \nu g(1-\tfrac{1}{8}g)^2$$

sed quia λ^{II} vnitate minor esse nequit, necesse est vt g capiatur vnitate maior euoluantur ergo quidam casus.

I. $g = \frac{5}{4}$; $0 = \frac{125}{4 \cdot 64} - \frac{\lambda''}{64} - \frac{5 \cdot 9}{4 \cdot 64}\nu$ et $\lambda^{II} = \frac{125-45\nu}{4}$

II. $g = \frac{6}{4}$; $0 = \frac{216}{4 \cdot 64} - \frac{8\lambda''}{64} - \frac{6 \cdot 4}{4 \cdot 64}\nu$ et $\lambda^{II} = \frac{216-24\nu}{4 \cdot 8} = \frac{27-3\nu}{4}$

III. $g = \frac{7}{4}$; $0 = \frac{343}{4 \cdot 64} - \frac{27\lambda''}{64} - \frac{7 \cdot 1}{4 \cdot 64}\nu$ et $\lambda^{II} = \frac{343-7\nu}{4 \cdot 27}$

IV. $g = \frac{8}{4}$; $0 = \frac{512}{4 \cdot 64} - \frac{64\lambda''}{64}$ et $\lambda^{II} = 2$

V. $g = \frac{9}{4}$; $0 = \frac{729}{4 \cdot 64} - \frac{125\lambda''}{64} - \frac{9 \cdot 1}{4 \cdot 64}\nu$ et $\lambda^{II} = \frac{729-0\nu}{4 \cdot 125}$

VI. $g = \frac{10}{4}$; $0 = \frac{1000}{4 \cdot 64} - \frac{216\lambda''}{64} - \frac{10 \cdot 4}{4 \cdot 64}\nu$ et $\lambda^{II} = \frac{1000-40\nu}{4 \cdot 216} = \frac{125-5\nu}{4 \cdot 27}$

In primo et secundo casu fit valor ipsius λ^{II} nimis magnus, quam vt ista lens commode in praxin recipi queat; ac si ipsi g multo maior tribuatur valor, leuis error ingentem effectum producit. Ponamus enim loco g per errorem sumi $g+\omega$, et cum λ^{II} ex g rite fuerit definitum, fiet

$$\lambda^{(3)} = \omega\left(\tfrac{3}{4}gg - 3\lambda^{II}(1-g)^2 - \nu(1-\tfrac{1}{8}g)(1-\tfrac{3}{8}g)\right)$$

et

et pro λ'' substituto valore:

$$\lambda^{(3)} = \frac{\omega}{4(1-g)}(3gg - \nu(4-gg))$$

vnde patet, quo minus g vnitatem excedat, ac simul quo maius fuerit g, valorem ipsius $\lambda^{(3)}$ ob errorem ω eo fieri maiorem. Intelligitur autem hunc errorem fieri minimum si capiatur

$g = 1 + \sqrt{\frac{3-3\nu}{\nu+3}}$, ex hoc autem valore elicitur,

$$\lambda'' = \frac{(3+\nu)\sqrt{3(1-\nu)(3+\nu)} + 3(3+\nu\nu)}{9(1-\nu)}$$

Capi ergo debebit $g = 1,84384$ vnde colligitur:

$$\lambda'' = \frac{g^3 - \nu g(2-g)^2}{4(g-1)^3} = 2,60372$$

et si hae mensurae exacte obseruentur, fiet $\lambda^{(3)} = 0$. At si in valore ipsius g particula ω aberretur, vt sit $g = 1,84384 + \omega$, prodibit ob hunc errorem:

$$\lambda^{(3)} = -2,981\,\omega,$$

ita si esset error $\omega = \pm\frac{1}{10}$, loco $\lambda^{(3)} = 0$, prodiret:

$\lambda^{(3)} = \mp 0,2981$ ideoque lens triplicata postponenda duplicatae, longe tamen praeferenda foret simplici.

In genere igitur pro quouis valore alio ipsius $\lambda^{(2)}$ idem commodum inuestigemus: ac primo cum sit

$$\lambda'' = \frac{\lambda^{(2)} g^3 + \nu g(g-1)}{(g-1)^3}$$

si loco iusti valoris g capiatur $g + \omega$ fiet

$$\lambda^{(3)} = \omega(3\lambda^{(2)} g^2 - 3\lambda''(g-1)^2 - \nu + 2\nu g)$$

vbi si pro λ'' valor substituatur, erit

$$\lambda^{(3)} = \frac{\omega(\nu - (3\lambda^{(2)} + \nu)gg)}{g-1}$$

qui error vt minimus reddatur capi debet

$$g = 1 + \sqrt{\frac{3\lambda^{(2)}}{3\lambda^{(1)} + \nu}}$$

qui eſt valor maxime idoneus pro g ſumendus, ex quo elicitur

$$\lambda'' = \frac{1}{3\lambda^{(1)}}\left(\sqrt{(3\lambda^{(2)}+\nu)} + \sqrt{3\lambda^{(2)}}\right)\left(\tfrac{1}{3}(6\lambda^{(2)}+\nu)\sqrt{3\lambda^{(2)}} + (2\lambda^{(2)}+\nu)\sqrt{(3\lambda^{(2)}+\nu)}\right)$$

et ſumto per errorem $g+\omega$ pro g erit

$$\lambda^{(3)} = -2\omega\left(3\lambda^{(2)} + \nu + \sqrt{3\lambda^{(2)}}\left(3\lambda^{(2)}+\nu\right)\right) \text{ ſeu}$$

$$\lambda^{(3)} = -2\omega\left(\sqrt{3\lambda^{(2)}} + \sqrt{(3\lambda^{(2)}+\nu)}\right)\sqrt{(3\lambda^{(2)}+\nu)}$$

Quo minor igitur iam fuerit valor ipſius $\lambda^{(2)}$ eo minus erit error metuendus, vnde ſolutio ante ex valore $\lambda^{(1)} = \frac{1-\nu}{4}$ eruta prae ceteris eſt commendanda. Tum autem erit $f = \frac{1}{2}$; $\lambda = 1$, $\lambda' = 1$, $\lambda'' = 2,60372$ et $g = 1,84384$, vnde pro lentium conſtructione habemus,

$$\frac{1}{a} = -\frac{1}{b} = \frac{-2+g}{2a} + \frac{g}{2\gamma}$$

$$\frac{1}{\mathfrak{a}} = -\frac{1}{c} = \frac{-1+g}{a} + \frac{g}{\gamma}$$

vnde ſingulae lentes per formulas §. 91 conſtruentur.

Coroll. 1.

146. Lens ergo prima PP conſtrui debet ex diſtantiis determinatricibus a et $\frac{2a\gamma}{ga-(2-g)\gamma}$ cum numero $\lambda = 1$

Lens vero ſecunda QQ ex diſtantiis determinatricibus

$\frac{-2a\gamma}{ga-(2-g)\gamma}$ cum numero $\lambda' = 1$

at

at lens tertia RR ex distantiis determinatricibus

$\frac{-a\gamma}{ga-(1-g)\gamma}$ et γ cum numero $\lambda''=2,60372$

existente $g=1,84384$.

Coroll. 2.

147. Quoniam in formula spatium diffusionis exprimente quae ob $\lambda^{(3)}=0$ est $\mu\gamma\gamma xx(\frac{1}{a}+\frac{1}{\gamma})\frac{\gamma}{a\gamma}$, reiecto factore primo $\mu\gamma\gamma xx$, cuius ratio in his inuestigationibus non est habita, distantiae a et γ inter se permutari possunt; hinc etiam alia lens triplicata quaesito aeque satisfaciens exhiberi poterit.

Coroll. 3.

148. Nempe pro hac altera lente triplicata lens prima PP construi debet ex distantiis determinatricibus

a et $\frac{+a\gamma}{(1-b)a-g\gamma}$ cum numero $\lambda=2,60372$

Lens secunda QQ ex distantiis determinatricibus

$\frac{-a\gamma}{(1-b)a-g\gamma}$ et $\frac{+2a\gamma}{(2-g)a-g\gamma}$ cum numero $\lambda'=1$

at lens tertia ex distantiis determinatricibus

$\frac{-2a\gamma}{(2-g)a-g\gamma}$ et γ cum numero $\lambda''=1$

existente vt ante $g=1,84384$.

Coroll. 4.

149. Hinc ergo duas nacti sumus lentes triplicatas pro distantiis a et γ, quae producunt spatium diffu-

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diffusionis $Hb = \mu\gamma\gamma xx(\frac{1}{a}+\frac{1}{\gamma})\frac{v}{a\gamma}$. Atque hae inter infinitas alias eundem effectum praestantes hac gaudent praerogatiua, vt leuis error in constructione commissus scopum minime perturbet.

Coroll. 5.

150. Si in constructione harum lentium per errorem numerus g parumper maior accipiatur, quam 1, 84384, tum pro lente triplicata numerus $\lambda^{(3)}$ prodit nihilo minor seu negatiuus. Sin autem numerus g in praxi aliquantillum maior sumatur numerus $\lambda^{(3)}$ fit nihilo maior, sicque lens triplicata ad naturam duplicatarum accedet.

Coroll. 6.

151. Si ergo opus fuerit lente, pro qua numerus λ valorem habeat negatiuum, huic scopo satisfieri commode poterit per lentes descriptas triplicatas, dummodo pro g numerus aliquanto maior quam 1, 84384 assumatur. Scilicet si sumatur

$$g = 1, 84384 + \omega \text{ fiet } \lambda^{(3)} = -2, 981\ \omega.$$

Supplementum II.

De lentibus triplicatis

Si pro singulis lentibus refractio sit diuersa, pro prima n : 1 pro secunda n' : 1, pro tertia n''

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$n^{(0)} : 1$, radiique facierum lentium sequenti modo definiantur

	Dist. determinatrices	Refractio et litterae inde pendentes	
I.	a et α	$n : 1, \mu, \nu, \varrho, \sigma, \tau$.	λ
II.	b et $\mathfrak{C}$	$n' : 1, \mu', \nu', \varrho', \sigma', \tau'$.	λ'
III.	c et γ	$n'' : 1, \mu'', \nu'', \varrho'', \sigma'', \tau''$.	λ''

scilicet si pro lente prima vocetur radius faciei anterioris $= F$; posterioris $= G$; erit

$$\tfrac{1}{F} = \tfrac{\varrho}{a} + \tfrac{\sigma}{\alpha} \mp \tau\left(\tfrac{1}{a} + \tfrac{1}{\alpha}\right)\sqrt{\lambda - 1}$$

$$\text{et } \tfrac{1}{G} = \tfrac{\varrho}{\alpha} + \tfrac{\sigma}{a} \pm \tau\left(\tfrac{1}{a} + \tfrac{1}{\alpha}\right)\sqrt{\lambda - 1}$$

similique modo pro reliquis lentibus, nempe pro secunda

$$\tfrac{1}{F'} = \tfrac{\varrho'}{b} + \tfrac{\sigma'}{\mathfrak{C}} \mp \tau'\left(\tfrac{1}{b} + \tfrac{1}{\mathfrak{C}}\right)\sqrt{\lambda' - 1}$$

$$\tfrac{1}{G'} = \tfrac{\varrho'}{\mathfrak{C}} + \tfrac{\sigma'}{b} \pm \tau'\left(\tfrac{1}{b} + \tfrac{1}{\mathfrak{C}}\right)\sqrt{\lambda' - 1}$$

tum vero pro tertia

$$\tfrac{1}{F''} = \tfrac{\varrho''}{c} + \tfrac{\sigma''}{\gamma} \mp \tau''\left(\tfrac{1}{c} + \tfrac{1}{\gamma}\right)\sqrt{\lambda'' - 1}$$

$$\tfrac{1}{G''} = \tfrac{\varrho''}{\gamma} + \tfrac{\sigma''}{c} \pm \tau''\left(\tfrac{1}{c} + \tfrac{1}{\gamma}\right)\sqrt{\lambda'' - 1}$$

Deinde quia distantiae lentium pro nihilo habentur scilicet $a + b = 0$ et $\mathfrak{C} + c = 0$, erit spatium diffusionis

$$= \gamma\gamma xx \left\{ \begin{array}{l} +\mu\left(\tfrac{1}{a} + \tfrac{1}{\alpha}\right)\left(\lambda\left(\tfrac{1}{a} + \tfrac{1}{\alpha}\right)^2 + \tfrac{\nu}{a\alpha}\right) \\ +\mu'\left(\tfrac{1}{b} + \tfrac{1}{\mathfrak{C}}\right)\left(\lambda'\left(\tfrac{1}{b} + \tfrac{1}{\mathfrak{C}}\right)^2 + \tfrac{\nu'}{b\mathfrak{C}}\right) \\ +\mu''\left(\tfrac{1}{c} + \tfrac{1}{\gamma}\right)\left(\lambda''\left(\tfrac{1}{c} + \tfrac{1}{\gamma}\right)^2 + \tfrac{\nu''}{c\gamma}\right) \end{array} \right.$$

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ſtatuatur nunc

$$\frac{1}{a}+\frac{1}{\alpha}=f\left(\frac{1}{a}+\frac{1}{\gamma}\right)$$
$$\frac{1}{b}+\frac{1}{\beta}=g\left(\frac{1}{a}+\frac{1}{\gamma}\right)$$
$$\frac{1}{c}+\frac{1}{\gamma}=h\left(\frac{1}{a}+\frac{1}{\gamma}\right)$$

vt fiat

$$\frac{1}{\alpha}=\frac{f-1}{a}+\frac{f}{\gamma}=\frac{-1}{b};\ \text{et}$$
$$\frac{1}{c}=\frac{h}{a}+\frac{h-1}{\gamma}=\frac{-1}{\beta}$$

vnde fit

$$\frac{1}{b}+\frac{1}{\beta}=\frac{1-f-h}{a}+\frac{1-f-h}{\gamma}=g\left(\frac{1}{a}+\frac{1}{\gamma}\right)$$

hincque $f+g+h=1$; quibus valoribus ſubſtitutis erit primo pro radiis facierum

$$\frac{1}{F}=\frac{\varrho+(f-1)\sigma}{a}+\frac{\sigma f}{\gamma}\mp\tau f\left(\frac{1}{a}+\frac{1}{\gamma}\right)\sqrt{\lambda-1}$$
$$\frac{1}{G}=\frac{\sigma+(f-1)\varrho}{a}+\frac{\varrho f}{\gamma}\mp\tau f\left(\frac{1}{a}+\frac{1}{\gamma}\right)\sqrt{\lambda-1}$$
$$\frac{1}{F'}=\frac{(1-f)\varrho'-h\sigma'}{a}+\frac{(1-h)\sigma'-f\varrho'}{\gamma}\mp\tau' g\left(\frac{1}{a}+\frac{1}{\gamma}\right)\sqrt{\lambda'-1}$$
$$\frac{1}{G'}=\frac{(1-f)\sigma'-h\varrho'}{a}+\frac{(1-h)\varrho'-f\sigma'}{\gamma}\mp\tau' g\left(\frac{1}{a}+\frac{1}{\gamma}\right)\sqrt{\lambda'-1}$$

et pro lente tertia

$$\frac{1}{F''}=\frac{h\cdot\varrho''}{a}+\frac{(h-1)\varrho''+\sigma''}{\gamma}\mp\tau''\cdot h\left(\frac{1}{a}+\frac{1}{\gamma}\right)\sqrt{\lambda''-1}$$
$$\frac{1}{G''}=\frac{h\cdot\sigma''}{a}+\frac{(h-1)\sigma''+\varrho''}{\gamma}\mp\tau''\cdot h\left(\frac{1}{a}+\frac{1}{\gamma}\right)\sqrt{\lambda''-1}$$

Spatium

Spatium vero diffusionis iam ita exprimetur

$$\gamma\gamma\, xx\left(\tfrac{1}{a}+\tfrac{1}{\gamma}\right)\begin{cases}\left(\tfrac{1}{a}+\tfrac{1}{\gamma}\right)^2(\mu\lambda f^3+\mu'\lambda' g^3+\mu''\lambda''.b^3)\\ +\frac{\nu\mu f}{a}\left(\frac{f-1}{a}+\frac{f}{\gamma}\right)\\ +\nu'\mu' g\left(\frac{1-f}{a}-\frac{f}{\gamma}\right)\left(\frac{1-b}{\gamma}-\frac{b}{a}\right)\\ +\frac{\nu''\mu''.b}{\gamma}\left(\frac{b}{a}+\frac{b-1}{\gamma}\right)\end{cases}$$

Nunc igitur circa has lentes triplicatas sequentia sunt obseruanda

I. Diuersa media refringentia eum tantum in finem adhiberi solent, vt non solum spatium diffusionis hic determinatum ad nihilum redigatur, sed etiam confusio a diuersa radiorum refrangibilitate oriunda tollatur, quippe quod per lentes eiusdem refractionis obtineri nequit. Infra autem videbimus ad hanc conditionem implendam requiri, vt sit $\zeta f+\eta g+\vartheta b=0$, existente $\zeta=\frac{dn}{n-1}$; $\eta=\frac{dn'}{n'-1}$ et $\vartheta=\frac{dn''}{n''-1}$; ex quibus formis iam perspicitur, si haec differentialia dn, dn' et dn'' essent ipsis $n-1$; $n'-1$; $n''-1$ proportionalia, vti *Newtonus* statuerat; tum proditura esse $\zeta=\eta=\vartheta$ siue $f+g+b=0$; at iam vidimus, esse debere $f+g+b=1$; quare si *Newtoni* sententia esset vera; tum ne quidem diuersis refractionibus adhibendis diffusioni a diuersa refrangibilitate oriundae remedium adferri posset. Eatenus igitur tantum hoc incommodum vitari poterit, quatenus litterae ζ, η et ϑ sunt diuersae, ita, vt simul esse possit, et $f+g+b=1$ et $\zeta f+\eta g+\vartheta b=0$ ex quo perspicuum est, quantitatum f, g et b vnam

vel adeo duas esse debere negatiuas sicque hac conditione superaddita casus ille principalis, quo omnes tres litterae f, g et h positiuae sunt assumtae hic locum inuenire nequit.

II. Vt ergo nostrum spatium diffusionis euanescat, satisfaciendum est huic aequationi

$$\left(\tfrac{1}{\alpha}+\tfrac{1}{\gamma}\right)^2\left(\mu\lambda f^3+\mu'\lambda'g^3+\mu''\lambda''h^3\right)$$
$$+\frac{2\mu f}{\alpha}\left(\frac{f-1}{\alpha}+\frac{f}{\gamma}\right)$$
$$+2\mu'\mu''g\left(\frac{\alpha-f}{\alpha}-\frac{g}{\gamma}\right)\left(\frac{\alpha-h}{\gamma}-\frac{h}{\alpha}\right)$$
$$+\frac{2\mu''\mu''h}{\gamma}\left(\frac{h}{\alpha}+\frac{h-1}{\gamma}\right)=0$$

Vnde vel λ vel λ' vel λ'' quaeri potest, dummodo caueatur, ne valor vel vnitate minor vel nimis magnus prodeat, quia prius naturae repugnat; alterum autem, quia constructio lentium fieret nimis lubrica. Hoc autem praestito circa binas reliquas litteras λ nihil amplius definitur neque etiam circa litteras f, g et h, praeterquam quod supra allatis conditionibus $f+g+h=1$ et $\zeta f+\eta g+\vartheta h=0$ continetur, vnde ex data vna harum litterarum duae reliquae sponte definiuntur sequenti scilicet modo:

$$g=\frac{\zeta(\vartheta-\zeta)f-\vartheta}{\eta-\vartheta}; \text{ et}$$
$$h=\frac{(\zeta-\eta)f+\eta}{\eta-\vartheta}$$

III. In praxi autem huiusmodi lentes triplicatae ideo potissimum quaeruntur, vt loco lentis obiectiuae

duae in telescopiis substitui queant, pro quibus est $\alpha = \infty$. Statuamus ergo statim $\alpha = \infty$ et pro radiis facierum singularum lentium habebimus

$$\frac{1}{F} = \frac{\varrho f}{\gamma} + \frac{\tau f}{\gamma}\sqrt{\lambda - 1}\,; \quad \frac{1}{G} = \frac{\varrho f}{\gamma} - \frac{\tau f}{\gamma}\sqrt{\lambda - 1}$$

pro secunda lente

$$\frac{1}{F'} = \frac{(1-b)\varrho' - f\varrho'}{\gamma} + \frac{\tau' g}{\gamma}\sqrt{\lambda' - 1}\,; \quad \frac{1}{G'} = \frac{(1-b)\varrho' - f\sigma'}{\gamma} - \frac{\tau' g}{\gamma}\sqrt{\lambda' - 1}$$

pro tertia lente

$$\frac{1}{F''} = \frac{(b-1)\varrho'' + \varrho''}{\gamma} + \frac{\tau'' b}{\gamma}\sqrt{\lambda'' - 1}\,; \quad \frac{1}{G''} = \frac{(b-1)\varrho'' + \varrho''}{\gamma} - \frac{\tau'' b}{\gamma}\sqrt{\lambda'' - 1}$$

Spatium autem diffusionis tum ita exprimetur

$$\frac{x^2}{\gamma}(\mu\lambda f^3 + \mu'\lambda' g^3 + \mu''\lambda'' b^3 - \nu'\mu' fg(1-b) - \nu''\mu'' b(1-b)$$

Ita vt satisfieri oporteat huic aequationi

$$\mu\lambda f^3 + \mu'\lambda' g^3 + \mu''\lambda'' b^3 - \nu'\mu' fg(1-b) - \nu''\mu'' b(1-b) = 0.$$

Vnde vnus valorum λ, λ', λ'', qui ad vsum commodissimus videtur, determinari debet.

IV. Si tantum lentibus vitreis vti velimus, sufficiet duas tantum vitri species adhiberi; si igitur statuamus, lentem tertiam et primam ex eadem vitri specie parari, vt $\mu'' = \mu$; $\nu'' = \nu$; $\varrho'' = \varrho$; $\sigma'' = \sigma$; $\tau'' = \tau$ et $\zeta = \mathfrak{P}$ ob $n'' = n$; pro litteris autem f, g et b hae determinationes habebuntur, 1°. ex aequatione

$$\zeta f + \eta g + \zeta b = 0 \text{ fit } f + b = -\frac{\eta}{\zeta}\, g,$$

quo valore substituto fiet

$$g\,\frac{(\zeta - \eta)}{\zeta} = 1\,; \; g = \frac{\zeta}{\zeta - \eta} \text{ ideoque } f + b = \frac{-\eta}{\zeta - \eta}\,;$$

vnde patet, prouti littera g fuerit vel positiua vel negatiua, fore viciſſim ſummam $f+b$ vel negatiuam vel poſitiuam et aequatio reſoluenda iam erit

$$\mu(\lambda f^3+\lambda'' b^3)+\mu'\lambda' g^3-\frac{\mu'\nu'.\zeta}{\zeta-\eta}.f(1-b)-\mu\nu.b(1-b)=0$$

Hinc igitur elicitur

$$\mu'\lambda' g^3=-\mu(\lambda f^3+\lambda'' b^3)+\frac{\mu'\nu'.\zeta}{\zeta-\eta}.f(1-b)+\mu\nu.b(1-b)$$

cui facile erit pro caſu quouis propoſito ſatisfacere.

IV. Dum lens prima et tertia ex eadem vitri ſpecie parantur, media conſtet ex aqua vel alia materia fluida, vt lens triplicata intra duas lentes vitreas contineat fluidum, et quia fluidum plerumque minorem refractionem patitur, quam vitrum; erit $n'<n$ indeque porro $\eta'<\zeta$. Quia igitur pro hoc caſu fit g poſitiuum ſeu lens aquea conuexa, lentes vitreae vel ambae vel vna ſaltem debent eſſe concauae.

Inprimis autem praeter determinationes iam inventas neceſſe eſt, vt radius faciei anterioris pro lente media aequalis et contrarius ſit radio faciei poſterioris lentis primae, eodemque modo radius faciei poſterioris aequalis et contrarius radio faciei anterioris lentis tertiae vnde hae aequalitates naſcentur

$\frac{1}{F}=\frac{-1}{G}$, ſeu $F'+G=0$; et $\frac{1}{G'}=\frac{-1}{F''}$ ſeu $F''+G'=0$.

Ideoque ſatisfieri oportet iſtis aequationibus:

$$(1-b)\sigma'-f\varrho'\mp\tau' g.\sqrt{\lambda'-1}=-\varrho f\mp\tau f\sqrt{\lambda-1} \text{ et}$$

$$(1-b)\varrho'-f\sigma'\mp\tau''.g.\sqrt{\lambda''-1}=(1-b)\varrho-\sigma\mp\tau'' b\sqrt{\lambda''-1}$$

Erit igitur duas conditiones, quibus satisfieri oportet, vnde vel numeri λ et λ'' vel alter eorum cum alterutra litterarum f et b definiri debent; quem in finem probe obseruandum est, formulas $\sqrt{\lambda - 1}$ et $\sqrt{\lambda'' - 1}$ pro lubitu siue positiuas siue negatiuas assumi posse, neque a se inuicem pendere. Pro formula autem $\sqrt{\lambda' - 1}$ notandum est, si ea in priore aequatione positiue ponatur, in posteriore necessario negatiue sumi debere et vicissim.

Problema 8.

152. Determinare eas lentes quadruplicatas ad datas distantias determinatrices $AE = a$ et $dF = \delta$ accommodatas, quae minimum spatium diffusionis producant. Tab. II. Fig. 9.

Solutio.

Prima lens PP ad distantias determinatrices $AE = a$ et $AF = \alpha$ cum numero λ construatur, secunda QQ ad distantias $b = -\alpha$ et $AG = \beta$ cum numero λ', tertia RR ad distantias $c = -\beta$ et $AH = \gamma$ cum numero λ'': et quarta SS ad distantias $d = -\gamma$ et $dI = \delta$ cum numero λ''' construatur: vbi scilicet crassitiem lentium vt euanescentem spectamus. Posito iam semidiametro aperturae lentis x, sola prima lens PP produceret spatium diffusionis

$$Ff = \mu\alpha\alpha xx\left(\tfrac{1}{a}+\tfrac{1}{\alpha}\right)\left(\lambda\left(\tfrac{1}{a}+\tfrac{1}{\alpha}\right)^2+\tfrac{\nu}{a\alpha}\right)$$

Adiun-

Adiuncta autem secunda lente QQ, positoque

$$\frac{1}{\alpha} = \frac{-1+f}{a} + \frac{f}{\varepsilon} \text{ et } \frac{1}{b} = \frac{1-f}{a} - \frac{f}{\varepsilon}$$

si breuitatis gratia statuamus

$$\lambda^{(2)} = \lambda f^3 + \lambda'(1-f)^3 - \nu f(1-f)$$

vidimus fore spatium diffusionis:

$$Gg = \mu \varepsilon \varepsilon x x \left(\tfrac{1}{a}+\tfrac{1}{\varepsilon}\right)\left(\lambda^{(2)}\left(\tfrac{1}{a}+\tfrac{1}{\varepsilon}\right)^2 + \tfrac{\nu}{a\varepsilon}\right)$$

Adiungatur insuper tertia lens RR, numerusque g ita sumatur vt sit

$$\frac{1}{\varepsilon} = \frac{-1+g}{a} + \frac{g}{\gamma} \text{ et } \frac{1}{c} = \frac{1-g}{a} - \frac{g}{\gamma}$$

ac si breuitatis ergo ponamus:

$$\lambda^{(3)} = \lambda^{(2)} g^3 + \lambda''(1-g)^3 - \nu g(1-g)$$

erit spatium diffusionis:

$$Hh = \mu \gamma \gamma x x \left(\tfrac{1}{a}+\tfrac{1}{\gamma}\right)\left(\lambda^{(3)}\left(\tfrac{1}{a}+\tfrac{1}{\gamma}\right)^2 + \tfrac{\nu}{a\gamma}\right)$$

Nunc denique adiungatur lens quarta SS, et numero b ita in calculum introducto, vt sit

$$\frac{1}{\gamma} = \frac{-1+b}{a} + \frac{b}{\delta} \text{ et } \frac{1}{\beta} = \frac{1-b}{a} - \frac{b}{\delta}$$

si simili modo ponamus

$$\lambda^{(4)} = \lambda^{(3)} b^3 + \lambda'''(1-b)^3 - \nu b(1-b)$$

erit spatium diffusionis a lente quadruplicata productum:

$$Ii = \mu \delta \delta x x \left(\tfrac{1}{a}+\tfrac{1}{\delta}\right)\left(\lambda^{(4)}\left(\tfrac{1}{a}+\tfrac{1}{\delta}\right)^2 + \tfrac{\nu}{a\delta}\right)$$

quod igitur minimum reddi debet. Hunc in finem considerentur numeri λ, λ', λ'' et λ''' vt dati, et quaerantur

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rantur idonei valores pro numeris f, g, et h: atque vt valor $\lambda^{(1)}$ minimus euadat, neceſſe eſt quoque valores $\lambda^{(3)}$ et $\lambda^{(2)}$ minimos fieri. Incipiamus ergo a valore $\lambda^{(2)}$ qui minimus redditur ſumendo

$$f=\frac{\sqrt{(3\lambda'+\nu)}}{\sqrt{(3\lambda+\nu)}+\sqrt{(3\lambda'+\nu)}} \text{ ſeu } \frac{1}{f}=1+\sqrt{\frac{3\lambda+\nu}{3\lambda'+\nu}}$$

vnde fit

$$\frac{1}{\sqrt{(3\lambda^{(1)}+\nu)}}=\frac{1}{\sqrt{(3\lambda+\nu)}}+\frac{1}{\sqrt{(3\lambda'+\nu)}}.$$

Deinde numerus $\lambda^{(3)}$ minimum induet valorem capiendo

$$g=\frac{\sqrt{(3\lambda''+\nu)}}{\sqrt{(3\lambda^{(2)}+\nu)}+\sqrt{(3\lambda''+\nu)}} \text{ ſeu } \frac{1}{g}=1+\sqrt{\frac{3\lambda^{(2)}+\nu}{3\lambda''+\nu}}$$

hincque colligitur

$$\frac{1}{\sqrt{(3\lambda^{(3)}+\nu)}}=\frac{1}{\sqrt{(3\lambda^{(2)}+\nu)}}+\frac{1}{\sqrt{(3\lambda''+\nu)}}.$$

Denique numerus $\lambda^{(4)}$ ideoque et ſpatium diffuſionis Ii minimum efficietur ſumendo

$$b=\frac{\sqrt{(3\lambda'''+\nu)}}{\sqrt{(3\lambda^{(3)}+\nu)}+\sqrt{(3\lambda'''+\nu)}} \text{ ſeu } \frac{1}{b}=1+\sqrt{\frac{3\lambda^{(3)}+\nu}{3\lambda'''+\nu}}$$

vnde obtinetur

$$\frac{1}{\sqrt{(3\lambda^{(4)}+\nu)}}=\frac{1}{\sqrt{(3\lambda^{(3)}+\nu)}}+\frac{1}{\sqrt{(3\lambda'''+\nu)}}.$$

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Quod si hic valores ante inuentos substituamus, nanciscemur

$$\frac{1}{V(3\lambda^{(1)}+\nu)} = \frac{1}{V(3\lambda+\nu)} + \frac{1}{V(3\lambda'+\nu)} + \frac{1}{V(3\lambda''+\nu)} + \frac{1}{V(3\lambda'''+\nu)}.$$

Pro praecedentibus vero erit

$$\frac{1}{V(3\lambda^{(3)}+\nu)} = \frac{1}{V(3\lambda+\nu)} + \frac{1}{V(3\lambda'+\nu)} + \frac{1}{V(3\lambda''+\nu)}$$

et

$$\frac{1}{V(3\lambda^{(2)}+\nu)} = \frac{1}{V(3\lambda+\nu)} + \frac{1}{V(3\lambda'+\nu)}.$$

Tum vero ex his porro consequimur:

$$f = V\frac{3\lambda^{(2)}+\nu}{3\lambda+\nu}, \quad g = V\frac{3\lambda^{(3)}+\nu}{3\lambda^{(1)}+\nu}, \quad b = V\frac{3\lambda^{(4)}+\nu}{3\lambda^{(2)}+\nu}$$

Superest ergo vt constructionem singularum lentium luculentius exponamus: et earum distantias determinatrices per solas propositas α et δ exprimamus: erit igitur

$$\frac{1}{\gamma} = \frac{-1+b}{\alpha} + \frac{b}{\delta}; \quad \frac{1}{\delta} = \frac{1-b}{\alpha} - \frac{b}{\delta}$$

$$\frac{1}{\varepsilon} = \frac{-1+gb}{\alpha} + \frac{gb}{\delta}; \quad \frac{1}{\zeta} = \frac{1-gb}{\alpha} - \frac{gb}{\delta}$$

$$\frac{1}{\alpha} = \frac{-1+fgb}{\alpha} + \frac{fgb}{\delta}; \quad \frac{1}{\beta} = \frac{1-fgb}{\alpha} - \frac{fgb}{\delta}$$

Ex superioribus vero formulis colligitur:

$$b = V\frac{3\lambda^{(4)}+\nu}{3\lambda^{(2)}+\nu}; \quad g = V\frac{3\lambda^{(3)}+\nu}{3\lambda^{(1)}+\nu}; \quad f = V\frac{3\lambda^{(2)}+\nu}{3\lambda+\nu}$$

vnde

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vnde fit

$$g\,b = \sqrt{\frac{3\lambda^{(4)}+\nu}{3\lambda^{(2)}+\nu}} \text{ et } fg\,b = \sqrt{\frac{3\lambda^{(4)}+\nu}{3\lambda+\nu}}.$$

et ſuperiores valores ita exprimi poterunt

$$\frac{1}{a} = -\frac{1}{b} = -\frac{\sqrt{(3\lambda^{(4)}+\nu)}}{a}\left(\frac{1}{\sqrt{(3\lambda'+\nu)}}+\frac{1}{\sqrt{(3\lambda''+\nu)}}+\frac{1}{\sqrt{(3\lambda'''+\nu)}}\right)+\frac{\sqrt{(3\lambda^{(4)}+\nu)}}{\delta}\cdot\frac{1}{\sqrt{(3\lambda+\nu)}}.$$

$$\frac{1}{e} = -\frac{1}{c} = -\frac{\sqrt{(3\lambda^{(4)}+\nu)}}{a}\left(\frac{1}{\sqrt{(3\lambda''+\nu)}}+\frac{1}{\sqrt{(3\lambda'''+\nu)}}\right)+\frac{\sqrt{(3\lambda^{(4)}+\nu)}}{\delta}\left(\frac{1}{\sqrt{(3\lambda+\nu)}}+\frac{1}{\sqrt{(3\lambda'+\nu)}}\right)$$

$$\frac{1}{\gamma} = -\frac{1}{d} = -\frac{\sqrt{(3\lambda^{(4)}+\nu)}}{a}\cdot\frac{1}{\sqrt{(3\lambda'''+\nu)}}+\frac{\sqrt{(3\lambda^{(4)}+\nu)}}{\delta}\left(\frac{1}{\sqrt{(3\lambda+\nu)}}+\frac{1}{\sqrt{(3\lambda'+\nu)}}+\frac{1}{\sqrt{(3\lambda''+\nu)}}\right).$$

Coroll. 1.

153. Si pro lentibus ſimplicibus numeri λ, λ' λ'' et λ''' ſumantur inter ſe aequales, fiet pro minimo ſpatio diffuſionis:

$$\sqrt{(3\lambda^{(2)}+\nu)} = \tfrac{1}{2}\sqrt{(3\lambda+\nu)};\ \lambda^{(2)} = \tfrac{3\lambda-1.3\nu}{3.4}$$

$$\sqrt{(3\lambda^{(3)}+\nu)} = \tfrac{1}{3}\sqrt{(3\lambda+\nu)};\ \lambda^{(3)} = \tfrac{3\lambda-2.4\nu}{3.9}$$

$$\sqrt{(3\lambda^{(4)}+\nu)} = \tfrac{1}{4}\sqrt{(3\lambda+\nu)};\ \lambda^{(4)} = \tfrac{3\lambda-3.5\nu}{3.16}$$

hinc $f = \frac{1}{2}$; $g = \frac{2}{3}$; $b = \frac{3}{4}$, et pro conſtructione lentium ſimplicium:

$$\tfrac{1}{a} = -\tfrac{3}{4a}+\tfrac{1}{4\delta};\ \tfrac{1}{b} = +\tfrac{3}{4a}-\tfrac{1}{4\delta}$$

$$\tfrac{1}{e} = -\tfrac{2}{4a}+\tfrac{2}{4\delta};\ \tfrac{1}{c} = +\tfrac{2}{4a}-\tfrac{2}{4\delta}$$

$$\tfrac{1}{\gamma} = -\tfrac{1}{4a}+\tfrac{3}{4\delta};\ \tfrac{1}{d} = +\tfrac{1}{4a}-\tfrac{3}{4\delta}$$

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Coroll. 2.

154. Hinc ex §. 91 sequens quatuor lentium simplicium constructio obtinetur:

Pro lente	radius faciei	
Prima PP	anterioris	$= \frac{4\alpha\delta}{(4\varrho-3\sigma)\delta+\sigma\alpha \pm \tau(\alpha+\delta)\sqrt{(\lambda-1)}}$
	posterioris	$= \frac{4\alpha\delta}{(4\sigma-3\varrho)\delta+\varrho\alpha \mp \tau(\alpha+\delta)\sqrt{(\lambda-1)}}$
Secunda QQ	anterioris	$= \frac{4\alpha\delta}{(3\varrho-2\sigma)\delta+(2\sigma-\varrho)\alpha \pm \tau(\alpha+\delta)\sqrt{(\lambda-1)}}$
	posterioris	$= \frac{4\alpha\delta}{(3\sigma-2\varrho)\delta+(2\varrho-\sigma)\alpha \mp \tau(\alpha+\delta)\sqrt{(\lambda-1)}}$
Tertia RR	anterioris	$= \frac{4\alpha\delta}{(2\varrho-\sigma)\delta+(3\sigma-2\varrho)\alpha \pm \tau(\alpha+\delta)\sqrt{(\lambda-1)}}$
	posterioris	$= \frac{4\alpha\delta}{(2\sigma-\varrho)\delta+(3\varrho-2\sigma)\alpha \mp \tau(\alpha+\delta)\sqrt{(\lambda-1)}}$
Quarta SS	anterioris	$= \frac{4\alpha\delta}{\varrho\delta+(4\sigma-3\varrho)\alpha \pm \tau(\alpha+\delta)\sqrt{(\lambda-1)}}$
	posterioris	$= \frac{4\alpha\delta}{\sigma\delta+(4\varrho-3\sigma)\alpha \mp \tau(\alpha+\delta)\sqrt{(\lambda-1)}}$

Coroll. 3.

155. Si praeterea numeri λ, λ^{I}, λ^{II}, λ^{III} vnitati aequales statuantur, qui est valor minimus, quem recipere possunt erit: ob $\nu = 0,232692$

$$\lambda^{(2)} = \frac{3-7\nu}{3\cdot 4} = 0,191827$$

$$\lambda^{(3)} = \frac{3-8\nu}{3\cdot 9} = 0,042165$$

$$\lambda^{(4)} = \frac{3-15\nu}{3\cdot 16} = -0,010216$$

sicque pro lente quadruplicata valor numeri $\lambda^{(4)}$ adeo infra nihilum deprimitur.

Coroll.

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Coroll. 4.

156. Maiorem ergo valorem numeris $\lambda, \lambda', \lambda'', \lambda'''$ tribuendo effici poterit, vt valor ipsius $\lambda^{(4)}$ praecise nihilo aequalis prodeat; quippe hoc fiet sumendo $\lambda = 5\nu = 1,163460$. Hinc habebitur $\lambda - 1 = 0,163460$ et $\tau\sqrt{(\lambda-1)} = 0,365947$, vnde singulae lentes simplices duplici modo per formulas exhibitas construi poterunt.

Scholion 1.

157. Si inter se comparemus hos duos casus, quibus est vel $\lambda^{(4)} = -0,010216$ vel $\lambda^{(4)} = 0$, videmus, etiamsi discrimen vix partem centesimam vnitatis superet, in constructione tamen lentium simplicium satis magnum discrimen deprehendi, cum denominatores formularum, (154) siue augeri siue diminui debeant quantitate: $0,365947\ (\alpha+\delta)$; quae differentia maior est, quam errores, qui forte ab artifice non nimis rudi committi queant. Ex quo vicissim colligimus, etiamsi in constructione harum lentium quadruplicatarum ab artifice leues errores committantur, inde vix perceptibilem effectum in spatio diffusionis vel valore ipsius $\lambda^{(4)}$ esse metuendum, quam ob causam hae lentes imprimis ad praxin accommodatae videntur. Si scilicet opus fuerit lente, pro qua valor ipsius λ in nihilum abeat, multo magis his lentibus quadruplicatis Coroll. 4 descriptis erit vtendum, quam triplicatis, quas supra definiui. Quin etiam eas lentes quadruplicatas adhibere licebit,

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in quibus est $\lambda^{(4)} = -0,010216$, quia hic numerus vix nihilo est minor; ac si in constructione a praescriptis mensuris aberretur, ille valor adhuc propius ad nihilum perducatur, ita vt hoc casu adeo errores commissi scopo magis attingendo inseruiant. Infra autem videbimus, plurimos dari casus, quibus eiusmodi lentibus vti conueniat, pro quibus valor ipsius λ non solum sit nihilo aequalis, sed etiam tantillum infra nihilum deprimatur; tunc optimo cum successu huiusmodi lentes quadruplicatas in vsum vocabimus. Sin autem eiusmodi lentes sufficiant, pro quibus valor ipsius λ sit $0,042165$ vel aliquantillum excedat, lentes triplicatae erunt commendandae, quarum constructionem supra (§. 136) dedimus; quemadmodum duplicatae negotium conficient si non opus fuerit minore valore ipsius λ quam $0,191827$. Tota autem instrumentorum dioptricorum perfectio in hoc maxime consistit, vt lentes habeantur, pro quibus valor ipsius λ sit quam minimus, cum eae sint aptissimae ad confusionem penitus tollendam; ex quo lentium quadruplicatarum hic descriptarum vsus erit amplissimus.

Scholion 2.

158. Si haec, quae de lentibus quadruplicatis hic tradidimus, attente considerentur, facile patebit, quomodo lentes quintuplicatae magisque multiplicatae ad vsum sint accommodandae. Eiusmodi scilicet semper constructione erit opus, quae a natura minimi parumper

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tumper recedat, quoniam hoc modo errores in praxi commissi effectum propositum minime perturbant. Cum autem vix vnquam vsu veniat, vt lentibus opus sit, pro quibus valor numeri λ magis vltra nihilum imminuatur, superfluum foret, constructionem lentium quintuplicatarum magisue multiplicatarum vlterius prosequi. Interim tamen iuuabit, si pro huiusmodi lentibus numeri λ lentium numero conuenienter signis $\lambda^{(5)}$ $^{(6)}$ etc. indicentur, eorum valores, quos ex natura minimi recipiunt, exposuisse. Sumamus omnes numeros λ, λ', λ'', λ''', λ'''', λ''''' etc. lentibus simplicibus respondentes vnitati aequales, et eorum qui lentibus multiplicatis conueniunt ex natura minimi ita se habebunt.

Pro Lente

Solitaria ... $\lambda^{(1)} = \frac{s-0v}{s.1} = 1,000000$

Duplicata ... $\lambda^{(2)} = \frac{s-3v}{s.4} = 0,191827$

Triplicata ... $\lambda^{(3)} = \frac{s-8v}{s.9} = 0,042165$

Quadruplicata $\lambda^{(4)} = \frac{s-15v}{s.16} = -0,010216$

Quintuplicata $\lambda^{(5)} = \frac{s-24v}{s.25} = -0,034461$

Sextuplicata $\lambda^{(6)} = \frac{s-35v}{s.36} = -0,047632$

Septuplicata $\lambda^{(7)} = \frac{s-48v}{s.49} = -0,055573$

Octuplicata $\lambda^{(8)} = \frac{s-63v}{s.64} = -0,060727$

Noncuplicata $\lambda^{(9)} = \frac{s-80v}{s.81} = -0,064261$

Decuplicata $\lambda^{(10)} = \frac{s-99v}{s.100} = -0,066788$

ac

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ac si huiusmodi lentes in infinitum multiplicentur, valor numeri respondentis $\lambda^{(\infty)}$ erit $= -\frac{\gamma}{\beta} = -0,67564$ ita vt nunquam infra hunc numerum deprimi possit: ex quo patet vix vnquam casum existere posse, quo lentes saltem quintuplicata opus esset. Ceterum etiam in genere, quicunque alii valores praeter vnitatem, numeris λ, λ', λ'', λ''' etc. tribuantur, ex formulis superioribus numeri $\lambda^{(5)}$, $\lambda^{(6)}$ etc. facile derivabuntur; quin etiam distantiae determinatrices singularum lentium simplicium indidem sine difficultate definientur, cum lex progressionis satis sit manifesta. Verum per totum hoc caput probe tenendum est, vbique crassitiem lentium tanquam evanescentem esse consideratum, in sequente autem capite iterum lentes in genere non neglecta crassitie sumus contemplaturi.

CAPVT

CAPVT IV.

DE CONFVSIONE VISIONIS NEC NON DE MAGNITVDINE APPARENTE ET CLARITATE.

Definitio 1.

159.

Visio est distincta, si omnes radii, qui ex quolibet obiecti puncto in oculum ingrediuntur in fundo oculi super retina iterum in vnum punctum congregantur.

Scholion.

160. Ad visionem distinctam requiritur, vt obiecta in certa quadam ab oculo distantia reperiantur, quae distantia pro varia oculorum indole maxime solet esse diuersa, dum Myopes exiguam, ii qui oculis valent ingentem, ac presbytes non solum infinitam sed quandoque etiam negatiuam exigunt; cuiusmodi distantia cum in veris obiectis locum habere nequeat, ope perspicillorum sibi satisfacere solent. Quilibet ergo oculus ad certam quandam distantiam obiectorum est instructus quam eius distantiam iustam appellabo; vbi quidem insignis latitudo locum habet,

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propterea quod ſtructura oculi ita eſt artificioſa, vt contractione ac elongatione quadam ſe ad diſtantias aliquanto maiores et minores accommodare poſſit. Quando ergo obiecta in diſtantia ab oculo iuſta reperiuntur, viſio eſt diſtincta, dum ſingula obiectorum puncta ſuper retina ſingulis punctis exprimuntur.

Definitio 2.

161. Viſio eſt confuſa, ſi radii ex quolibet obiecti puncto in oculum immiſſi non in vno retinae puncto congregantur, ſed per aliquod ſpatium retinam afficiunt.

Corollarium 1.

162. Eo maior ergo erit confuſio, quo maius fuerit hoc ſpatium in retina, per quod radii ex eodem obiecti puncto emiſſi diſſipantur. Ex quo huius ſpatii magnitudo veram confuſionis menſuram ſuppeditabit.

Coroll. 2.

163. Viſio ergo erit confuſa, cum obiecti viſi diſtantia multum fuerit diuerſa ab oculi diſtantia iuſta. Paruum enim diſcrimen vel per ſe nullam confuſionem parit vel oculus ſe ad obiecti diſtantiam, ſua qua pollet, volubilitate accommodare valet.

Scholion.

164. Si obiecta per lentes diſtincte repraeſentarentur, viſio imaginum eadem lege teneretur atque ipſo-

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ipsorum obiectorum; iusta scilicet earum ab oculo distantia visionem distinctam, admodum autem diuersa confusam produceret. Verum si imago per aliquod spatium fuerit diffusa etiamsi ab oculo ad distantiam iustam sit remota, inde tamen in visione confusio oriatur necesse est. Quod si nempe non obiecta ipsa, sed earum imagines per vnam pluresue lentes repraesentatas intueamur ob duplicem causam visio poterit esse confusa; altera, si distantia imaginis ab oculo multum fuerit diuersa a distantia iusta, altera vero si ipsa imago per aliquod spatium diffundatur. Priorem quidem causam tollere in nostra est potestate siquidem lentes ita disponere licet, vt imaginem in iusta ab oculo distantia exhibeant; quam lentium dispositionem propterea hic perpetuo assumamus. Quamobrem in hoc capite inuestigare constitui quanta confusio in visione imaginum a spatio diffusionis oriri debeat, eiusque quantitate determinata deinceps in hoc erit elaborandum quemadmodum lentes formatas ac dispositas esse oporteat, vt confusio inde in visione nata datum limitem, quo adhuc est tolerabilis, non excedat. In genere quidem certum est, quo maius fuerit spatium diffusionis, eo maiorem inde in visionem induci debere confusionem, verum tamen mox videbimus, confusionem visionis non esse spatio diffusionis proportionalem, sed aliam omnino legem sequi, quam accurate determinasse maximi erit momenti, cum ex hoc fonte constructio omnium instrumentorum Dioptricorum ad visionem accommodatorum sit repetenda.

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Problema 1.

Tab. II. Fig. 10.

165. Si oculus aſpiciat imaginem cuiuspiam obiecti ab vna pluribusue lentibus per ſpatium Ll diffuſam, atque imago principalis L in iuſta ab oculo diſtantia reperiatur, definire confuſionem, qua viſio huius imaginis afficietur.

Solutio.

In ſpatio diffuſionis Ll, in quo punctum L a radiis per lentium medium, punctum l vero a radiis per lentium extremitates transmiſſis exhibeatur, conſiderari oportet primo eius magnitudinem Ll deinde radiorum in l concurrentium inclinationem ad axem. Supra autem vidimus has res a primae lentis apertura, cuius ſemidiameter ſit $= x$, ita pendere vt ſit ipſum ſpatium $Ll = \mathfrak{V} x x$, et angulus $OlT = \mathfrak{B} x$ pro quouis ſcilicet lentium numero valores horum coefficientium $\mathfrak{V}$ et $\mathfrak{B}$ determinauimus. Repraeſentet iam circulus OV oculum, quem hic vt exiguam cameram obſcuram ſpectare licet, ſed ita perfectam, vt radios ex vno puncto emiſſos iterum in vno puncto colligat, etſi enim in oculo plures fiunt refractiones, tamen conditione illa ſeruata vnica lens earum loco conſiderari poteſt, quae ſit in TOT, a qua retina remota ſit interuallo $OV = u$. Cum nunc imago principalis L in debita ab oculo diſtantia, quae vocetur $OL = l$ exiſtat, puncti L imago in oculo in ipſam retinam incidet, et in V diſtincte

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cte depingetur: puncti vero l imago non in V, sed ante retinam in v referetur, quod interuallum Vv, si lentis in TOT conceptae crassities euanescat, secundum §. 62 ita exprimitur, vt sit $Vv = \frac{OV^2}{OL^2}$. $Ll = \frac{uu}{ll}$. Vxx. Cum autem radii punctum l formantes ad axem inclinati sint angulo $Ol T = \mathfrak{V}x$, ii per lentis oculi puncta TT ita intrabunt, vt neglecto interuallo Ll prae distantia $OL = l$, sit $OT = l\mathfrak{V}x$, ex quo ii in puncto v concurrentes cum axe angulum facient $Ov T = \frac{OT}{Ov} = \frac{OT}{OV} = \frac{l}{u}\mathfrak{V}x$. Hinc ergo vltra ad retinam pergentes super ea circulum UU effingent cuius radius erit $VU = \frac{l}{u}\mathfrak{V}x$. $Vv = \frac{l}{u}$. $\mathfrak{V}x$. $\frac{uu}{ll}$. Vxx et a punctis inter L et l mediis hoc spatium circulare super retina replebitur. Quare imago per spatium Ll diffusa super retina circello repraesentabitur, cuius radius $VU = \frac{u}{l}\mathfrak{V}$. Vx^2 qui circellus veram confusionis mensuram supeditat. Quodlibet scilicet obiecti punctum, quod lentibus per spatium Ll diffusum exhibetur, in oculo super retina non puncto sed circello exprimetur cuius radius erit $= \frac{u}{l}$. $\mathfrak{V}$. Vx^2.

Coroll. 1.

166. Videmus ergo confusionem, qua visio afficitur non solum a quantitate spatii diffusionis $Ll = Vxx$, sed insuper ab angulo, quo radii in l concurrentes ad axem inclinantur, qui est $= \mathfrak{V}x$ pendere; radiumque circelli confusionem metientis producto illius spatii per hunc angulum esse proportionalem.

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Coroll. 2.

167. Cum igitur posito semidiametro aperturae primae lentis $= x$, spatium diffusionis sit vt eius quadratum xx; radius circelli confusionem metientis est vt eius cubus x^3. Ac si confusio ipsa areae huius circelli proportionalis aestimetur erit ea vt x^6, seu vt cubus aperturae primae lentis.

Coroll. 3.

168 Hinc ergo intelligitur quanti interfit per idoneam lentium dispositionem spatium diffusionis Vxx diminuisse non solum enim in eadem ratione qua spatium diffusionis Vxx seu quantitas V diminuitur sed adeo in ratione duplicata, ipsa confusio visa minor redditur.

Scholion.

169. Assumsi hic pupillam tam late patere, vt radios ab l diuergentes recipiat; at si apertura pupillae minor esset, radios a puncto l venientes nequidem caperet: hoc ergo casu res eodem rediret ac si contracta primae lentis apertura spatium diffusionis Ll eo vsque diminueretur quoad pupilla omnes radios ab imagine emissos recipere posset hocque casu manifestum est, confusionem minorem esse prodituram. Verum in sequentibus ostendetur, tam in Telescopiis quam Microscopiis hunc casum vix vnquam locum inuenire cum plerumque conus radiosus ex puncto l emissus circa ingressum in oculum multo

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to tenuior sit quam pupillae apertura; quamobrem ne opus quidem est hunc casum etsi per se facile expediretur, hic expendere. Ceterum tametsi oculus non commode cum lente simplici cuius crassities evanescat, conferri possit hincque expressio spatii $V\omega$ secundum §. 84. aliquantum diuersa prodire potuisset hic ad istam circumstantiam non attendimus propterea quod spatium Vv secundum certam quandam rationem auctum vel diminutum prodiisset; Hic autem non adeo necesse est ipsam quantitatem absolutam confusionis cognouisse, dummodo rationem quam sequitur accurate definiuerimus. Cum enim nostrum calculum cum experientia contulerimus, terminum cognoscemus, quem si confusio nostro more expressa superauerit, intolerabilis euadat; hincque perpetuo sufficiet confusionem simili modo expressam infra hunc terminum reduxisse. Interim tamen notasse iuvabit, per oculi conformationem confusionem adhuc multo minorem effici posse.

Problema 2.

170. Positis iisdem, quae in praecedente problemate sunt assumta, eam definire oculi conformationem, qua minima confusio percipiatur. Tab. II. Fig. II.

Solutio.

Oculus omnis facultate praeditus est sese aliquantillum aliter conformandi, vt etiam obiecta, quorum distantia a iusta non nimis differt, distincte videat:

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videat: quod quomodocunque perficiatur, id ita fieri concipere licet, ac si retina ad pupillam seu potius eam lentem, quam oculi loco consideramus, propius admoueretur seu longius ab ea detorqueretur. Cum ergo ante retinam in V sitam simus contemplati, hic primo sumamus retinam per ipsum punctum v transire, ac manifestum est super ea punctum l distincte expressum iri. Verum quia punctum L alios radios nisi axi proximos non emittit etiam punctum L sine vlla confusione in v repraesentabitur; ita vt tantum puncta media inter L et l confusionem sint paritura. Consideretur ergo quoduis punctum intermedium λ, quod esset extremum in spatio confusionis, si semidiameter aperturae primae lentis minor foret quam x, qui ergo ponatur $=z$: eritque $L\lambda = V zz$, et inclinatio radiorum in λ concurrentium ad axem $= \mathfrak{V} z$: hinc istius puncti imago intra oculum referetur in υ, vt fit $V\upsilon = \frac{uu}{ll} V zz$, ideoque ob $Vv = \frac{uu}{ll} Vxx$ erit interuallum $v\upsilon = \frac{uu}{ll} V(xx - zz)$: et radiorum in υ conuergentium inclinatio ad axem $= \frac{l}{u} \mathfrak{V} z$; ex quo circelli super retina in v existentis radiis puncti λ expressi semidiameter erit $= \frac{u}{l} \mathfrak{V} V z(xx - zz)$ qui euanescit, vti iam monuimus, siue sit $z = 0$, siue $z = x$. Quaeratur iam ille valor ipsius z, quo ille circellus fiat maximus; id quod eueniet, si $xx - 3zz = 0$ seu $z = \frac{x}{\sqrt{3}}$; sicque circelli confusionem hoc casu metientis semidiameter erit $= \frac{2u}{3l\sqrt{3}} \mathfrak{V} V x^3$, quod est multo minus quam casu praecedente, quo retina erat in V.

At

At si retina aliquantillum a v versus V remoueatur, hic circellus adhuc minor effici poterit. Ponatur enim spatium $VS = s$, vt sit $vS = \frac{uu}{ll} V xx - s$, et $S\upsilon = s - \frac{uu}{ll} V zz$. Punctum ergo l, cuius effigies in u exhibetur, super retina S circello referetur, cuius radius est $= \frac{l}{u} \mathfrak{B} x (\frac{uu}{ll} V xx - s)$, punctum vero λ, cuius effigies est in υ, super retina S circello, cuius radius erit $= \frac{l}{u} \mathfrak{B} z (s - \frac{uu}{ll} V zz)$; nunc igitur id punctum λ inuestigemus, vnde iste circellus minimus euadat: quod fit si sit $s = \frac{3uu}{ll} V zz$; eritque huius circuli radius $= \frac{2u}{l} \mathfrak{B} V z^3$, qui simul confusionem exhiberet, si modo a puncto l, non maior circulus oriretur: sed substituto pro s valore inuento huius circuli radius erit, $\frac{u}{l} \mathfrak{B} V x (xx - 3zz)$, qui ergo illi $\frac{2u}{l} \mathfrak{B} V z^3$ aequalis statuatur, vnde nascitur haec aequatio:

$$x^3 - 3xzz = 2z^3, \text{ hincque } z = \tfrac{1}{2} x.$$

Quare circelli, quo minima confusio mensuratur, radius erit $= \frac{u}{4l} \mathfrak{B} V x^3$, quadruplo minor, quam si retina esset in V; indeque ipsa confusio sedecies minor. Etsi autem illa aequatio cubica tres habet radices, praeter $z = \frac{1}{2} x$ binae reliquae sunt aequales et $z = -x$, sicque $zz = xx$, vnde non minima sed quasi maxima confusio nasceretur.

Coroll. 1.

171. Cum igitur, vt oculus minimam confusionem sentiat, debeat esse $z = \frac{1}{2} x$, ideoque $s = \frac{3uu}{4ll} V xx$,

patet retinam in eum locum S cogi debere, vt sit $VS = \frac{3}{4} Vv$ et $vS = \frac{1}{4} Vv$.

Coroll. 2.

172. Cum ergo oculus non solum praeditus sit facultate sese parumper immutandi sed etiam hac facultate vti soleat ad confusionem euitandam, nullum est dubium quin circellorum confusionem producentium radius sit $= \frac{n}{4l} \mathfrak{B} V x^3$, ideoque quadruplo minor, quam problemate praecedente inueneramus dummodo imago spectanda Ll propemodum in distantia iusta reperiatur.

Coroll. 3.

173. Haec igitur expressio $\frac{n}{4l} \mathfrak{B} V x^3$ iustam nobis exhibet mensuram confusionis, cum exprimat radium circellorum, quibus singula obiecti puncta, quatenus id per lentes spectatur, super retina repraesentantur. Si modo imago in distantia iusta ab oculo reperiatur.

Scholion.

174. Si hi circelli euanescerent, visio plane esset distincta hoc autem fieri nequit, nisi ipsum interuallum diffusionis Vxx euanescat: vnde patet si apertura primae lentis ad nihilum reduceretur, nullam confusionem sentiri debere. Verum visio non ita est delicata, vt prorsus nullam confusionem pati possit sed dummodo confusio certum quendam termi-

num

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num non superet, quasi esset nulla considerari potest. Iste terminus, seu valor quem expressio $\frac{n}{n-1}\mathfrak{V}v^{\frac{1}{n}}$ excedere non debet, ex experientia potius peti debet, quam ex Theoria isque idcirco insigni adhuc latitudine continetur: vnde fit vt pro diuerso scopo modo maiore gradu confusionis contenti esse soleamus, modo autem minorem gradum exigamus: quas circumstantias cum ad praxin propius accedemus, accuratius sumus examinaturi. Ceterum notandum est pro interuallo u, quo quasi profunditas oculi exhibetur, vnum circiter pollicem assumi posse, quae quidem mensura fere erit arbitraria, cum deinceps limites confusionis per experientiam constituemus.

Problema. 3.

175. Si oculus per vnicam lentem obiectum aspiciat ita vt imago visa ab oculo in distantia iusta reperiatur, definire confusionem, qua visio afficietur. Tab. I. Fig. 3.

Solutio.

Sit distantia obiecti Ee ante lentem $AE = a$, imaginis vero principalis $F\zeta$ post lentem $BF = \alpha$, lentisque crassities $AB = v$, quantitas vero arbitraria qua cum binis distantiis determinatricibus a et α lentis facies determinatur, sit $= k$, ponatur autem breuitatis gratia $\frac{k-v}{k+v} = i$. Tum autem posita ratione refractionis $= n$, debet esse lentis PP

radius faciei anterioris $= \frac{(n-1)a(k+v)}{k+v+2na}$

radius faciei posterioris $= \frac{(n-1)\alpha(k-v)}{k-v-2n\alpha}$

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ac si semidiameter aperturae faciei anterioris sit $=x$, posterioris vero $>ix$ spatium diffusionis Ff erit $=P\alpha\alpha xx$ existente

$$P=\frac{n}{2(n-1)^2}\left(\frac{1}{ii}\left(\frac{n}{a}+\frac{2}{k+v}\right)\left(\frac{1}{a}+\frac{2}{k+v}\right)^2+ii\left(\frac{n}{a}-\frac{2}{k-v}\right)\left(\frac{1}{a}-\frac{2}{k-v}\right)^2\right)$$

radiorum vero in f concurrentium inclinatio ad axem $=i.\frac{x}{a}$. His, quae §. 86 sunt stabilita, praemissis, sit distantia oculi post lentem $BO=O$, et quia imago $F\zeta$ ante oculum in distantia iusta existere assumitur, erit $O=a+l$, ideoque $a=O-l$, si locus oculi vt datus consideretur. Hinc ergo habebimus $V=P\alpha\alpha$ et $\mathfrak{V}=\frac{i}{a}$ ideoque $\mathfrak{V}V=ia\mathrm{P}$. Consequenter radius circellorum in oculo confusionem metientium, seu vt in posterum loquemur mensura confusionis erit

$$=\frac{n}{+l}.iax^2.P=\frac{in}{+l}(O-l)x^2P.$$

Si lentis crassities v euanescat, et pro determinatione lentis numerus arbitrarius λ loco k introducatur, erit ex §. 91 $P=\mu\left(\frac{1}{a}+\frac{1}{a}\right)\left(\lambda\left(\frac{1}{a}+\frac{2}{a}\right)^2+\frac{v}{aa}\right)$, et $i=1$ ex quo loco citato etiam ipsa lentis constructio est petenda.

Coroll. 1.

176. Si igitur lens fuerit data, locus oculi post lentem ita definitur, vt debeat esse distantia $BO=O=a+l$, existente l distantia oculi iusta: Sin autem locus oculi detur, pro lentis constructione distantia determinatrix a ita capi debet, vt sit $a=O-l$.

Coroll.

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Coroll. 2.

177. Quare si oculus ita fuerit compasatus, vt exigat distantiam iustam $l=\infty$, fiet $\alpha=-\infty$, et mensura confusionis erit $=-\frac{1}{4}iux^2$. P seu $=\frac{1}{4}iux^2$. P, quia signum — nihil mutat in magnitudine circellorum confusionem producentium.

Coroll. 3.

178. Etsi ergo hoc casu quo $\alpha=\infty$ spatium diffusionis Ff est infinitum, tamen inde confusio in visione orta est finita, quia hoc non obstante valor ipsius P manet finitus: erit enim

$$P=\frac{n}{2(n-1)^2}\left(\frac{1}{ii}\left(\frac{n}{a}+\frac{2}{k+v}\right)\left(\frac{i}{a}+\frac{2}{k+v}\right)^2-\frac{uii}{(k-v)^2}\right)$$

Radius autem faciei posterioris fit $=-\frac{(n-1)}{2n}(k-v)$.

Coroll. 4.

179. Quia est $i=\frac{k-v}{k+v}$, valor ipsius P etiam in genere ita exprimi potest, vt sit:

$$P=\frac{n}{2(n-1)^2}\left(\left(\frac{n}{a}+\frac{2}{k+v}\right)\left(\frac{1}{1a}+\frac{2}{k-v}\right)^2+\left(\frac{n}{a}-\frac{2}{k-v}\right)\left(\frac{1}{a}-\frac{2}{k+v}\right)^2\right)$$

vbi est $\frac{n}{2(n-1)^2}=\frac{330}{121}=2,561983$ ob $n=\frac{21}{20}$. Sicque etiam valores reliquarum litterarum Q, R, S etc. in §. 86 transformari poterunt.

Problema 4.

180. Si oculus per duas lentes obiectum Ee aspiciat, ita vt imago per eas repraesentata $G\eta$ in distantia iusta $OG=l$ ob oculo sit remota, definire confusionem, qua visio afficietur. Tab. I. Fig. 5.

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Solutio.

Sint vt hactenus pro lente PP distantiae determinatrices $AE=a$, $aF=\alpha$, crassities $Aa=v$, et distantia arbitraria $=k$, ita vt sit

radius faciei anterioris $=\frac{(n-1)a(k+v)}{k+v+2na}$

radius faciei posterioris $=\frac{(n-1)\alpha(k-v)}{k-v-2n\alpha}$

tum vero posito $\frac{k-v}{k+v}=i$ ponatur

$$P=\frac{n}{2(n-1)^2}\left(\left(\frac{n}{a}+\frac{2}{k+v}\right)\left(\frac{1}{a}+\frac{2}{k+v}\right)^2+\left(\frac{n}{\alpha}-\frac{2}{k-v}\right)\left(\frac{i}{\alpha}-\frac{2}{k+v}\right)^2\right)$$

Deinde pro lente posteriori QQ sint distantiae determinatrices $BF=b$, $bG=\mathfrak{G}$, crassities lentis $Bb=v'$ et distantia arbitraria $=k'$ vt sit

radius faciei anterioris $=\frac{(n-1)b(k'+v')}{k'+v'+2nb}$

radius faciei posterioris $=\frac{(n-1)\mathfrak{G}(k'-v')}{k'-v'-2n\mathfrak{G}}$

tum vero posito $\frac{k'-v'}{k'+v'}=i'$ ponatur

$$Q=\frac{n}{2(n-1)^2}\left(\left(\frac{n}{b}+\frac{2}{k'+v'}\right)\left(\frac{1}{b}+\frac{2}{k'+v'}\right)^2+\left(\frac{n}{\mathfrak{G}}-\frac{2}{k'-v'}\right)\left(\frac{i'}{\mathfrak{G}}-\frac{2}{k'+v'}\right)^2\right)$$

Atque iam erit spatium diffusionis

$$Gg=\mathfrak{G}\mathfrak{G}xx\left(\frac{1}{ii'}\cdot\frac{\alpha\alpha}{bb}P+ii\cdot\frac{bb}{\alpha\alpha}Q\right)=Vxx$$

et radiorum in g concurrentium inclinatio ad axem $ii'\cdot\frac{bx}{\alpha\mathfrak{G}}=\mathfrak{V}x$,

Sit iam oculus in O existente $OG=l$, ac ponatur eius post lentem QQ distantia $bO=O$ erit $O=\mathfrak{G}+l$, ideoque

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ideoque $\mathfrak{E} = O - l$; quibus positis cum sit mensura confusionis $= \frac{u}{4l} \mathfrak{V} x$. $V x x$ erit ea pro nostro casu:

$$\frac{ii'u}{4l} \cdot \frac{b\mathfrak{E}}{a} x^3 \left(\frac{1}{i'i'} \cdot \frac{aa}{bb} P + ii \cdot \frac{bb}{aa} Q \right)$$

vel pro $\mathfrak{E}$ posito valore $O - l$ et signo mutato

$$\frac{1}{4} ii'u \left(1 - \frac{O}{l}\right) \cdot \frac{b}{a} x^3 \left(\frac{1}{i'i'} \cdot \frac{aa}{bb} P + ii \cdot \frac{bb}{aa} Q \right).$$

Coroll. 1.

181. Si ergo pro oculo fuerit $l = \infty$, ideoque et $\mathfrak{E} = \infty$, etsi spatium diffusionis Gg fit infinitum tamen ob $\frac{O}{l} = 0$ confusio visionem afficiens nihilominus erit finita: neque enim P neque Q ob $\mathfrak{E} = \infty$ fit infinita.

Coroll. 2.

182. Si crassities lentium euanescat, vt sit $v = 0$ et $v' = 0$ erit $i = 1$ et $i' = 1$, ideoque hoc casu mensura confusionis

$$\frac{1}{4} u \left(1 - \frac{O}{l}\right) \cdot \frac{b}{a} x^3 \left(\frac{aa}{bb} P + \frac{bb}{aa} Q \right)$$

At si loco k et k' introducantur numeri λ et λ' erit

$$P = \mu \left(\frac{1}{a} + \frac{1}{\alpha}\right) \left(\lambda \left(\frac{1}{a} + \frac{1}{\alpha}\right)^2 + \frac{\nu}{a\alpha}\right) \text{ et}$$

$$Q = \mu \left(\frac{1}{b} + \frac{1}{\beta}\right) \left(\lambda' \left(\frac{1}{b} + \frac{1}{\beta}\right)^2 + \frac{\nu}{b\beta}\right)$$

existente

$$\mu = 0,938191 \text{ et } \nu = 0,232692$$

Coroll.

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Coroll. 3.

183. Eodem autem hoc casu constructio binarum lentium ita se habebit:

radius faciei

Pro lente PP $\begin{cases} \text{anterioris} = \frac{\alpha\alpha}{\varrho\alpha+\sigma\alpha\pm\tau(\alpha+\alpha)\sqrt{(\lambda-1)}} \\ \text{posterioris} = \frac{\alpha\alpha}{\varrho\alpha+\sigma\alpha\mp\tau(\alpha+\alpha)\sqrt{(\lambda-1)}} \end{cases}$

Pro lente QQ $\begin{cases} \text{anterioris} = \frac{b\beta}{\varrho\beta+\sigma b\pm\tau(b+\beta)\sqrt{(\lambda'-1)}} \\ \text{posterioris} = \frac{b\beta}{\varrho b+\sigma\beta\mp\tau(b+\beta)\sqrt{(\lambda'-1)}} \end{cases}$

existente $\varrho = 0,190781$; $\sigma = 1,627401$ et $\tau = 0,905133$.

Problema 5.

Tab. II. Fig. 6.

184. Si oculus per tres lentes PP, QQ et RR obiectum $E\varepsilon$ aspiciat, ita vt imago per eas repraesentata $H\theta$ in distantia iusta $OH = l$ ab oculo O sit remota, definire confusionem, qua visio afficietur.

Solutio.

Positis duabus prioribus lentibus PP et QQ vt in problemate praecedente, indeque determinatis valoribus P et Q cadat imago per has duas lentes repraesentata principalis in $G\eta$, post quam tertia lens RR ita collocata sit, vt sint eius distantiae determinatrices $CG = c$, $cH = \gamma$, crassities $Cc = v''$ et quantitas arbitraria $= k''$ vt sit:

radius faciei anterioris $= \frac{(n-1)c(k''+v'')}{k''+v''+2nc}$

radius faciei posterioris $= \frac{(n-1)\gamma(k''-v'')}{k''-v''-2n\gamma}$

Tum

Tum vero posito $\frac{k^{\alpha}-v^{\alpha}}{k^{\alpha}+v^{\alpha}}=i^{\alpha}$ ponatur

$$R=\frac{n}{2(n-1)^2}\left(\left(\frac{n}{c}+\frac{2}{k^{\alpha}+v^{\alpha}}\right)\left(\frac{1}{i^{\alpha}c}+\frac{2}{k^{\alpha}+v^{\alpha}}\right)^2+\left(\frac{n}{v}-\frac{2}{k^{\alpha}+v^{\alpha}}\right)\left(\frac{i^{\alpha}}{v}-\frac{2}{k^{\alpha}+v^{\alpha}}\right)^{\alpha}\right)$$

atque iam spatium diffusionis erit

$$Hb=\gamma\gamma xx\left(\frac{1}{i'i'.i''i''},\frac{\alpha\alpha\varepsilon\varepsilon}{bbcc}P+\frac{ii}{i''i''}.\frac{bb\varepsilon\varepsilon}{\alpha\alpha cc}Q+ii.i'i'.i''i''.\frac{bbcc}{\alpha\alpha\varepsilon\varepsilon}R\right)$$

quod est valor ipsius Vxx. Radiorum vero in b concurrentium inclinatio ad axem est

$$i\,i'\,i''.\frac{bcx}{\alpha\varepsilon\gamma}=\mathfrak{B}x$$

sit iam oculus in O, ac ponatur eius distantia post lentem $RR=O$ erit $O=\gamma+l$, ideoque $\gamma=O-l$. Hinc mensura confusionis in oculo ortae colligitur

$$\tfrac{1}{2}ii'i''u\left(1-\tfrac{O}{l}\right)\frac{bc}{\alpha\varepsilon}x^3\left(\frac{1}{i'i'.i''i''}.\frac{\alpha\alpha\varepsilon\varepsilon}{bbcc}P+\frac{ii}{i''i''}.\frac{bb\varepsilon\varepsilon}{\alpha\alpha cc}Q+ii.i'i'.\frac{bbcc}{\alpha\alpha\varepsilon\varepsilon}R\right)$$

Coroll. 1.

185. Hic iterum vt ante patet, si fuerit $l=\infty$ ideoque et $\gamma=\infty$, quo casu diffusionis spatium in infinitum extenditur, mensuram confusionis terminis finitis contineri, quod etiam locum habet pro quouis lentium numero.

Coroll. 2.

186. Si lentium crassities pro euanescente habeatur, ob $i=1$ $i'=1$, $i''=1$, mensura confusionis ita simplicius exprimetur, vt sit

$$=\tfrac{1}{2}u\left(1-\tfrac{O}{l}\right)\frac{bc}{\alpha\varepsilon}x^3\left(\frac{\alpha\alpha\varepsilon\varepsilon}{bbcc}P+\frac{bb\varepsilon\varepsilon}{\alpha\alpha cc}Q+\frac{bbcc}{\alpha\alpha\varepsilon\varepsilon}R\right)$$

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At hoc casu loco k, k', k'' introductis numeris $\lambda, \lambda', \lambda''$ erit

$$P = \mu\left(\tfrac{1}{a}+\tfrac{1}{a}\right)\left(\lambda\left(\tfrac{1}{a}+\tfrac{1}{a}\right)^2+\tfrac{2}{aa}\right)$$

$$Q = \mu\left(\tfrac{1}{b}+\tfrac{1}{\varepsilon}\right)\left(\lambda''\left(\tfrac{1}{b}+\tfrac{1}{\varepsilon}\right)^2+\tfrac{2}{b\varepsilon}\right)$$

$$R = \mu\left(\tfrac{1}{c}+\tfrac{1}{\gamma}\right)\left(\lambda''\left(\tfrac{1}{c}+\tfrac{1}{\gamma}\right)^2+\tfrac{2}{c\gamma}\right)$$

Coroll. 3.

187. Eodem vero casu constructio lentium per istos numeros $\lambda, \lambda', \lambda''$ ita erit dirigenda vt sit

Radius faciei

Pro lente P P
- anterioris $= \dfrac{aa}{\varrho a+\sigma a \pm \tau(a+a)\sqrt{(\lambda-1)}}$
- posterioris $= \dfrac{aa}{\varrho a+\sigma a \mp \tau(a+a)\sqrt{(\lambda-1)}}$

Pro lente Q Q
- anterioris $= \dfrac{b\varepsilon}{\varrho\varepsilon+\sigma b \pm \tau(b+\varepsilon)\sqrt{(\lambda'-1)}}$
- posterioris $= \dfrac{b\varepsilon}{\varrho b+\sigma\varepsilon \mp \tau(b+\varepsilon)\sqrt{(\lambda'-1)}}$

Pro lente R R
- anterioris $= \dfrac{c\gamma}{\varrho\gamma+\sigma c \pm \tau(c+\gamma)\sqrt{(\lambda''-1)}}$
- posterioris $= \dfrac{c\gamma}{\varrho c+\sigma\gamma \mp \tau(c+\gamma)\sqrt{(\lambda''-1)}}$

Scholion.

188. Hinc satis manifestum est, quemadmodum hae formulae pro pluribus lentibus progrediantur: verum antequam eas exponam, conueniet alias quoque circumstantias, quae hinc facillime deducuntur, perpendi, scilicet magnitudinem obiecti visam, et copiam radiorum a singulis eius punctis in oculum transmissorum vt hae simul cum confusione visionis dein-

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deinceps coniunctim pro quouis lentium numero exhiberi queant; quo pacto plures taedioſas repetitiones euitabimus. Duae autem res ex hactenus allatis facile definiri poſſunt, quarum altera eſt quantitas, qua imago obiecti per lentes repraeſentata ab oculo cernitur, quae quantitas aeſtimanda eſt ex angulo, ſub quo imago videtur, vt is deinceps comparari poſſit, cum eo angulo ſub quo ipſum obiectum in data diſtantia a nudo oculo ſpectaretur, vnde qua ratione magnitudo per lentes viſa augeatur, intelligetur. Altera res in copia radiorum, a ſingulis obiecti punctis in oculum transmiſſorum verſatur, qua claritas viſione percepta continetur, a quolibet ſcilicet puncto conus ſeu cylindrus radioſus in oculum ingreditur, qui ſi pupillam penitus expleat, claritas ad ſummum gradum erit euecta, niſi forte maiori illuſtratione ipſi obiecto maius lumen concilietur. At ſi ſectio illius coni aut cylindri, qua in oculum intrat, minor fuerit pupilla, in eadem ratione claritas decreſcet; quod cum in omnibus inſtrumentis dioptricis, quibus magnitudinem viſam vehementer augere propoſitum eſt, vſu venire ſoleat, plurimum intererit amplitudinem illius coni ſeu cylindri, qua in oculum penetrat, accurate determinaſſe.

Problema 6.

189 Definire quantitatem, ſub qua quaeuis obiecti portio per lentes quotcunque ab oculo in diſtantia iuſta ab imagine vltima remoto cernetur.

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Solutio.

Sit z linea in obiecto concepta, quae quanta per lentes oculo sit apparitura, definiri oporteat. Ostensum autem est in praecedentibus, quotcunque fuerint lentes, imaginem principalem huius lineae iterum esse lineam, cuius longitudo ad z certam teneat rationem a distantiis determinatricibus lentium et numeris i, i', i'', i''' etc. pendentem (86). Sit ergo haec longitudo imaginis $= Mz$, quae cum ab oculo in distantia l remoto aspiciatur, apparebit sub angulo $= \frac{Mz}{l}$; vel cuius tangens potius sit $= \frac{Mz}{l}$; sed quia hic angulus rarissime vltra aliquot gradus assurgere solet, tangens tuto pro ipso arcu assumitur. Effigies autem quae ab hac linea Mz in oculo exprimitur, erit $= \frac{Muz}{l}$; hic enim cogitationes a confusione abstraho, qua vtique fit, vt effigies maior imprimatur, propterea quod singula puncta circellis exhibentur. Iam positis iisdem lentium determinationibus, quibus supra §. 86 sum vsus, pro vario lentium numero angulus, sub quo linea z in obiecto sumta cernetur ita se habebit:

	Angulus visionis	situ
Pro vnica lente	$\ldots 1 \cdot \frac{\alpha}{a} \cdot \frac{z}{l} \ldots$	inuerso
Pro duabus lentibus	$\ldots \frac{1}{ii'} \cdot \frac{\alpha\varepsilon}{ab} \cdot \frac{z}{l} \ldots$	erecto
Pro tribus lentibus	$\ldots \frac{1}{ii'i''} \cdot \frac{\alpha\varepsilon\gamma}{abc} \cdot \frac{z}{l} \ldots$	inuerso
Pro quatuor lentibus	$\ldots \frac{1}{ii'i''i'''} \cdot \frac{\alpha\varepsilon\gamma\delta}{abcd} \cdot \frac{z}{l} \ldots$	erecto

etc.

Scilicet

Scilicet si hae formulae sint positiuae, oculus lineam z situ siue erecto siue inuerso videbit, prout est notatum sin autem fuerint negatiuae, situm indicatum in contrarium verti oportet.

Coroll. 1.

190. Si eadem obiecti linea z in distantia $=b$ ab oculo nudo cerneretur, ea apparitura esset sub angulo $=\frac{z}{b}$, vnde perspicitur, quanto ea vel maior vel minor per lentes videatur.

Coroll. 2.

191. Si distantia oculi a postrema lente ponatur $=O$, erit casu vnius lentis $\alpha=O-l$, ideoque $\frac{\alpha}{l}=-(1-\frac{O}{l})$ vnde angulus opticus lineae obiecti z respondens erit $=\frac{1}{l}(1-\frac{O}{l})\frac{z}{a}$ pro situ erecto, quia signum mutauimus.

Coroll. 3.

192. Simili modo casu duarum lentium ob $\beta=O-l'$ erit iste angulus $=\frac{1}{ll'}(1-\frac{O}{l})\frac{\alpha z}{ab}$ pro situ inuerso.

Casu vero trium lentium ob $\gamma=O-l''$ erit

iste angulus $=\frac{1}{ll'l''}(1-\frac{O}{l})\frac{\alpha\beta z}{abc}$ pro situ erecto

Casu quatuor lentium ob $\delta=O-l'''$ erit

iste angulus $\frac{1}{ll'l''l'''}(1-\frac{O}{l})\frac{\alpha\beta\gamma z}{abcd}$ pro situ inuerso, et ita porro pro pluribus lentibus.

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Scholion.

193. Hinc etiam modus se offert confusionem ob lentium aperturam in oculo natam distinctius aestimandi. Scilicet cum singula obiecti puncta in oculo exprimantur circulis, quorum radius est $= \frac{u}{4l} \mathfrak{V} V x^3$, verus autem circulus cuius radius $= z$, oculo in distantia l expositus in oculo referatur circulo cuius radius $= \frac{u}{l} z$ singula puncta illius obiecti per lentes spectati, aeque magna apparebunt, ac orbes circulares radii $z = \frac{1}{4} \mathfrak{V} V x^3$, si in distantia ab oculo $= l$ spectarentur. Vel cum horum orbium semidiameter apparens sit $= \frac{z}{l}$, ob confusionem singula obiecti puncta instar circulorum videbuntur, quorum semidiameter apparens esset $= \frac{1}{4l} \mathfrak{V} V x^3$. In expressionibus igitur ante pro confusione inuentis deleatur quantitas u, et habebitur semidiameter apparens circulorum confusionem exprimentium. Hinc iudicari poterit quam parua esse debeat confusio, vt non amplius sentiatur, scilicet si oculus non amplius percipere valeat spatium circulare, cuius semidiameter esset $1''$, seu $\frac{1}{600}$ pars radii circiter, euidens est, si fuerit nostra expressio $\frac{1}{4l} \mathfrak{V} V x^3 = \frac{1}{600}$, confusionem fore imperceptibilem. Ac experientiam consulentes deprehendimus multo maiores angulos non amplius percipi posse, ita vt confusio non sit metuenda etiamsi expressio $\frac{1}{4l} \mathfrak{V} V x^3$ notabiliter maior fuerit, quam $\frac{1}{600}$; ne autem hic temere quicquam statuamus, ponamus limitem, quem formula $\frac{1}{4l} \mathfrak{V} V x^3$ excedere non

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non debeat, esse $= \frac{1}{4x^2}$, ita vt esse oporteat $\frac{1}{4}\mathfrak{V}Vx^2 < \frac{1}{x^2}$. Postea igitur ad praxin descendentes poterimus pro x numerum vel 40 vel minorem assumere prout experientia quouis casu postulauerit. Quod si ergo hoc modo confusionis rationem habeamus, profunditas oculi a non amplius in computum ingredietur.

Definitio 3.

194. Semidiameter confusionis, est semidiameter apparens circuli, qui ab oculo aeque magnus videtur, ac singula obiecti puncta ipsi ob confusionem apparent.

Corollarium.

195. Inueniemus igitur facile semidiametrum confusionis si formulas supra pro confusione repertas per profunditatem oculi a diuidamus quo pacto eae formulae ad numeros absolutos reducentur.

Definitio 4.

196. Multiplicatio per lentes producta ex ratione quantitatis, qua obiecta per lentes spectantur, ad quantitatem, qua eadem obiecta in data distantia ab oculo nudo cernerentur aestimatur. Exponens autem multiplicationis inuenitur, si magnitudo, qua linea quaecunque in obiecto concepta per lentes videtur, diuidatur per magnitudinem, qua eadem linea in data distantia ab oculo nudo spectata esset apparitura.

Coroll.

Coroll. 1.

197. Inuoluit ergo diiudicatio multiplicationis distantiam quandam fixam, in qua eadem obiecta a nudo oculo aspici assumimus quae prout diuersa assumatur, multiplicatio alio atque alio modo exprimetur.

Coroll. 2.

198. Si haec distantia fixa, ex qua multiplicatio diiudicatur ponatur $= b$ et exponens multiplicationis $= m$, sit linea quaepiam in obiecto concepta $= z$, quae ergo nudo oculo in distantia b appareret sub angulo $= \frac{z}{b}$; eadem autem linea per lentes spectetur sub angulo $= \frac{mz}{l}$ (88) ex quo erit exponens multiplicationis $m = \frac{mb}{l}$.

Coroll. 3.

199. In ratione ergo m: 1 dimensiones lineares per lentes augeri sunt censendae; vnde superficies auctae apparebunt in ratione mm: 1, et ipsa corpora in ratione m^3: 1. Cum exponente autem multiplicationis coniungi debet situs, quo obiecta apparent siue is sit erectus siue inuersus.

Scholion.

200. Distantia haec fixa b ad quam multiplicatio refertur non eodem modo perpetuo assumi solet, quippe quod etiam pro diuersitate obiectorum omnino fieri non posset. Nam si per lentes obiecta valde

valde remota veluti coeleſtia contuemur, quoniam ea nunquam in diſtantia modica ſpectare ſolemus, conuenit vtique magnitudinem per lentes viſam cum ea comparare, qua in ea ipſa a nobis diſtantia nudis oculis cernerentur: ideoque his caſibus diſtantia fixa b ipſi diſtantiae a, qua a lentibus obiecta ſunt remota, aequalis conſtitui ſolet. Scilicet ſi diſtantia a fuerit valde magna, qui eſt caſus Teleſcopiorum, ſtatuitur $b = a$; et magnitudo per haec inſtrumenta viſa, cum magnitudine per nudos oculos viſa in eadem diſtantia commodiſſime comparatur. Sic de Teleſcopiis dicitur, quoties diametri corporum coeleſtium multiplicentur, eoque caſu littera m exponentem huius rationis indicabit. Sin autem obiecta propiora contemplamur, qui eſt vſus Microſcopiorum ea plerumque ita prope ad inſtrumentum admouentur, vt in tam exigua diſtantia nudis oculis nunquam diſtincte cerni poſſent: neque ergo his caſibus ſtatui $b = a$ conueniret. Aliam ergo rationem ineundo pro b ſumi ſolet eiusmodi diſtantia modica, in qua obiecta commode ac diſtincte cernere liceat; quae etſi vtique pro diuerſa oculorum indole diuerſa ſumi deberet; tamen vt aliquid fixi ſtatuatur, pro b diſtantia 8 pollicum vtpote maximae oculorum parti conueniens accipi ſolet, ita vt his caſibus definiamus, quoties huiusmodi obiecta maiora *appareant*, quam ſi eadem nudis oculis in diſtantia 8 digitorum aſpicerentur. Interim tamen ſi multiplicatio ad hanc diſtantiam fuerit relata, non difficile erit eam ad quam-

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libet aliam referre, ita vt haec hypotheſis naturam horum inſtrumentorum afficere non ſit cenſenda.

Problema 7.

Tab. II. Fig. 10.

201. Si obiectum per lentes quotcunque aſpiciatur, definire amplitudinem coni ſeu cylindri luminoſi, qui a ſingulis obiecti punctis in oculum transmittitur.

Solutio.

Reperiatur vt ſupra oculus in diſtantia iuſta l poſt imaginem poſtremam per lentes repraeſentatam quae etſi eſt per ſpatium aliquod Ll diffuſa, hic tamen a confuſione inde nata mentem abſtrahimus, quoniam confuſionem iam ſeorſim determinauimus. Ex puncto igitur l oculum verſus diffunditur conus luminoſus, cuius radii extremi ad axem inclinati ſunt angulo Olt quem poſuimus ſupra $= \mathfrak{V}x$. Huius igitur coni ſectio circa ingreſſum in oculum conſideretur, cuius ſemidiameter erit $= \mathfrak{V}lx$, vnde pro vario lentium numero hic ſemidiameter ſequenti modo definietur.

Pro vnica lente . . . . . $il\frac{x}{\alpha}$

Pro duabus lentibus . . . . . $ii'l\frac{bx}{\alpha\beta}$

Pro tribus lentibus . . . . $ii'i''l\frac{bcx}{\alpha\beta\gamma}$

Pro quatuor lentibus . . . $ii'i''i'''l\frac{bcdx}{\alpha\beta\gamma\delta}$

etc.

hicque perinde eſt ſiue hae formulae ſint poſitiuae ſiue negatiuae; quia circulus ſiue radio poſitiuo ſiue negatiuo deſcribatur eiusdem prodit magnitudinis.

Coroll.

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Coroll. 1.

202. Si iſte ſemidiameter $\mathfrak{B}lx$ maior fuerit ſemidiametro pupillae, tota pupillae apertura radiis impletur neque propterea viſio clarior procurari poterit, niſi forte ipſum obiectum fortiori illuſtratione ſplendidius reddatur.

Coroll. 2.

203. Sin autem haec quantitas $\mathfrak{B}lx$ minor fuerit ſemidiametro pupillae claritatis menſura exiſtet, quae eo maior erit quo maior fuerit iſta quantitas: dum contra minuta hac quantitate claritas tam exigua euadere poteſt, vt non amplius ſenſui viſus excitando ſufficiat.

Definitio 5.

204. Gradus claritatis per lentes perceptae commodiſſime definietur ſemidiametro coni luminoſi, qui a quouis obiecti puncto in oculum transmittitur.

Coroll. 1.

205. Gradus ergo claritatis definitur quantitate ſupra inuenta $\mathfrak{B}lx$, ita vt ſi gradum claritatis ponamus $=y$ habeamus $y = \mathfrak{B}lx$, quo cognito facillime iudicabimus, quonam gradu viſio ſit clara habenda.

Coroll. 2.

206. Scilicet ſi ſemidiametrum pupillae ponamus $= \omega$, quamdiu fuerit $y > \omega$, claritate plena

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fruemur, quae nullius augmenti est capax nisi forte ipsam pupillam magis dilatare valeamus.

Coroll. 3.

207. At si fuerit $y < \omega$, claritatem vtique minorem percipiemus: ac si claritatem plenam vnitate designemus, claritas ex casu $y < \omega$ resultans erit $= \frac{yy}{\omega\omega}$; propterea quod copia radiorum in oculum immissorum est vt quadratum semidiametri y.

Coroll. 4.

208. Quod si gradus claritatis y eousque decrescat, vt copia radiorum nimis sit parua, quam vt sensum visus excitare possit, nihil ob summam caliginem percipi poterit, vnde manifestum est ad visionem requiri, vt gradus claritatis certum quempiam limitem superet.

Scholion.

209. Tam in Telescopiis quam Microscopiis maxime necesse est, vt obiecta certo claritatis gradu exhibeantur ne repraesentatio nimis fiat obscura. Hic autem gradus plurimum a lumine proprio obiectorum pendet, quae quo fuerint illustriora minor claritatis gradus iis satis clare videndis sufficit; ideoque stellas illustriores contemplantes minori gradu claritatis contenti esse possumus: terrestria vero obiecta multo maiorem claritatis gradum postulant. Quo igitur haec

ad

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ad omnes casus accommodare valeamus gradum claritatis hic littera indefinita y contentum in computum sum ducturus. Quamobrem his de multiplicatione et claritate praemissis, haec duo elementa simul cum confusione pro quouis lentium numero exhibebo: ac primo quidem non neglecta lentium crassitie, tum vero eadem seorsim lentium crassitie neglecta exponi conueniet.

Problema 8.

210. Si oculus per quotcunque lentes PP, QQ, RR, SS etc. obiectum Ee aspiciat; ita vt imago postrema per eas repraesentata ante oculum in iusta distantia $= l$ reperiatur, determinare tam multiplicationem et claritatem, quam confusionem, qua visio perturbabitur.

Solutio

Quotcunque fuerint lentes, sint pro singulis distantiae determinatrices, vt et crassities cum quantitate arbitraria, vt sequitur.

Pro Lente	Dist: determinatr:	Crassities	quant: arb:
Prima PP	$EA=a$; $aF=\alpha$;	$Aa=v$;	k
Secunda QQ	$FB=b$; $bG=\beta$;	$Bb=v'$;	k'
Tertia RR	$GC=c$; $cH=\gamma$;	$Cc=v''$;	k''
Quarta SS	$HD=d$; $dI=\delta$;	$Dd=v'''$;	k'''

etc.

atque hinc posita ratione refractionis $\frac{31}{20} = n$ constructio lentium ita se habebit;

Pro Lente	Radius Faciei anterioris	Faciei posterioris
Prima P P	$\frac{(n-1)\alpha(k+v)}{k+v+2n\alpha}$	$\frac{(n-1)\alpha(k-v)}{k-v-2n\alpha}$
Secunda Q Q	$\frac{(n-1)\beta(k'+v')}{k'+v'+2n\beta}$	$\frac{(n-1)\beta(k'-v')}{k'-v'-2n\beta}$
Tertia R R	$\frac{(n-1)\gamma(k''+v'')}{k''+v''+2n\gamma}$	$\frac{(n-1)\gamma(k''-v'')}{k''-v''-2n\gamma}$
Quarta S S	$\frac{(n-1)\delta(k'''+v''')}{k'''+v'''+2n\delta}$	$\frac{(n-1)\delta(k'''-v''')}{k'''-v'''-2n\delta}$

etc.

Tum posito brevitatis gratia:

$$\frac{k-v}{k+v} = i;\ \frac{k'-v'}{k'+v'} = i';\ \frac{k''-v''}{k''+v''} = i'';\ \frac{k'''-v'''}{k'''+v'''} = i''' \text{ etc.}$$

si aperturae primae lentis in facie anteriori semidiameter fuerit $= x$, tam pro facie posteriori quam pro vtraque facie singularum lentium sequentium aperturae maiores esse debent vel saltem non minores, quam sequens tabula ostendit:

Pro Lente	Semidiameter aperturae in facie anteriori	Semidiameter aperturae in facie posteriori
Prima P P	x	$i\,x$
Secunda Q Q	$i.\frac{bx}{\alpha}$	$ii'.\frac{bx}{\alpha}$
Tertia R R	$ii'i''.\frac{bcx}{\alpha\beta}$	$ii'i''.\frac{bcx}{\alpha\beta}$
Quarta S S	$ii'i''i'''.\frac{bcdx}{\alpha\beta\gamma}$	$ii'i''i'''.\frac{bcdx}{\alpha\beta\gamma}$

etc.

Denique

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Denique ad abbreuiandum ponatur:

$$P = \frac{n}{2(n-1)^2}\left(\left(\frac{n}{a}+\frac{2}{k+v}\right)\left(\frac{i}{a}+\frac{2}{k-u}\right)^2+\left(\frac{n}{a}-\frac{2}{k-v}\right)\left(\frac{i}{a}-\frac{2}{k+v}\right)^2\right)$$

$$Q = \frac{n}{2(n-1)^2}\left(\left(\frac{n}{b}+\frac{2}{k'+v'}\right)\left(\frac{i'}{b}+\frac{2}{k'-v'}\right)^2+\left(\frac{n}{b}-\frac{2}{k'-v'}\right)\left(\frac{i'}{b}-\frac{2}{k'+v'}\right)^2\right)$$

$$R = \frac{n}{2(n-1)^2}\left(\left(\frac{n}{c}+\frac{2}{k''+v''}\right)\left(\frac{i''}{c}+\frac{2}{k''-v''}\right)^2+\left(\frac{n}{c}-\frac{2}{k''-v''}\right)\left(\frac{i''}{c}-\frac{2}{k''+v''}\right)^2\right)$$

$$S = \frac{n}{2(n-1)^2}\left(\left(\frac{n}{d}+\frac{2}{k'''+v'''}\right)\left(\frac{i'''}{d}+\frac{2}{k'''-v'''}\right)^2+\left(\frac{n}{d}-\frac{2}{k'''-v'''}\right)\left(\frac{i'''}{d}-\frac{2}{k'''+v'''}\right)^2\right)$$

etc.

His positis ponamus oculum in distantia $= O$ ab vltima lente locari ita vt post imaginem vltimam reperiatur in distantia $= l$: magnitudinem autem visam comparari cum magnitudine qua idem obiectum nudo oculo in distantia fixa $= b$ cerneretur; ac ponatur exponens multiplicationis $= m$.

Deinde pro claritate sit gradus claritatis $= y$, ita vt y indicet semidiametrum coni luminosi in oculum intrantis.

Confusio autem aestimetur per semidiametrum confusionis supra (194) definitum.

Iam pro quouis lentium numero hae tres res ita se habebunt.

I. Pro vnica lente $O = \alpha + l$:

1. Exponens multiplicationis $m = \frac{1}{i}\cdot\frac{\alpha b}{\alpha i}$ situ inuerso
2. Gradus claritatis $y = il.\ \frac{x}{\alpha}$
3. Semid: Confusionis $= \frac{1}{4} i \frac{\alpha}{l} x^2 . P$

II.

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II. Pro duabus Lentibus $O = \varepsilon + l$

1. Exp. mult: $m = \frac{1}{ii'} \cdot \frac{\alpha\varepsilon b}{abl}$ situ erecto

2. Gradus claritatis $y = ii'l \cdot \frac{bx}{a\varepsilon}$

3. Semid. Confusionis $= \frac{1}{4} ii' \cdot \frac{\varepsilon}{l} \cdot \frac{b}{a} x^2 \left(\frac{1}{ii'} \cdot \frac{aa}{bb} P + ii' \frac{bb}{aa} Q \right)$.

III. Pro tribus Lentibus $O = \gamma + l$

1. Exp. mult: $m = \frac{1}{ii'i''} \cdot \frac{a\varepsilon\gamma b}{abcl}$ situ inuerso:

2. Gradus claritatis $y = ii'i''l \cdot \frac{bcx}{a\varepsilon\gamma}$.

3. Semidiameter confusionis:

$$\frac{3}{4} ii'i''l \cdot \frac{\gamma}{l} \cdot \frac{bc}{a\varepsilon} x^2 \left(\frac{1}{ii'i''i'''} \cdot \frac{aa\varepsilon\varepsilon}{bbcc} P + \frac{ii}{i''i'''} \cdot \frac{bb\varepsilon\varepsilon}{aacc} Q + ii'i''i''' \cdot \frac{bbcc}{aa\varepsilon\varepsilon} R \right).$$

IV. Pro quatuor Lentibus $O = \delta + l$

1. Exp. multipl: $m = \frac{1}{ii'i''i'''} \cdot \frac{a\varepsilon\gamma\delta b}{abcdl}$ situ erecto

2. Gradus Claritatis $y = ii'i''i'''l \cdot \frac{bcdx}{a\varepsilon\gamma\delta}$

3. Semidiameter Confusionis:

$$\frac{4}{5} ii'i''i'''l \cdot \frac{\delta}{l} \cdot \frac{bcd}{a\varepsilon\gamma} x^3 \left\{ \begin{array}{l} \frac{1}{ii'i''i'''i^{IV}} \cdot \frac{aa\varepsilon\varepsilon\gamma\gamma}{bbccdd} P + \frac{ii}{i''i'''i^{IV}} \cdot \frac{bb\varepsilon\varepsilon\gamma\gamma}{aaccdd} Q \\ + \frac{ii'i''}{i'''i^{IV}} \cdot \frac{bbcc\gamma\gamma}{aa\varepsilon\varepsilon dd} R + ii'i''i'''l \cdot \frac{bbccdd}{aa\varepsilon\varepsilon\gamma\gamma} S \end{array} \right\}$$

V.

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V. Pro quinque Lentibus $O = \varepsilon + l$

1. Exp. multipl: $m = \frac{1}{i\,i'\,i''\,i'''\,i''''} \cdot \frac{\alpha\beta\gamma\delta\varepsilon b}{a b c d e l}$ ſitu inuerſo

2. Gradus claritatis $y = i\,i'\,i''\,i'''\,i''''\,l \cdot \frac{b c d e x}{\alpha\beta\gamma\delta\varepsilon}$

3. Semidiameter confuſionis

$$\tfrac{1}{4} i\,i'\,i''\,i'''\,i''''\,l \cdot \frac{\varepsilon}{l} \cdot \frac{b c d e}{\alpha\beta\gamma\delta} x^3 \left\{ \begin{array}{ll} + \frac{1}{i\,i'\,i''\,i'''\,i''''} \cdot \frac{\alpha\alpha\beta\beta\gamma\gamma\delta\delta}{b b c c d d e e} & P \\ + \frac{i\,i}{i''\,i'''\,i''''\,l} \cdot \frac{b b\beta\beta\gamma\gamma\delta\delta}{\alpha\alpha c c d d e e} & Q \\ + \frac{i\,i \cdot i'\,i''}{i''\,i'''\,i''''} \cdot \frac{b b c c\gamma\gamma\delta\delta}{\alpha\alpha\beta\beta d d e e} & R \\ + \frac{i\,i \cdot i'\,i' \cdot i''\,i'''}{i''\,i''''} \cdot \frac{b b c c d d\delta\delta}{\alpha\alpha\beta\beta\gamma\gamma e e} & S \\ + i\,i \cdot i'\,i' \cdot i''\,i''' \cdot i'''\,i'''' \cdot \frac{b b c c d d e e}{\alpha\alpha\beta\beta\gamma\gamma\delta\delta} & T \end{array} \right.$$

atque hinc etiam progreſſus ad plures lentes eſt manifeſtus.

Coroll. 1.

211. His omnibus caſibus euidens eſt fore generatim $my = \frac{bx}{a}$.

Datis ſcilicet multiplicatione m cum claritate y ſtatim definitur apertura primae lentis nempe $x = my \cdot \frac{a}{b}$. Quo maior ſcilicet tam multiplicatio quam claritas deſideratur, eo maiorem eſſe oportet aperturam lentis primae.

Coroll. 2.

212. Quum autem x maius accipere non liceat, quam vt confuſio infra certum limitem contineatur, dato x cum exponente multiplicationis m

definitur claritatis gradus $y = \frac{bx}{ma}$, vnde patet reliquis paribus, quo maior multiplicatio exigatur, eo minori claritate contentos nos esse oportere.

Coroll. 3.

213. Imprimis autem hic obseruandum est, has formulas aeque negotium conficere, quamcunque crassitiem lentes habuerint. *Euadent autem* tractabiliores, si lentium crassities negligatur, qui casus seorsim tractari meretur.

Problema 9.

214. Iisdem positis, quae in problemate praecedente, si lentium crassities vt euanescens consideretur, determinare tam multiplicationem et claritatem, quam confusionem, qua visio perturbabitur.

Solutio.

Haec tractatio a praecedenti in isto differt, quod lentium crassities v, v', v'' etc. euanescant, et loco quantitatum arbitrariarum k, k', k'' etc. numeri arbitrarii λ, λ', λ'' etc. in calculum introducantur. Ponatur ergo

Pro Lente		Dist. determinatr.	num. arb.
Prima	PP	$EA = a$; $aF = \alpha$	λ
Secunda	QQ	$FB = b$; $bG = \beta$	λ'
Tertia	RR	$GC = c$; $cH = \gamma$	λ''
Quarta	SS	$HD = d$; $dI = \delta$	λ'''

etc.

Hinc

Hinc si sit breuitatis gratia!

$\varrho = 0,190781$; $\sigma = 1,627401$; et $\tau = 0,905133$

lentium constructio ita est instituenda

Pro Lente	Radius Faciei anterioris	Radius Faciei posterioris
Prima PP	$\frac{a\,\alpha}{\varrho a + \sigma\alpha \pm \tau(a+\alpha)\sqrt{(\lambda-1)}}$	$\frac{a\,\alpha}{\varrho a + \sigma\alpha + \tau(a+\alpha)\sqrt{(\lambda-1)}}$
Secunda QQ	$\frac{b\,\beta}{\varrho b + \sigma\beta \pm \tau(b+\beta)\sqrt{(\lambda'-1)}}$	$\frac{b\,\beta}{\varrho b + \sigma\beta + \tau(b+\beta)\sqrt{(\lambda'-1)}}$
Tertia RR	$\frac{c\,\gamma}{\varrho c + \sigma\gamma \pm \tau(c+\gamma)\sqrt{(\lambda''-1)}}$	$\frac{c\,\gamma}{\varrho c + \sigma\gamma + \tau(c+\gamma)\sqrt{(\lambda''-1)}}$
Quarta SS	$\frac{d\,\delta}{\varrho d + \sigma\delta \pm \tau(d+\delta)\sqrt{(\lambda'''-1)}}$	$\frac{d\,\delta}{\varrho d + \sigma\delta + \tau(d+\delta)\sqrt{(\lambda'''-1)}}$

Cum iam lentis primae PP semidiameter aperturae sit $= x$, et in qualibet lente vtriusque faciei eadem sit ratio, vt omnes radii per primam ingressi simul per reliquas transmittantur, apertura reliquarum sequentes limites superare debet:

Semid. aperturae

Lentis secundae QQ $> \frac{b}{a}x$

Lentis tertiae RR $> \frac{bc}{a\beta}x$

Lentis quartae SS $> \frac{bcd}{a\beta\gamma}x$

etc.

Tum vero posito breuitatis ergo $\mu = 0,938191$ et $\nu = 0,232692$ statuatur:

$P = \mu\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{a\alpha}\right)$

$Q = \mu\left(\frac{1}{b}+\frac{1}{\beta}\right)\left(\lambda'\left(\frac{1}{b}+\frac{1}{\beta}\right)^2+\frac{\nu}{b\beta}\right)$

$R = \mu\left(\frac{1}{c}+\frac{1}{\gamma}\right)\left(\lambda''\left(\frac{1}{c}+\frac{1}{\gamma}\right)^2+\frac{\nu}{c\gamma}\right)$

$S = \mu\left(\frac{1}{d}+\frac{1}{\delta}\right)\left(\lambda'''\left(\frac{1}{d}+\frac{1}{\delta}\right)^2+\frac{\nu}{d\delta}\right)$

etc.

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Sit iam O distantia oculi post vltimam lentem, ita vt ab imagine postrema distet interuallo $= l$, comparetur magnitudo visa cum ea, qua idem obiectum in distantia fixa b nudo oculo cerneretur, sitque exponens multiplicationis $= m$; et gradus claritatis $= y$, quibus positis erit pro quouis lentium numero vt sequitur.

I. Pro vnica lente $O = \alpha + l$

1. Exponens multiplicationis $m = \frac{\alpha b}{\alpha l}$ situ inuerso
2. Gradus claritatis $y = l. \frac{x}{\alpha}$ hinc $my = \frac{bx}{a}$
3. Semid: confusionis $= \frac{\alpha}{+l}. x^3$ P

II. Pro duabus lentibus $O = \varepsilon + l$

1. Exponens multiplicationis $m = \frac{\alpha\varepsilon b}{ab l}$ situ erecto
2. Gradus claritatis $y = l. \frac{bx}{a\varepsilon}$, hinc $my = \frac{bx}{a}$
3. Semid: confusionis $= \frac{\varepsilon}{+l}. \frac{b}{a} x^3 (\frac{aa}{bb} P + \frac{bb}{aa} Q)$

III. Pro tribus lentibus $O = \gamma + l$

1. Exponens multiplicationis $m = \frac{a\varepsilon\gamma b}{abcl}$ situ inuerso
2. Gradus claritatis $y = l. \frac{bcx}{a\varepsilon\gamma}$ hinc $my = \frac{bx}{a}$
3. Semidiameter confusionis

$\frac{\gamma}{+l}. \frac{bc}{a\varepsilon} x^3 (\frac{aa\varepsilon\varepsilon}{bbcc} P + \frac{bb\varepsilon\varepsilon}{aacc} Q + \frac{bbcc}{aa\varepsilon\varepsilon} R)$.

IV. Pro quatuor lentibus $O = \delta + l$

1. Exponens multiplicationis $m = \frac{a\varepsilon\gamma\delta b}{abcdl}$ situ erecto
2. Gradus claritatis $y = l. \frac{bcdx}{a\varepsilon\gamma\delta}$ hinc $my = \frac{bx}{a}$
3. Semidiameter confusionis:

$\frac{\delta}{+l}. \frac{bcd}{a\varepsilon\gamma} x^3 (\frac{aa\varepsilon\varepsilon\gamma\gamma}{bbccdd} P + \frac{bb\varepsilon\varepsilon\gamma\gamma}{aaccdd} Q + \frac{bbcc\gamma\gamma}{aa\varepsilon\varepsilon dd} R + \frac{bbccdd}{aa\varepsilon\varepsilon\gamma\gamma} S)$

V.

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V. Pro quinque lentibus $O=\varepsilon+l$

1. Exponens multipl. $m=\frac{\alpha\beta\gamma\delta eb}{abcdel}$ ſitu inuerſo

2. Gradus claritatis $y=l.\frac{bcdex}{\alpha\beta\gamma\delta\varepsilon}$ hinc $my=\frac{b^4x}{a}$

3. Semidiameter confuſionis

$$\frac{\varepsilon}{4l}\cdot\frac{bcde}{\alpha\beta\gamma\delta}x^3\begin{cases}+\frac{aa\beta\beta\gamma\gamma\delta\delta}{bbccddee}\,P+\frac{bb\beta\beta\gamma\gamma\delta\delta}{aaccddee}\,Q\\+\frac{bbcc\gamma\gamma\delta\delta}{aa\beta\beta ddee}\,R\\+\frac{bbccdd\delta\delta}{aa\beta\beta\gamma\gamma ee}\,S+\frac{bbccddee}{aa\beta\beta\gamma\gamma\delta\delta}\,T\end{cases}$$

vnde non difficile erit has formulas ad lentes etiam plures continuare.

Coroll. 1.

215. Lentes ſimplices adhibendo numeros $\lambda,\lambda',\lambda'',$ etc. vnitate minores accipi non poſſunt. Verum hic nihil obſtat quo minus loco lentium ſimplicium lentes duplicatae, triplicatae vel etiam quadruplicatae in vſum vocentur, quo pacto in his formulis numeri $\lambda,\lambda',\lambda'',$ etc. non ſolum infra vnitatem diminui ſed ad nihilum vſque perduci poterunt. Tum autem conſtructio lentium harum multiplicatarum ex ſuperiori capite peti et ad diſtantias determinatrices hîc poſitas accommodari debet.

Coroll. 2.

216. Veluti ſi pro lente prima PP debeat eſſe

$\lambda = 0,191827$, hanc lentem ita ex duabus componi oportet, vt sit

	Radius faciei anterioris	Radius faciei posterioris
Pro lente Priori . .	$\frac{2a\alpha}{(2\varrho-\sigma)\alpha+\sigma a}$;	$\frac{2a\alpha}{(2\sigma-\varrho)\alpha+\varrho a}$
Posteriori	$\frac{2a\alpha}{\varrho\alpha+(2\sigma-\varrho)a}$;	$\frac{2a\alpha}{\sigma\alpha+(2\varrho-\sigma)a}$.

Coroll. 3.

217. Sin autem velimus, vt pro lente prima sit $\lambda = 0,042165$, ea erit triplicanda, hoc modo

	Radius faciei anterioris	Radius faciei posterioris
Pro lente Priori	$\frac{3a\alpha}{(3\varrho-2\sigma)\alpha+\sigma a}$;	$\frac{3a\alpha}{(3\sigma-2\varrho)\alpha+\varrho a}$
Media	$\frac{3a\alpha}{(2\varrho-\sigma)\alpha+(2\sigma-\varrho)a}$;	$\frac{3a\alpha}{(2\sigma-\varrho)\alpha+(2\varrho-\sigma)a}$
Posteriori	$\frac{3a\alpha}{\varrho\alpha+(3\sigma-2\varrho)a}$;	$\frac{3a\alpha}{\sigma\alpha+(3\varrho-2\sigma)a}$

Coroll. 4.

218. At si pro lente prima requiratur $\lambda = -0,010216$, quadruplicata erit vtendum ita construenda:

	Radius faciei anterioris	Radius faciei posterioris
Pro lente Prima	$\frac{4a\alpha}{(4\varrho-3\sigma)\alpha+\sigma a}$;	$\frac{4a\alpha}{(4\sigma-3\varrho)\alpha+\varrho a}$
Secunda	$\frac{4a\alpha}{(\varrho-2\sigma)\alpha+(2\sigma-\varrho)a}$;	$\frac{4a\alpha}{(3\sigma-2\varrho)\alpha+(2\varrho-\sigma)a}$
Tertia	$\frac{4a\alpha}{(2\varrho-\sigma)\alpha+(3\sigma-2\varrho)a}$;	$\frac{4a\alpha}{(2\sigma-\varrho)\alpha+(3\varrho-2\sigma)a}$
Quarta	$\frac{4a\alpha}{\varrho\alpha+(4\sigma-3\varrho)a}$;	$\frac{4a\alpha}{\sigma\alpha+(4\varrho-3\sigma)a}$

Coroll.

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Coroll. 5.

219. Simili autem modo lens ſecunda QQ ex ſuis diſtantiis determinatricibus b et $\mathfrak{b}$ per multiplicationem erit conſtruenda, numerus que ei reſpondens λ' debet eſſe vel $0,191827$, vel $0,042165$ vel $-0,019216$; quod idem de reliquis lentibus eſt intelligendum.

Scholion 1.

220. Alias lentium ſpecies hic nollem in praxi adhiberi, cum hae ſolae ſine metu enormis erroris confici queant; tum vero etſi aliae ab his non admodum diſcrepantes fere aequo ſucceſſu in praxin introduci poſſent, tamen quia diſcrimen non admodum eſt notabile, iis facile carere poterimus. Cum igitur ſit $\varrho = 0,190781$ et $\sigma = 1,627401$, ideoque

$$\varrho = 0,190781; \quad \sigma = 1,627401$$

$$2\varrho - \sigma = -1,245839; \quad 2\sigma - \varrho = 3,064021$$

$$3\varrho - 2\sigma = -2,682459; \quad 3\sigma - 2\varrho = 4,500641$$

$$4\varrho - 3\sigma = -4,119079; \quad 4\sigma - 3\varrho = 5,937261$$

eaſdem conſtructiones in numeris euolutis exhiberi conueniet.

I. Si igitur pro lente PP debeat eſſe $\lambda = 1$, ea erit ſimplex hoc modo conſtruenda

Radius faciei anterioris	poſterioris
$\frac{aa}{+0,190781a+1,627401c}$	$\frac{aa}{+1,627401a+0,190781c}$

II.

II. Si pro lente PP debeat esse $\lambda = 0,191827$ ea erit duplicata hoc modo construenda

Pro lente	Radius Faciei anterioris	Radius Faciei posterioris
Priori	$\frac{aa}{-0,622915a+0,813700a}$	$\frac{aa}{+1,532010a+0,095390a}$
Posteriori	$\frac{aa}{+0,095390a+1,532010a}$	$\frac{aa}{+0,813700a-0,622915a}$

III. Si pro lente PP debeat esse $\lambda = 0,042165$ ea erit triplicata hoc modo construenda

Pro lente	Radius Faciei anterioris	Radius Faciei posterioris
Priori	$\frac{aa}{-0,894153a+0,542467a}$	$\frac{aa}{+1,500214a+0,063594a}$
Media	$\frac{aa}{-0,415280a+1,021340a}$	$\frac{aa}{+1,021340a-0,415280a}$
Posteriori	$\frac{aa}{+0,063594a+1,500214a}$	$\frac{aa}{+0,542467a-0,894153a}$

IV. Si pro lente PP debeat esse $\lambda = -0,010216$ ea erit quadruplicata hoc modo construenda

Pro lente	Radius Faciei anterioris	Radius Faciei posterioris
Prima	$\frac{aa}{-1,025770a+0,406850a}$	$\frac{aa}{+1,484315a+0,047695a}$
Secunda	$\frac{aa}{-0,670615a+0,766005a}$	$\frac{aa}{+1,125160a-0,311460a}$
Tertia	$\frac{aa}{-0,311460a+1,125160a}$	$\frac{aa}{+0,766005a-0,670615a}$
Quarta	$\frac{aa}{+0,047695a+1,484315a}$	$\frac{aa}{+0,406850a-1,025770a}$

Scholion

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Scholion 2.

221. Interim tamen si lentes desideremus, in quibus valor ipsius λ maior sit, quam hic est assumtus, constructio earum leui additione ex his ipsis formulis concinnari poterit. In binis scilicet fractionibus, quibus radii facierum cuiusque lentis designantur, alter denominator augeri alter vero diminui debet eadem quantitate, quae quantitas semper $=\tau(a+\alpha)\sqrt{v}$ denotante v excessum valoris ipsius λ supra ante assumtum. Ita si esse debeat

I. $\lambda=1+v$; II. $\lambda=0,191827+v$; III. $\lambda=0,042165+v$
vel IV. $\lambda=-0,010216+v$

denominatores fractionum in Scholio praecedente traditarum pro quauis lente simplici alternatim sunt augendi et minuendi quantitate $0,905133\,(a+\alpha)\sqrt{v}$. Vnde si lens quadruplicata desideretur, pro qua sit praecise $\lambda=0$, erit $v=0,010216$ et $\tau\sqrt{v}=0,091487$; hincque pro quauis lente simplici denominatorum alter augeri alter vero diminui debet hac quantitate

$$0,091487\alpha+0,091487a$$

ex quo talis nascitur constructio huiusmodi lentis quadruplicatae pro qua est $\lambda=0$

Pro lente	Radius faciei anterioris	Radius faciei posterioris
Prima	$\frac{aa}{-1,121257\alpha+0,315363a}$	$\frac{aa}{+1,575802\alpha+0,139182a}$
Secunda	$\frac{aa}{-0,762102\alpha+0,674518a}$	$\frac{aa}{+1,216647\alpha-0,219973a}$
Tertia	$\frac{aa}{-0,402542\alpha+1,033673a}$	$\frac{aa}{+0,857452\alpha-0,579128a}$
Quarta	$\frac{aa}{-0,043752\alpha+1,352828a}$	$\frac{aa}{+0,498337\alpha-0,938283a}$

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Sed hoc casu quaelibet lens adhuc alio modo construi potest: sic quarta ordine retrogrado exposita permutatis a et α dabit alteram formam lentis primae.

Supplementum III.

Ad Problem. 8.

Si lentes ratione refractionis discrepent, vt sit ratio refractionis pro prima lente $= n$, pro secunda $= n'$, pro tertia $= n''$; inde neque in multiplicatione m neque in gradu claritatis quicquam mutatur; at vero in semidiametro confusionis valores litterarum P, Q, R sequenti modo immutari debent.

$$P = \frac{n}{2(n-1)^2}\left\{\begin{array}{l}\left(\frac{n}{\alpha}+\frac{2}{k+v}\right)\left(\frac{1}{1\alpha}+\frac{2}{k-v}\right)^2\\ +\left(\frac{n}{\alpha}-\frac{2}{k-v}\right)\left(\frac{1}{\alpha}-\frac{2}{k+v}\right)^2\end{array}\right.$$

$$Q = \frac{n'}{2(n'-1)^2}\left\{\begin{array}{l}\left(\frac{n'}{\beta}+\frac{2}{k'+v'}\right)\left(\frac{1}{1'\beta}+\frac{2}{k'-v'}\right)^2\\ +\left(\frac{n'}{\beta}-\frac{2}{k'-v'}\right)\left(\frac{1}{\beta}-\frac{2}{k'+v'}\right)^2\end{array}\right.$$

$$R = \frac{n''}{2(n''-1)^2}\left\{\begin{array}{l}\left(\frac{n''}{\gamma}+\frac{2}{k''+v''}\right)\left(\frac{1}{1''\gamma}+\frac{2}{k''-v''}\right)^2\\ +\left(\frac{n''}{\gamma}-\frac{2}{k''-v''}\right)\left(\frac{1}{\gamma}-\frac{2}{k''+v''}\right)^2\end{array}\right.$$

$$S = \frac{n'''}{2(n'''-1)^2}\left\{\begin{array}{l}\left(\frac{n'''}{\delta}+\frac{2}{k'''+v'''}\right)\left(\frac{1}{1'''\delta}+\frac{2}{k'''-v'''}\right)^2\\ +\left(\frac{n'''}{\delta}-\frac{2}{k'''-v'''}\right)\left(\frac{1}{\delta}-\frac{2}{k'''+v'''}\right)^2\end{array}\right.$$

etc.

Ad Problem. 9.

Si lentes ratione refractionis discrepent, loco litterarum ϱ, σ, τ in radiis faciendum scribi oportet pro

pro ſecunda lenti ϱ', σ', τ'; pro tertia $\varrho'', \sigma'', \tau''$, etc. tum vero expreſſiones pro multiplicatione et gradu claritatis nulla mutatione egent; pro confuſione autem notari oportet, fore

$$P = \mu\left(\tfrac{1}{a}+\tfrac{1}{\alpha}\right)\left(\lambda\left(\tfrac{1}{a}+\tfrac{1}{\alpha}\right)^2+\tfrac{\nu}{a\alpha}\right)$$

$$Q = \mu'\left(\tfrac{1}{b}+\tfrac{1}{\beta}\right)\left(\lambda'\left(\tfrac{1}{b}+\tfrac{1}{\beta}\right)^2+\tfrac{\nu'}{b\beta}\right)$$

$$R = \mu''\left(\tfrac{1}{c}+\tfrac{1}{\gamma}\right)\left(\lambda''\left(\tfrac{1}{c}+\tfrac{1}{\gamma}\right)^2+\tfrac{\nu''}{c\gamma}\right)$$

etc.

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CAPVT V.

DE CAMPO APPARENTE OCVLIQVE LOCO MAXIME IDONEO.

Problema 1.

222.

Tab. II. Fig. 12. Si ex obiecti puncto extra axem sumto ε radius quicunque εM per ~~lentem~~ PP ~~transmittatur~~, definire eius concursum cum axe O.

Solutio.

Sit $F\zeta$ imago principalis per lentem proiecta, a diffusione enim hic mentem abstrahimus, ac ponamus pro lente eius distantias determinatrices $AE = a$, $aF = \alpha$, eius crassitiem $A\alpha = v$, et quantitatem arbitrariam $= k$, sitque breuitatis ergo $\frac{k-v}{k+v} = i$. His positis si puncti ε imago cadat in ζ, voceturque $E\varepsilon = z$, erit $F\zeta = \frac{1}{i} \cdot \frac{\alpha z}{a}$ ~~at pro puncto lentis~~ M statuatur eius distantia ab axe $AM = x$; ac supra ostendimus si radius a puncto E per punctum M transmitteretur, eum ita per m ad punctum F progressurum esse, vt foret $am = ix$. Radius igitur εM per punctum N ad ζ feretur, ita vt, cum obliquitas radiorum

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rum in facies lentis incidentium perpetuo valde parva ſtatuatur, ſit proxime angulus EMε ad NMm vt n ad 1 poſito $n = \frac{\mu}{\nu}$. Hinc erit Eε ad mN in ratione compoſita iſtorum angulorum, et diſtantiarum AE ad Aα, ſeu Eε : mN $= n\alpha : \omega$, vnde fit mN $= \frac{\nu z}{n\alpha}$, ideoque αN $= ix - \frac{\nu z}{n\alpha}$. Iam vero radius a puncto N recta ad ζ pergit, et propterea axem ita in O ſecabit, vt ſit αN + Fζ : αF $= \alpha$N : αO, ſicque

$$\alpha O = (i\alpha x - \frac{\alpha\nu z}{n\alpha}) : (ix - \frac{\nu z}{n\alpha} + \frac{\alpha z}{i\alpha}) = \frac{n\mu i\alpha\alpha x - i\alpha\nu z}{n\mu i\alpha x - i\nu z + n\alpha z}$$

et $FO = \frac{n\alpha\alpha z}{n\mu i\alpha x - i\nu z + n\alpha z}$

Si x ſit ſemidiameter aperturae lentis in facie anteriori, haec interſectio O reſpondet caſui, quo radius ab ε per lentis terminum ſummum transeat: ſin autem is per terminum imum transmittatur, ſumto x negatiuo fiet

$$FO = \frac{n\alpha\alpha z}{-n\mu i\alpha x - i\nu z + n\alpha z}$$

At ſi radius ex ε per centrum lentis A transeat, punctum interſectionis ita cadet in O vt ſit

$$FO = \frac{n\alpha\alpha}{n\alpha - i\nu}$$

Coroll. 1.

223. Si igitur x denotet ſemidiametrum aperturae lentis in facie anteriori PMAP, vt omnes radii a puncto ε in hanc faciem incidentes per lentem transmittantur, neceſſe eſt, vt faciei poſterioris

PNaP semidiameter sit maior quam $+ix-\frac{vx}{na}$; sumto x tam negatiuo quam positiuo. Vnde hic semidiameter minor esse nequit, quam $ix+\frac{vx}{na}$.

Coroll. 2.

224. Si apertura in facie anteriori euanescat, vt sit $x=0$ radii a puncto ε, cuius ab axe distantia $E\varepsilon=z$, per lentem non transmittentur, nisi in facie posteriori semidiameter aperturae sit $=\frac{vz}{na}$ vel maior. Vnde patet quo maior fuerit lentis crassities v eo maiori apertura in facie posteriori esse opus.

Coroll. 3.

225. Vicissim ergo si detur lentis apertura in facie posteriori, cuius semidiameter sit $=\frac{vz}{na}$; inde simul in obiecto extremum punctum ε determinatur, a quo radius in centrum lentis A incidens per eam transmittatur.

Coroll. 4.

226. Hic autem radius per A immissus post transitum cum axe in O occurret, vt ob $x=0$ sit interuallum $FO=\frac{naa}{na-iv}$. seu $aO=\frac{-iav}{na-iv}$. Nisi ergo oculus in hoc axis loco teneatur, radius transmissus non in oculum ingredietur; si quidem apertura pupillae vt infinite parua spectetur.

Coroll.

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Coroll. 5.

227. At si semidiameter pupillae ponatur $= \omega$, oculus etiam in o positus illum radium excipiet, si fuerit $o\,\omega = \omega$. Cum autem casu $x = 0$ sit

$$\alpha N\left(\frac{vz}{n\alpha}\right) : \alpha O\left(\frac{i\alpha v}{n\alpha - iv}\right) = \sigma\omega(\omega) : Oo \text{ erit}$$

$$Oo = \frac{ni\alpha\alpha\omega}{z(n\alpha - iv)}.$$

Quod interuallum cum aeque positiuum ac negatiuum accipi possit, pro loco oculi o habebimus

$$\alpha o = -\frac{i\alpha vz \mp ni\alpha\alpha\omega}{z(n\alpha \mp iv)} = \frac{-i\alpha(vz \mp n\alpha\omega)}{z(n\alpha - iv)}$$

Scholion.

228. In hac tractatione, vbi campum apparentem et locum oculi idoneum inuestigamus, tam aperturam lentis obiectiuae in facie anteriori quam pupillae amplitudinem pro nihilo habebimus, vt quaestiones obtineamus determinatas. Quare horum elementorum, quae in instrumentis dioptricis ad visionem accommodatis maximi sunt momenti, sequentes definitiones constituemus.

Definitio 1.

229. Campus apparens est spatium in obiecto, ex cuius singulis punctis radii in centrum lentis obiectiuae incidentes per reliquas lentes omnes transmittuntur. Quod spatium cum sit circulare, eius radius vocatur semidiameter campi apparentis.

Coroll.

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Coroll. 1.

230. Si ergo $Ee = z$ fuerit ſemidiameter campi apparentis, erit e punctum obiecti extremum, ſed ab axe maxime remotum, ex quo adhuc radii in centrum A lentis obiectiuae incidentes per omnes lentes transmittuntur.

Coroll. 2.

231. Determinatur igitur magnitudo campi apparentis per aperturam ſequentium facierum refringentium, ac fortaſſe per aperturam vnius, ſi ſcilicet radii a puncto quodam magis remoto quam e venientes nullum transitum per eam inuenirent, etiamſi per reliquas facies transmitterentur.

Scholion.

232. Si radii ex ſolo puncto E, quod eſt centrum campi apparentis, conſiderentur, ii quidem qui in A incidunt, quoniam perpetuo ſecundum axem progrediuntur, per omnes reliquas facies refringentes, quantumuis parua fuerit earum apertura, certe transmittentur; hincque campus apparens nunquam penitus euaneſcere poteſt. At quo magis punctum e ab axe diſtans accipitur, vt radii ab eo per omnes facies transmittantur, eo maior earum apertura requiritur, quae cum ab earum curuatura pendeat, neque certum limitem ſuperare debeat, hinc vltimum punctum e, vnde radii etiam nunc transmittuntur, ac propterea ſemidiameter campi apparentis determinatur.

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natur. In sequentibus quidem propositionibus campum apparentem seu eius semidiametrum $Ee = z$ vt datum assumam, et quanta esse debeat cuiusque faciei apertura, inuestigabo: hinc enim facile vicissim, si quaeque apertura fuerit cognita, campum apparentem ipsum definire licebit. Ceterum in hac definitione assumsi aperturam primae faciei esse euanescentem: ex quo facile intelligitur ea aucta etiam campum aliquantum extendi oportere; verum hoc augmentum postea in lucrum cedet, quod cum nunquam soleat esse notabile, eius rationem hic non habendam censui; quemadmodum etiam in sequente definitione aperturae pupillae rationem non habebo.

Definitio 2.

233. Locus oculi idoneus est id punctum in axe, in quo radii ab extremitate campi apparentis per lentes transmissi axem intersecant. Oculus scilicet in hoc loco constitutus totum campum apparentem conspiciet.

Coroll. 1.

234. Hinc igitur idoneus locus oculo assignabitur, si illa intersectio radiorum extremorum cum axe post lentem vltimam cadat, sin autem haec intersectio ante lentem vltimam reperiatur, fieri nequit vt oculus in eo loco teneatur neque propterea totum campum contueri poterit.

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Coroll. 2.

235. At si ista intersectio pone lentem vltimam cadat, oculus totum campum apparentem perspiciet, etiamsi pupilla maxime esset contracta: neque tamen ob maiorem pupillae amplitudinem maiorem campum percipere valet.

Coroll. 3.

236. Verum ob amplitudinem pupillae hoc commodi assequimur, vt oculus etiamsi extra locum idoneum constituatur, dummodo distantia non sit nimis magna, tamen totum campum apparentem conspicere possit: id quod egregie vsu veniet iis casibus, quibus locus idoneus oculi ante faciem refringentem extremam cadit. Tum enim fieri poterit, vt oculus huic faciei immediate applicatus tamen totum campum percipiat.

Scholion.

237. Quando hic de visione loquor, id ita in genere est interpretandum, vt a puncto viso radius in oculum ingrediatur, neque hic curo, vtrum visio sit distincta nec ne? in sequentibus enim docebitur, quomodo lentes disponi conueniat, vt oculus in loco idoneo positus etiam in iusta ab vltima imagine distantia reperiatur, quo visio distincta reddatur. Hic igitur sine vllo respectu ad visionem distinctam habito, eum oculo locum assigno, vbi ab omnibus punctis

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ctis in campo apparente contentis radios recipiat, et quoniam singula momenta, quae ad visionem pertinent, seorsim expediri conuenit; hic etiam non ad spatium diffusionis respicio, quod quidem semper per se euanescit, si apertura faciei primae euanescens statuatur.

Problema 2.

238. Si obiectum per vnicam lentem aspiciatur, determinare tam campum apparentem, quam locum idoneum oculi. Tab. III. Fig. 12.

Solutio.

Sint vt in problemate superiori distantiae determinatrices huius lentis, scilicet distantia obiecti ante lentem $AE = a$, et imaginis post lentem $aF = \alpha$, tum vero lentis crassities $Aa = v$, et distantia arbitraria constructionem lentis plene determinans $= k$. Deinde ponamus semidiametrum campi apparentis $E\varepsilon = z$, ita vt posita faciei anterioris PAP apertura infinite parua etiamnum a puncto ε radii per lentem transmittantur. Sit porro breuitatis gratia $\frac{k-v}{k+v} = i$, atque supra demonstrauimus campum apparentem ad ε vsque extendi, si pro facie posteriori PaP fuerit semidiameter aperturae $aN = \frac{vz}{na}$ existente $n = \frac{1}{i}$; perinde enim est, siue hic semidiameter affirmatiue accipiatur siue negatiue. Hinc ergo vicissim si semidiameter huius aperturae ponatur $= a$, erit semidiameter campi apparentis $E\varepsilon = z = \frac{na}{v}a$.

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Quod ad locum idoneum oculi attinet, qui sit in O, quoniam inuenimus $FO = \frac{n\alpha\alpha}{n\alpha - iv}$, erit distantia $aO = \frac{-i\alpha v}{n\alpha - iv}$; ideoque negatiua, nisi sit vel i numerus negatiuus, vel $iv > n\alpha$. Sin autem haec distantia aO fuerit positiua, oculus in O positus totum campum apparentem perspiciet.

Coroll. 1.

239. Si crassities lentis v euanescat, ob $z = \frac{n\alpha}{v}\alpha$, campus apparens euadet infinitus, seu potius indeterminatus; distantia vero aO euanescet. Oculus igitur lenti immediate applicatus tantum spatium conspiciet, quantum per propriam indolem complecti valebit.

Coroll. 2.

240. Sin autem ob lentis crassitiem $Aa = v$ distantia $aO = \frac{-i\alpha v}{n\alpha - iv}$ prodeat positiua, oculus in O positus totum campum apparentem aspicere valebit, seu in obiecto spatium circulare spectabit, cuius semidiameter $E\varepsilon = z = \frac{n\alpha}{v}\alpha = \frac{31\alpha\alpha}{20v}$, quod ergo eo erit maius, quo tenuior fuerit lens.

Coroll. 3.

241. At si distantia aO resultet negatiua, oculum in loco idoneo constitui non licet, quoniam is necessario post lentem teneri debet. Vbicunque autem is post lentem collocetur, non vniuersum campum apparentem contuebitur, sed tantum eius partem, et quidem

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quidem eo minorem, quo magis post lentem remoueatur, propterea quod hoc modo magis a loco idoneo recedit.

Coroll. 4.

242. Hoc igitur casu conueniet oculum immediate ad faciem lentis posteriorem applicare, quo situ eatenus tantum radios accipiet, quatenus pupilla patet; sicque campus visus ab apertura pupillae pendebit; quae si esset nulla etiam campus apparens euaneſceret.

Coroll. 5.

243. Hinc patet si pupilla excedat aperturam faciei PaP seu si sit $\omega > a$, denotante ω semidiametrum pupillae, quia tum oculus huic faciei applicatus omnes radios transmissos recipit, eum totum campum apparentem esse visurum. Sin autem sit $\omega < a$, partem tantum totius campi perspiciet, cuius semidiameter erit $= \frac{na}{v}\omega = \frac{s \cdot a\omega}{2\Delta}$, scilicet non maiorem quam si apertura faciei PaP aequalis esset pupillae.

Scholion.

244. Interim tamen si hoc casu postremo apertura faciei lentis PaP, cui oculus est applicatus maior sit quam pupilla, nihil obstat quominus ea succeſſiue totam aperturam peragret, sicque pedetentim totum campum apparentem con-

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ſpicere poterit, etiamſi eum ſimul contueri non valeat. Ceterum notandum eſt, ſi etiam faciei anteriori apertura tribuatur, inde campum apparentem aliquantum augeri, ſed partes obiecti vlteriores, quia non per medium lentis radios transmittunt, obſcuriores apparebunt, vnde merito a campo apparente excluduntur. At ſi faciei anteriori tribuatur apertura, cuius ſemidiameter $= x$, vt facies poſterior omnes radios in illam incidentes transmittat, eius apertura tanto maior eſſe debet, ſecundum regulas ſupra traditas; ſcilicet eius ſemidiametrum eſſe oportet $= ix + a$ ſeu $= ix + \frac{vz}{na}$.

Problema 3.

245. Si inſtrumentum dioptricum duabus conſtet lentibus, definire campum apparentem, et locum oculi idoneum.

Solutio.

Tab. III. Fig. 13.

Sit pro his lentibus vt hactenus:

pro PP: $AE = a, aF = \alpha, Aa = v$; diſt. arb: $= k$, et $\frac{k-v}{k+v} = i$

pro QQ: $BF = b, bG = \mathfrak{G}, Bb = v'$; diſt. arb: $= k'$ et $\frac{k'-v'}{k'+v'} = i'$

ac poſito ſemidiametro campi apparentis $E\varepsilon = z$, in imaginibus erit $F\zeta = \frac{1}{i}\cdot\frac{\alpha}{a}z$ et $G\eta = \frac{1}{i'}\cdot\frac{\alpha\mathfrak{G}}{ab}z$. Conſideretur iam radius a puncto ε in centrum lentis primae A incidens et per lentes transmiſſus qui vtique per extremitates imaginum ζ et η transibit. Iam

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Iam ex problemate praecedente patet esse $aN = \frac{\alpha z}{na}$, vnde in altera lente punctum M' ita definietur, vt sit

$$F\zeta - aN : aF = BM' - F\zeta : BF \text{ siue}$$
$$BM' = F\zeta + \frac{BF(F\zeta - aN)}{aF} = \frac{aB.F\zeta - BF.aN}{aF}$$

vnde fit

$$BM' = \frac{1}{i}.\frac{a+b}{a}z - \frac{bvz}{naa}.$$

Nunc punctum N' hinc perinde definietur, atque ex problemate primo ex puncto M determinabatur punctum N; erit quippe

$$bN' = i'.BM' - \frac{v'}{v}.F\zeta \text{ ideoque}$$
$$bN' = \frac{i'}{i}.\frac{a+b}{a}z - \frac{i'bv}{naa}z - \frac{1}{i}.\frac{av'}{nab}z$$

Hinc autem punctum O, vbi est locus oculi idoneus facile assignabitur erit enim $bN' + G\eta : bG = bN' : bO$, indeque

$$bO = \frac{bG.bN'}{bN' + G\eta} = \frac{\frac{i'}{i}.\frac{a+b}{a} - \frac{i'bv}{naa} - \frac{1}{i}.\frac{av'}{nab}}{\frac{i'}{i}.\frac{a+b}{a} - \frac{i'bv}{naa} - \frac{1}{i}.\frac{av'}{nab} + \frac{1}{ii'}.\frac{a}{a+b}\varepsilon}\varepsilon$$

Quod si ergo ponamus semidiametrum aperturae

pro lente PP { faciei anterioris $= \mathfrak{A} = x$ / faciei posterioris $= \mathfrak{a}$

pro lente QQ { faciei anterioris $= \mathfrak{B}$ / faciei posterioris $= \mathfrak{b}$

habe-

habebimus

$$\mathfrak{A} = 0$$

$$\mathfrak{a} = \frac{v}{n\,a}z$$

$$\mathfrak{B} = \left(\frac{1}{i}\cdot\frac{a+b}{a} - \frac{b\,v}{n\,a\,a}\right)z$$

$$\mathfrak{b} = \left(\frac{i'}{i}\cdot\frac{a+b}{a} - \frac{i'\,b\,v}{n\,a\,a} - \frac{1}{i}\cdot\frac{a\,v'}{n\,a\,b}\right)z$$

ac si distantia oculi post lentem QQ ponatur $b\mathrm{O} = \mathrm{O}$ erit

$$\mathrm{O} = \frac{\mathfrak{b}}{\mathfrak{b} + \frac{1}{i i'}\cdot\frac{a\,e}{a\,b}z}\cdot\mathfrak{E}$$

Coroll. 1.

246. Si ambarum lentium crassities euanescat erit $v = 0$. $v' = 0$, et $i = i' = 1$; quo ergo casu nostrae formulae in sequentes abibunt:

$$\mathfrak{A} = 0;\ \mathfrak{a} = 0;\ \mathfrak{B} = \frac{a+b}{a}z \text{ et } \mathfrak{b} = \frac{a+b}{a}z$$

Coroll. 2.

247. Datis ergo vicissim aperturis lentium ex aequationum traditarum ea, pro qua quantitas z minimum valorem adipiscitur, definietur campus adparens.

Coroll. 3.

248. Si igitur crassities lentium euanescat, campus apparens ex apertura lentis posterioris facillime determinatur. Erit enim $z = \frac{a\cdot\mathfrak{B}}{a+b}$; id quod intelligendum est, si distantia $b\mathrm{O} = \mathrm{O}$ fuerit positiua oculusque in O collocetur. Coroll.

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Coroll. 4.

249. Sin autem distantia $bO = O$ prodeat negatiua oculusque vltimae lenti immediate applicetur, tum eius apertura plus non praestat quam si amplitudini pupillae esset aequalis. Quare si $\mathfrak{b}$ maior fuerit semidiametro pupillae ω loco $\mathfrak{b}$ scribatur ω et ex vltima aequatione verus valor ipsius z elicietur nisi forte ex aliqua reliquarum aequationum adhuc minor valor pro z esset proditurus.

Problema 4.

250. Si instrumentum dioptricum ex tribus constet lentibus determinare cum campum apparentem, tum locum oculi idoneum. Tab. III. Fig. 14.

Solutio.

Existentibus imaginibus per has lentes successiue repraesentatis in $F\zeta$, $G\eta$ et $H\theta$, obiecto vero ipso in $E\varepsilon$, ponamus vt hactenus:

Pro Lente Distantias crassitiem dist. arb: et

Prima PP .. $AE = a$; $aF = \alpha$; $Aa = v$; k; $\frac{k-v}{k+v} = i$

Secunda QQ .. $BF = b$; $bG = \mathfrak{b}$; $Bb = v'$; k'; $\frac{k'-v'}{k'+v'} = i'$

Tertia RR .. $CG = c$; $cH = \gamma$; $Cc = v''$; k''; $\frac{k''-v''}{k''+v''} = i''$

Semidiametros vero aperturarum

Pro Lente

PP faciei $\begin{cases} \text{anterioris} = \mathfrak{A} = \mathfrak{o} \\ \text{posterioris} = \mathfrak{a} \end{cases}$

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QQ faciei $\begin{cases} \text{anterioris} = \mathfrak{B} \\ \text{posterioris} = \mathfrak{b} \end{cases}$

RR faciei $\begin{cases} \text{anterioris} = \mathfrak{C} \\ \text{posterioris} = \mathfrak{c} \end{cases}$

Tum vero sit semidiameter campi apparentis $E\varepsilon = z$, supraque ostendimus fore:

$$F\zeta = \frac{1}{i}\frac{\alpha}{a}z;\ G\eta = \frac{1}{i\,i'}\cdot\frac{\alpha\varepsilon}{a\,b}z;\ H\theta = \frac{1}{i\,i'\,i''}\cdot\frac{\alpha\varepsilon\gamma}{a\,b\,c}z.$$

His positis habebimus $a\,N = \mathfrak{a} = \frac{\nu z}{n\alpha}$; vnde si per F recta ipsi $N\,M'$ parallela ducta intelligatur erit

$$F\zeta - aN : aF = BM' - F\zeta : BF \text{ sive } BM' = \frac{\alpha B.F\zeta - BF.\alpha N}{\alpha F}$$

ideoque

$$B\,M' = \frac{\alpha+b}{a}\cdot\frac{1}{i}\cdot\frac{\alpha}{a}z - \frac{b\nu z}{n\alpha\alpha} \text{ seu}$$

$$\mathfrak{B} = \frac{1}{i}\cdot\frac{\alpha+b}{a}z - \frac{b\,\nu}{n\,\alpha\,\alpha}z$$

Porro vero est ex problemate primo $b\,N' = i'.BM' - \frac{\nu'}{n\,b}.F\zeta$,

hincque $\mathfrak{b} = \frac{i'}{i}\cdot\frac{\alpha+b}{a}z - \frac{i'b\nu}{n\alpha\alpha}z - \frac{1}{i}\cdot\frac{\alpha\nu'}{n\alpha b}z$

simili modo per G ducta intelligatur recta ipsi $N'M''$ parallela, eritque $b\,G : b\,N' + G\eta = CG : C\,M'' - G\eta$ sive

$$C\,M'' = \mathfrak{C} = G\eta + \frac{c}{\mathfrak{c}}(\mathfrak{b} + G\eta) = \frac{\mathfrak{c}+c}{\mathfrak{c}}\cdot G\eta + \frac{c}{\mathfrak{c}}\mathfrak{b},$$

vnde fit

$$\mathfrak{C} = \frac{1}{i\,i'}\cdot\frac{\alpha(\mathfrak{c}+c)}{a\,b}z + \frac{i''}{i}\cdot\frac{c(\alpha+b)}{a\,\mathfrak{c}}z - \frac{i''bc\nu}{n\alpha\alpha\mathfrak{c}}z - \frac{1}{i}\cdot\frac{\alpha c\nu'}{n\alpha b\mathfrak{c}}z$$

Deinde

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Deinde ex CM'' ita definitur cN'' per problema primum, vt sit

$$cN'' = i''.CM'' - \frac{v''}{nc}.G\gamma \text{ seu } \mathfrak{c} = i''.\mathfrak{C} - \frac{1}{i\,i'}.\frac{\alpha\varepsilon v''}{n\,a\,b\,c}z$$

hincque

$$\mathfrak{c} = \frac{i''}{i\,i'}.\frac{\alpha(\varepsilon+c)}{a\,b}z + \frac{i'i''}{i}.\frac{c(\alpha+b)}{a\,\varepsilon}z - i'i''.\frac{b\,c\,v}{n\,a\,a\,\varepsilon}z - \frac{i''}{i}.\frac{\alpha c v}{n\,a\,b\,\varepsilon}z - \frac{1}{i\,i'}.\frac{\alpha\varepsilon v''}{n\,a\,b\,c}z$$

Isti ergo valores sequenti modo determinantur:

$$\mathfrak{A} = 0$$

$$\mathfrak{a} = \frac{v}{n\,a}.E\varepsilon$$

$$\mathfrak{B} = \frac{\alpha+b}{a}.F\zeta - \frac{b}{a}.\mathfrak{a}$$

$$\mathfrak{b} = i'.\mathfrak{B} - \frac{v'}{n\,b}.F\zeta$$

$$\mathfrak{C} = \frac{\varepsilon+c}{\varepsilon}.G\eta + \frac{c}{\varepsilon}.\mathfrak{b}$$

$$\mathfrak{c} = i''.\mathfrak{C} - \frac{v''}{n\,c}.G\eta$$

Cum iam punctum O praebeat locum oculi iustum, si ponamus $cO = O$, erit $cN'' + H\theta : cH = cN'' : cO$ vnde reperitur

$$O = \frac{\gamma c}{1 + H\theta}, \text{ vel } \frac{1}{O} = \frac{1}{\gamma} + \frac{1}{i\,i'\,i''}.\frac{\alpha\varepsilon}{a\,b}.\frac{z}{c\,c}$$

Coroll. 1.

251. Ex his aequationibus sequitur fore

$$\frac{\mathfrak{B}}{b} + \frac{\mathfrak{a}}{a} = \frac{\alpha+b}{a\,b}.F\zeta = \frac{1}{i}\left(\frac{\alpha}{b}+1\right)\frac{z}{a} \text{ et}$$

$$\frac{\mathfrak{C}}{c} - \frac{\mathfrak{b}}{\varepsilon} = \frac{\varepsilon+c}{\varepsilon\,c}.G\eta = \frac{1}{i\,i'}\left(\frac{\alpha\varepsilon}{b\,c}+\frac{\alpha}{b}\right)\frac{z}{a}.$$

hincque porro:

$$\frac{i\mathfrak{a}}{a} + \frac{i\mathfrak{B}}{b} + \frac{i\,i'\mathfrak{b}}{\varepsilon} - \frac{i\,i''\mathfrak{C}}{c} = \left(1 - \frac{\alpha\varepsilon}{b\,c}\right)\frac{z}{a}.$$

Coroll. 2.

252. Si ergo craſſities lentium euaneſcat, cum ſit $a=0$, $b=\mathfrak{B}$ et $c=\mathfrak{C}$, definitio campi apparentis reducitur ad has duas aequationes.

I. $\mathfrak{B}.\frac{1}{b}=(1+\frac{a}{b})\frac{n}{a}$

II. $\mathfrak{B}\,(\frac{1}{b}+\frac{1}{c})-\mathfrak{C}.\frac{1}{c}=(1-\frac{a\mathfrak{C}}{bc})\frac{n}{a}$

vnde minor valor ipſius x praebet ſemidiametrum campi apparentis.

Coroll. 3.

253. Deinde ſi diſtantia $cO=O$ prodeat negatiua, vt oculum cogamur lenti vltimae immediate applicare, pro c ſcribi oportet ſemidiametrum pupillae ω, et ex vltima aequatione definietur ſemidiameter campi x, niſi forte ex alia aequatione adhuc minor valor pro x prodeat.

Problema 5.

254. Si inſtrumentum Dioptricum ex quatuor lentibus ſuper eodem axe diſpoſitis conſtet, determinare cum campum apparentem, tum locum oculi idoneum.

Solutio.

Tab. III. Fig. 15. Exiſtente obiecto $Ee=z$, fint imagines per lentes ſucceſſiue repraeſentatae $F\zeta$, $G\eta$, $H\theta$, et $I\iota$; ponamusque

musque pro lentium singularum determinatione vt hactenus:

	Pro lente	distantias	crassitiem	dist. arb.	et
Prima	PP	$AE=a;$	$aF=\alpha;$	$Aa=v;$	$..k;$	$\frac{k-v}{k+v}=i$
Secunda	QQ	$BF=b;$	$bG=\mathfrak{b};$	$Bb=v';$	$..k';$	$\frac{k'-v'}{k'+v'}=i'$
Tertia	RR	$CG=c;$	$cH=\gamma;$	$Cc=v'';$	$..k'';$	$\frac{k''-v''}{k''+v''}=i''$
Quarta	SS	$DH=d;$	$dI=\delta;$	$Dd=v'';$	$..k'';$	$\frac{k''-v''}{k''+v''}=i''$

Semidiametri vero aperturarum sint:

Pro Lente

Prima PP faciei { anterioris $=\mathfrak{A}=\mathfrak{a}$ / posterioris $=\mathfrak{a}$

Secunda QQ faciei { anterioris $=\mathfrak{B}$ / posterioris $=\mathfrak{b}$

Tertia RR faciei { anterioris $=\mathfrak{C}$ / posterioris $=\mathfrak{c}$

Quarta SS faciei { anterioris $=\mathfrak{D}$ / posterioris $=\mathfrak{d}$.

Si iam $E\varepsilon=z$ exhibeat semidiametrum campi apparentis erit, vt iam supra ostendimus:

$$F\zeta=\frac{1}{i}\cdot\frac{\alpha}{a}z;\ G\eta=\frac{1}{ii'}\cdot\frac{\alpha\mathfrak{b}}{ab}z;\ H\theta=\frac{1}{ii'i''}\cdot\frac{\alpha\mathfrak{b}\gamma}{abc}z$$

$$\text{et } I\iota=\frac{1}{ii'i''i'''}\cdot\frac{\alpha\mathfrak{b}\gamma\delta}{abcd}z$$

Quod si ratiocinium nunc vt ante instituamus, obtinebimus sequentes aequationes:

$$\mathfrak{A} = 0; \qquad \mathfrak{a} = \frac{v}{n a} \cdot E\varepsilon$$

$$\mathfrak{B} = \left(1 + \frac{b}{a}\right) F\zeta - \frac{b}{a}\mathfrak{a}: \qquad \mathfrak{b} = i' \mathfrak{B} - \frac{v'}{n b} \cdot F\zeta$$

$$\mathfrak{C} = \left(1 + \frac{c}{\beta}\right) G\eta + \frac{c}{\beta} \cdot \mathfrak{b}: \qquad \mathfrak{c} = i'' \mathfrak{C} - \frac{v''}{n c} \cdot G\eta$$

$$\mathfrak{D} = \left(1 + \frac{d}{\gamma}\right) H\theta + \frac{d}{\gamma} \cdot \mathfrak{c}: \qquad \mathfrak{d} = i''' \mathfrak{D} - \frac{v'''}{n d} \cdot H\theta$$

Ex quarum ordine priori consequimur:

$$\frac{\mathfrak{B}}{b} + \frac{\mathfrak{a}}{a} = \left(\frac{1}{a} + \frac{1}{b}\right) F\zeta = \frac{1}{i}\left(1 + \frac{a}{b}\right)\frac{z}{a}$$

$$\frac{\mathfrak{C}}{c} - \frac{\mathfrak{b}}{\beta} = \left(\frac{1}{\beta} + \frac{1}{c}\right) G\eta = \frac{1}{i\,i'}\left(\frac{a}{b} + \frac{a\varepsilon}{b c}\right)\frac{z}{a}$$

$$\frac{\mathfrak{D}}{d} - \frac{\mathfrak{c}}{\gamma} = \left(\frac{1}{\gamma} + \frac{1}{d}\right) H\theta = \frac{1}{i\,i'\,i''}\left(\frac{a\varepsilon}{b c} + \frac{a\varepsilon\gamma}{b c d}\right)\frac{z}{a}$$

ex ordine vero posteriori

$$\mathfrak{a} = \frac{v z}{n a}$$

$$\mathfrak{b} = i' \mathfrak{B} - \frac{1}{i} \cdot \frac{a}{b} \cdot \frac{v' z}{n a}$$

$$\mathfrak{c} = i'' \mathfrak{C} - \frac{1}{i\,i'} \cdot \frac{a\varepsilon}{b c} \frac{v'' z}{n a}$$

$$\mathfrak{d} = i''' \mathfrak{D} - \frac{1}{i\,i'\,i''} \cdot \frac{a\varepsilon\gamma}{b c d} \cdot \frac{v''' z}{n a}$$

Si denique pro loco oculi idoneo ponamus $d\mathfrak{O} = \mathfrak{O}$ erit

$$\mathfrak{O} = \frac{\delta \mathfrak{d}}{\mathfrak{d} + 1i} \text{ seu } \frac{1}{\mathfrak{O}} = \frac{1}{\delta} + \frac{1}{i\,i'\,i''\,i'''} \cdot \frac{a\varepsilon\gamma}{b c d} \cdot \frac{z}{a\mathfrak{d}}.$$

Coroll.

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Coroll. 1.

255. Ex aequationibus prioribus deducimus sequentes:

$$\frac{ia}{\alpha}+\frac{i\mathfrak{B}}{b}=\left(1+\frac{\mathfrak{z}}{b}\right)\frac{z}{a}$$

$$\frac{ia}{\alpha}+i\frac{\mathfrak{B}}{b}+\frac{ii'\mathfrak{b}}{\mathfrak{c}}-\frac{ii'\mathfrak{C}}{c}=\left(1-\frac{\alpha\mathfrak{c}}{b\,c}\right)\frac{z}{a}$$

$$\frac{ia}{\alpha}+\frac{i\mathfrak{B}}{b}+\frac{ii'\mathfrak{b}}{\mathfrak{c}}-\frac{ii'\mathfrak{C}}{c}-\frac{ii'i''c}{\gamma}+\frac{ii'i''\mathfrak{D}}{d}=\left(1+\frac{\alpha\mathfrak{c}\gamma}{b\,c\,d}\right)\frac{z}{a}.$$

quae quomodo ad plures lentes sint continuandae, facile perspicitur.

Coroll. 2.

256. Si lentium crassities evanescat, fiet $\mathfrak{a}=\sigma$; $\mathfrak{b}=\mathfrak{B}$; $\mathfrak{c}=\mathfrak{C}$ et $\mathfrak{d}=\mathfrak{D}$, porro $i=i'=i''=i'''=1$, unde hae aequationes in sequentes formas abibunt:

$$\mathfrak{B}.\frac{1}{b}=\left(1+\frac{\alpha}{b}\right)\frac{z}{a}$$

$$\mathfrak{B}\left(\frac{1}{b}+\frac{1}{\mathfrak{c}}\right)-\mathfrak{C}\frac{1}{c}=\left(1-\frac{\alpha\mathfrak{c}}{b\,c}\right)\frac{z}{a}$$

$$\mathfrak{B}\left(\frac{1}{b}+\frac{1}{\mathfrak{c}}\right)-\mathfrak{C}\left(\frac{1}{c}+\frac{1}{\gamma}\right)+\mathfrak{D}\frac{1}{d}=\left(1+\frac{\alpha\mathfrak{c}\gamma}{b\,c\,d}\right)\frac{z}{a}$$

ex quibus tribus aequationibus, vti in genere, valor ipsius z, qui prodierit minimus verum semidiametrum campi apparentis praebebit.

Coroll. 3.

257. Totus iste campus apparens revera spectabitur ab oculo in puncto O constituto, dummodo distantia $dO=O$ fuerit positiva. Sed si ea sit negativa, oculusque lenti vltimo SS immediate applicetur, pona-

ponatur $\mathfrak{d} = \omega$, scilicet semidiametro pupillae, et ex vltima aequatione elicietur semidiameter spatii in obiecto reuera conspicui.

Scholion 1.

258. Hinc igitur perspicitur, quomodo campus apparens a singularum lentium apertura pendeat; simulque patet quanta esse debeat cuiusque lentis apertura, vt campus apparens datae magnitudinis obtineatur. Si enim quantitas z cum quantitatibus ad lentium determinationem pertinentibus pro data assumatur, per nostras formulas successiue semidiametri aperturarum pro singulis lentibus definiuntur: vbi quidem deinceps est dispiciendum, num lentes tantae aperturae sint capaces. Hinc scilicet campo apparenti limites praefiniuntur, quos transgredi non liceat; vnde sequitur campum apparentem maiorem assumi non posse, quam vt aperturae inde pro singulis lentibus oriundae admitti queant. His autem definitis perinde est siue cuique lenti ea ipsa, quae fuerit inuenta apertura tribuatur, siue maior, dum ne sit minor quandoquidem hic aperturam lentis obiectiuae euanescentem assumsimus. Verum si insuper claritatis ratio habeatur, necesse est vt vera apertura cuiusque lentis eam, quam hic assignauimus, aliquantum superet, et quidem ea quantitate, quam supra pro limitibus ob claritatem requisitis exhibuimus, nisi enim hoc augmentum accesserit, extremitas in campo apparente minore lumine praedita erit quam

quam medium. Tum autem campus apparens latius patebit oramque obscuriorem complectetur; quamobrem si circa extremitates minori lumine contenti esse velimus, ne opus quidem est, vt lentibus maior apertura, quam quidem per formulas nostras definitur, tribuatur; superfluumque foret aperturas vltra hos limites augere, ita vt hinc cuique lenti conueniens apertura constituatur.

Scholion 2.

259. Etsi pro casu, quo lentium crassities negligitur formulae nostrae non multo simpliciores euadunt, tamen in iis percommode vsu venit, vt litterarum $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, coefficientes, scilicet $\frac{1}{b}+\frac{1}{\beta}$; $\frac{1}{c}+\frac{1}{\gamma}$; $\frac{1}{d}+\frac{1}{\delta}$; ipsam distantiam focalem, inuoluant cuiusque lentis; in praxi autem apertura satis tuto ex distantia focali colligi solet. Nam si lentis QQ distantia focalis ponatur $= q$, erit $\frac{1}{b}+\frac{1}{\beta} = \frac{1}{q}$, sicque $\mathfrak{B}(\frac{1}{b}+\frac{1}{\beta}) = \frac{\mathfrak{B}}{q}$; ac ne arcus nimis magni in apertura comprehendantur, necesse est vt sit $\mathfrak{B} < \frac{1}{8}q$; et pro varia lentis forma valor fractionis $\frac{\mathfrak{B}}{q}$ vsque ad $\frac{1}{4}$, vel $\frac{1}{3}$ diminui debet. Quare si ponamus

$$\mathfrak{B}(\tfrac{1}{b}+\tfrac{1}{\beta}) = \pi;\ \mathfrak{C}(\tfrac{1}{c}+\tfrac{1}{\gamma}) = \pi';\ \mathfrak{D}(\tfrac{1}{d}+\tfrac{1}{\delta}) = \pi''$$

hae litterae π, π', π'' eiusmodi denotabunt fractiones, quarum valor vt plurimum erit vel $\frac{1}{3}$ vel $\frac{1}{4}$ vel $\frac{1}{5}$, id scilicet tantum cauendum est, ne his litteris nimis magnus valor tribuatur. Quo obseruato cum arbitrio nostro relinquantur, imprimis conueniet has ipsas lit-

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teras in calculum introduci, ex iisque reliquas determinari; earum enim beneficio campus apparens facillime definitur. Quin etiam ipse campus apparens statim quoque in calculum induci poterit, quippe cuius determinatio deinceps per formulam simplicissimam expedietur. Hunc in finem vt tota inuestigatio ad meros numeros redigatur, ponam $\frac{\mathfrak{B}}{a} = \Phi$ ita vt Φ sit angulus, sub quo semidiameter campi apparentis ab oculo ad lentem obiectiuam collocato spectaretur. Videamus ergo quomodo per hos numeros π, π', π'', etc. et Φ reliquae quantitates definiantur.

Definitio 3.

260. Ratio aperturae cuiusque lentis mihi vocabitur quotus, qui oritur, si semidiameter aperturae diuidatur per distantiam focalem lentis; eius crassitie pro nihilo habita.

Coroll. 1.

261. Ita si b et $\mathfrak{C}$ sint distantiae determinatrices lentis, et $\mathfrak{B}$ semidiameter aperturae eiusdem, quia distantia focalis est $= \frac{b\mathfrak{C}}{b+\mathfrak{C}}$, ratio aperturae erit $= \mathfrak{B}\left(\frac{1}{b}+\frac{1}{\mathfrak{C}}\right)$

Coroll. 2.

262. Ratio igitur aperturae cuiusuis lentis est fractio minor quam $\frac{1}{2}$, quandoquidem hanc legem sanciuimus vt neutrius faciei arcus 60 gradibus maior in apertura contineatur.

Coroll.

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Coroll. 3.

263. Si scilicet ambae facies fuerint aeque curuae ratio aperturae per hanc legem vsque ad $\frac{1}{3}$ augeri poterit; sin autem altera facies fuerit plana, ratio aperturae $\frac{1}{4}$ superare vix poterit: ac si lens sit meniscus, ea adhuc minor statui debebit.

Coroll. 4.

264. Cum autem nihil sit, quo apertura lentium accuratius definiatur, ea fere arbitrio nostro relinquitur, et quouis casu commodissime per experientiam determinatur, sufficietque notasse, eam fractioni siue $\frac{1}{3}$ siue $\frac{1}{4}$ siue etiam $\frac{1}{5}$ pro forma lentis aequalem statui debere.

Scholion.

265. Vt scilicet quouis casu ratio aperturae recte definiatur, radios vtriusque faciei lentis contemplari oportet, qui si fuerint f et g, erit distantia focalis $= \frac{fg}{(n-1)(f+g)} = \frac{20}{11}\cdot\frac{fg}{f+g}$. Iam semidiameter aperturae minor esse debet quam $\frac{1}{3}f$ vel quam $\frac{1}{3}g$, prout vel f vel g fuerit minor. Sit $g < f$, et cum semidiameter aperturae minor esse debeat quam $\frac{1}{3}g$, ratio aperturae minor accipienda est quam $\frac{11}{40}\left(1+\frac{g}{f}\right)$. Vnde patet, si lens sit vtrinque aeque conuexa, seu $g = f$, rationem aperturae capi debere infra $\frac{11}{20}$; sin autem sit altera facies plana seu $f = \infty$, illum limitem esse $\frac{11}{40}$, qui adhuc minor fiet, si lens sit

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meniſcus, ſeu $\frac{g}{f}$ numerus negatiuus. Ceterum ſi ratio aperturae ſit $= \pi$, eique hoc modo idoneus valor tribuatur, perinde eſt ſiue is negatiue ſiue poſitiue accipiatur: ſemper autem conducet rationi aperturae minorem valorem tribui, quam ſecundum hanc regulam; partim vt obliquitas radiorum incidentium diminuatur, partim vero potiſſimum, vt ob claritatem aperturas lentium adhuc vltra augere liceat.

Problema 6.

266. Si inſtrumentum dioptricum ex quotcunque lentibus, quarum craſſitiem vt nullam ſpectare liceat, ſit compoſitum, dataque ſit ratio aperturae pro ſingulis lentibus vna cum campo apparente, definire diſtantias determinatrices ſingularum lentium.

Solutio.

Sit diſtantia obiecti ante lentem primam $AE = a$, imaginisque per eam repraeſentatae $aF = \alpha$, ac pro ſequentibus lentibus ponatur.

Pro Lente	Diſt: determinatrices	ratio aperturae
Secunda . .	$BF = b$; $bG = \beta$;	π
Tertia . .	$CG = c$; $cH = \gamma$;	π'
Quarta . .	$DH = d$; $dI = \delta$;	π''
Quinta . .	$EI = e$; $eK = \varepsilon$;	π'''

etc.

Hinc

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Hinc ergo si semidiametri aperturarum harum lentium vt ante indicentur literis $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, $\mathfrak{E}$, etc. erit $\pi = \mathfrak{B}(\frac{1}{b}+\frac{1}{\beta})$; $\pi' = \mathfrak{C}(\frac{1}{c}+\frac{1}{\gamma})$; $\pi'' = \mathfrak{D}(\frac{1}{d}+\frac{1}{\delta})$; $\pi''' = \mathfrak{E}(\frac{1}{e}+\frac{1}{\varepsilon})$ etc. Tum vero si sit semidiameter campi apparentis $= z$ ponatur etiam $\frac{z}{a} = \Phi$. Cum igitur hinc sit:

$$\mathfrak{B} = \frac{\pi b\beta}{b+\beta};\ \mathfrak{C} = \frac{\pi' c\gamma}{c+\gamma};\ \mathfrak{D} = \frac{\pi'' d\delta}{d+\delta};\ \mathfrak{E} = \frac{\pi''' e\varepsilon}{e+\varepsilon} \text{ etc.}$$

habebimus ex §. 256 sequentes aequationes:

$$\frac{\pi\beta}{b+\beta} = (1+\frac{\alpha}{b})\Phi$$
$$\pi - \frac{\pi'\gamma}{c+\gamma} = (1-\frac{\alpha\beta}{bc})\Phi$$
$$\pi - \pi' + \frac{\pi''\delta}{d+\delta} = (1+\frac{\alpha\beta\gamma}{bcd})\Phi$$
$$\pi - \pi' + \pi'' - \frac{\pi'''\varepsilon}{e+\varepsilon} = (1-\frac{\alpha\beta\gamma\delta}{bcde})\Phi$$
etc.

Quo iam facilius hinc per π, π', π'', π''' etc. et Φ distantiae determinatrices lentium definiri queant, ponatur $\alpha = Aa$; $\beta = Bb$; $\gamma = Cc$; $\delta = Dd$; $\varepsilon = Ee$ etc.

Ita vt litterae A, B, C, D, E etc. denotent numeros absolutos, ac nostrae aequationes induent has formas:

$$\frac{B\pi}{B+1} = (1+\frac{Aa}{b})\Phi$$
$$\pi - \frac{C\pi'}{C+1} = (1-\frac{ABa}{c})\Phi$$
$$\pi - \pi' + \frac{D\pi''}{D+1} = (1+\frac{ABCa}{d})\Phi$$
$$\pi - \pi' + \pi'' - \frac{E\pi'''}{E+1} = (1-\frac{ABCDa}{e})\Phi$$
etc.

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vnde eliciuntur sequentes determinationes:

$$b = \frac{A(B+1)a\Phi}{B\pi-(B+1)\Phi}$$

$$c = \frac{AB(C+1)a\Phi}{C\pi'-(C+1)(\pi-\Phi)}$$

$$d = \frac{ABC(D+1)a\Phi}{D\pi''-(D+1)(\pi'-\pi+\Phi)}$$

$$e = \frac{ABCD(E+1)a\Phi}{E\pi'''-(E+1)(\pi''-\pi'+\pi-\Phi)}$$

etc.

Datis ergo praeter numeros $\Phi, \pi, \pi', \pi'', \pi'''$ etc. numeris A, B, C, D, E etc. cum distantia obiecti $AF = a$, per has formulas distantiae b, c, d, e etc. determinantur indeque insuper alterae $\alpha, \beta, \gamma, \delta, \varepsilon$ etc. hoc modo:

$$\alpha = Aa$$

$$\beta = \frac{AB(B+1)a\Phi}{B\pi-(B+1)\Phi}$$

$$\gamma = \frac{ABC(C+1)a\Phi}{C\pi'-(C+1)(\pi-\Phi)}$$

$$\delta = \frac{ABCD(D+1)a\Phi}{D\pi''-(D+1)(\pi'-\pi+\Phi)}$$

$$\varepsilon = \frac{ABCDE(E+1)a\Phi}{E\pi'''-(E+1)(\pi''-\pi'+\pi-\Phi)}$$

etc.

Hinc nanciscimur distantias focales lentium

Primae $PP = \frac{Aa}{A+1}$

Secundae $QQ = \frac{ABa\Phi}{B\pi-(B+1)\Phi}$

Tertiae $RR = \frac{ABCa\Phi}{C\pi'-(C+1)(\pi-\Phi)}$

Quartae $SS = \frac{ABCDa\Phi}{D\pi''-(D+1)(\pi'-\pi+\Phi)}$

Quintae $TT = \frac{ABCDEa\Phi}{E\pi'''-(E+1)(\pi''-\pi'+\pi-\Phi)}$

etc.

Coroll.

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Coroll. 1.

267. Ex angulo Φ cum distantia obiecti ante lentem primam $AE = a$, ita definitur semidiameter campi apparentis z, vt sit $z = a\Phi$: neque tamen campus apparens pro lubitu assumi potest, sed is per multiplicationem determinabitur, vt mox videbimus.

Coroll. 2.

268. Cum omnes numeri hic in calculum introducti aeque negatiue ac positiue accipi queant, obseruandum est eos perpetuo ita assumi debere, vt interualla lentium quae sunt $\alpha+b$; $\beta+c$; $\gamma+d$; $\delta+e$; etc. omnia prodeant positiua.

Coroll. 3.

269. Quod ad aperturam cuiusque lentis attinet eius semidiameter habebitur si eius distantia focalis multiplicetur per rationem aperturae litteram π insignitam.

Scholion.

270. Quo formulas hic inuentas simpliciores reddamus quoniam litteris $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$; etc. non amplius indigebimus, ponamus ad abbreuiandum;

$$\frac{A}{A+1} = \mathfrak{A}; \frac{B}{B+1} = \mathfrak{B}; \frac{C}{C+1} = \mathfrak{C}; \frac{D}{D+1} = \mathfrak{D}; \frac{E}{E+1} = \mathfrak{E} \text{ etc.}$$

vt sit

$$A = \frac{\mathfrak{A}}{1-\mathfrak{A}}; B = \frac{\mathfrak{B}}{1-\mathfrak{B}}; C = \frac{\mathfrak{C}}{1-\mathfrak{C}}; D = \frac{\mathfrak{D}}{1-\mathfrak{D}}; E = \frac{\mathfrak{E}}{1-\mathfrak{E}} \text{ etc.}$$

at-

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atque habebimus:

$$\alpha = A\alpha; \qquad \beta = \frac{AB\alpha\Phi}{\mathfrak{B}\pi - \Phi}$$

$$\gamma = \frac{ABC\alpha\Phi}{\mathfrak{C}\pi' - \pi + \Phi} \qquad \delta = \frac{ABCD\alpha\Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi}$$

$$\varepsilon = \frac{ABCDE\alpha\Phi}{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi} \qquad \text{etc.}$$

Hincque porro definientur distantiae focales lentium:

$$\text{Primae } PP = \mathfrak{A}\alpha$$

$$\text{Secundae } QQ = \frac{AB\alpha\Phi}{\mathfrak{B}\pi - \Phi}$$

$$\text{Tertiae } RR = \frac{AB\mathfrak{C}\alpha\Phi}{\mathfrak{C}\pi' - \pi + \Phi}$$

$$\text{Quartae } SS = \frac{ABC\mathfrak{D}\alpha\Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi}$$

$$\text{Quintae } TT = \frac{ABCD\mathfrak{E}\alpha\Phi}{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi}$$

etc.

et lentium interualla:

$$\text{I et II} = \frac{A\mathfrak{B}\alpha\pi}{\mathfrak{B}\pi - \Phi}$$

$$\text{II et III} = \frac{AB\alpha\Phi(\mathfrak{C}\pi' - (1-\mathfrak{B})\pi)}{(\mathfrak{B}\pi - \Phi)(\mathfrak{C}\pi' - \pi + \Phi)}$$

$$\text{III et IV} = \frac{ABC\alpha\Phi(\mathfrak{D}\pi'' - (1-\mathfrak{C})\pi')}{(\mathfrak{C}\pi' - \pi + \Phi)(\mathfrak{D}\pi'' - \pi' + \pi - \Phi)}$$

$$\text{IV et V} = \frac{ABCD\alpha\Phi(\mathfrak{E}\pi''' - (1-\mathfrak{D})\pi'')}{(\mathfrak{D}\pi'' - \pi' + \pi - \Phi)(\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi)}$$

Quae interualla debent esse positiua.

Problema 7.

271. Positis iisdem, quae in problemate praecedente sunt assumta, definire locum idoneum oculi, vnde totus campus apparens conspici queat.

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Solutio.

Maneant omnes denominationes vt ante, et quia apertura lentis PP vt nulla ſpectatur, pro reliquis lentibus ex data aperturae ratione, ſemidiameter aperturae cuiusque ita ſe habebit

Lentis	ſemidiameter aperturae
Secundae QQ . .	$\frac{\mathfrak{B}a\pi}{\mathfrak{B}\pi-\Phi}\Phi$
Tertiae RR . .	$\frac{AB\mathfrak{C}a\pi'}{\mathfrak{C}\pi'-\pi'+\Phi}\Phi$
Quartae SS . .	$\frac{ABC\mathfrak{D}a\pi''}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}\Phi$
Quintae TT . .	$\frac{ABCD\mathfrak{E}a\pi'''}{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi}\Phi$

vbi litterae $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, $\mathfrak{E}$, etc. valores in praecedente Scholio aſſignatos obtinent.

Deinde magnitudines ſingularum imaginum conſiderari conuenit, quae ob $E\varepsilon = z = a\Phi$ et

$\alpha = Aa$, $\mathfrak{C} = Bb$; $\gamma = Cc$; $\delta = Dd$ etc.

erunt

$F\zeta = \alpha\Phi = Aa\Phi$

$G\eta = ABa\Phi$

$H\theta = ABCa\Phi$

$I\iota = ABCDa\Phi$

etc.

Iam pro quolibet lentium numero locus oculi idoneus ſeorſim definiri debet; denotante ergo O diſtantiam oculi poſt vltimam lentem

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I. Pro vnica lente

Quia crassities lentis vt nulla spectatur, euidens est pro loco oculi idoneo fore $O = o$.

II. Pro duabus Lentibus

Tab. III. Fig. 13. Cum hic sit $bN' + G\eta$: $bG = bN'$: bO erit $bO = O = \frac{bN'}{bN' + G\eta}\mathfrak{E}$. Sed est $bN' = \frac{A\mathfrak{B}\pi}{\mathfrak{B}\pi - \Phi}a\Phi$ et $G\eta = AB a\Phi$ vnde fit

$$bN' + G\eta = A a\Phi . \frac{(B+1)\mathfrak{B}\pi - B\Phi}{\mathfrak{B}\pi - \Phi} = \frac{AB a\Phi(\pi - \Phi)}{\mathfrak{B}\pi - \Phi}$$

ob $(B+1)\mathfrak{B} = \mathfrak{B}$. Erit ergo $\frac{bN'}{bN' + G\eta} = \frac{\mathfrak{B}\pi}{\mathfrak{B}(\pi - \Phi)}$, quae fractio per $\mathfrak{E} = \frac{AB a\Phi}{(\mathfrak{B}\pi - \Phi)}$ multiplicata dat locum oculi idoneum

$$O = \frac{A\mathfrak{B} a\pi\Phi}{(\pi - \Phi)(\mathfrak{B}\pi - \Phi)}$$

III. Pro tribus Lentibus

Fig. 14. Cum hic sit $cN'' + H\theta$: $cH = cN''$: cO, erit $cO = O = \frac{cN''}{cN'' + H\theta}\cdot\gamma$. Sed est

$$cN'' = \frac{AB\mathfrak{C} a\pi'\Phi}{\mathfrak{C}\pi' - \pi + \Phi} \text{ et } H\theta = AB\mathfrak{C}Ca\Phi$$

hincque ob $(C+1)\mathfrak{C} = C$ fiet

$$cN'' + H\theta = \frac{AB\mathfrak{C}Ca\Phi(\pi' - \pi + \Phi)}{\mathfrak{C}\pi' - \pi + \Phi}$$

et $\frac{cN''}{cN'' + H\theta} = \frac{\mathfrak{C}\pi'}{C(\pi' - \pi + \Phi)}$. Nunc igitur ob $\gamma = \frac{AB\mathfrak{C}Ca\Phi}{\mathfrak{C}\pi' - \pi + \Phi}$ habebimus distantiam oculi idoneam:

$$O = \frac{AB\mathfrak{C} a\pi'\Phi}{(\pi' - \pi + \Phi)(\mathfrak{C}\pi' - \pi + \Phi)}$$

IV.

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IV. Pro quatuor Lentibus

Cum sit $dN''' + I_4 : dI = dN''' : dO$, erit $dO = O = \frac{dN'''}{dN''' + I_4}\cdot\delta$. Sed est Tab. III. Fig. 15.

$$dN''' = \frac{ABCD\alpha\pi''\Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi} \text{ et } I_4 = AB\mathfrak{C}D\alpha\Phi$$

hincque ob $(D+1)\mathfrak{D} = D$ fiet

$$dN''' + I_4 = \frac{ABCD\alpha\Phi(\pi'' - \pi' + \pi - \Phi)}{\mathfrak{D}\pi'' - \pi + \pi - \Phi} \text{ et}$$

$$\frac{dN'''}{dN''' + I_4} = \frac{\mathfrak{D}\pi''}{D(\pi'' - \pi' + \pi - \Phi)}. \text{ Ergo ob } \delta = \frac{AB\mathfrak{C}D\alpha\mathfrak{A}}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi}$$

prodit distantia oculi idonea

$$O = \frac{ABCD\alpha\pi''\Phi}{(\pi'' - \pi' + \pi - \Phi)(\mathfrak{D}\pi'' - \pi' + \pi - \Phi)}.$$

V. Pro quinque Lentibus

Si ratiocinium simili modo ad casum quinque lentium extendatur, reperiemus distantiam oculi idoneam

$$O = \frac{ABCDE\alpha\pi'''\Phi}{(\pi''' - \pi'' + \pi' - \pi + \Phi)(\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi)}.$$

VI. Pro sex Lentibus

Eodemque modo progrediendo colligitur fore pro casu sex lentium distantiam oculi idoneam

$$O = \frac{ABCDE\mathfrak{F}\alpha\pi''''\Phi}{(\pi'''' - \pi''' + \pi'' - \pi' + \pi - \Phi)(\mathfrak{F}\pi'''' - \pi''' + \pi'' - \pi' + \Phi)}$$

sicque vlterius, quousque libuerit progredi licet.

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Coroll. 1.

272. Quouis ergo casu necesse est, vt distantia oculi idonea prodeat positiua: si enim fieret negatiua totus campus apparens nusquam conspici posset.

Coroll. 2.

273. Iis autem casibus, quibus distantia O fit negatiua oculum immediate vltimae lenti applicari conueniet; Tum vero oculus plus non cernet, quam si vltimae lentis apertura aequalis esset amplitudini pupillae.

Coroll. 3.

274. Hoc ergo casu statuatur semidiameter aperturae vltimae lentis $= \omega$ semidiametro pupillae, ex eaque aequatione eliciatur valor ipsius Φ, quo invento erit $a\Phi$ semidiameter campi apparentis, qui in obiecto reuera conspicietur.

Problema 8.

275. Positis iisdem atque in problematibus praecedentibus, eam conditionem in lentium dispositione definire, vt oculus in loco idoneo positus obiectum simul distincte videat.

Solutio.

Quia aperturam lentis obiectiuae euanescentem assumimus, in visione alia confusio locum habere nequit nisi quatenus oculus non in distantia iusta ab vltima

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vltima imagine, quam intuetur exiſtit, quae ergo tolletur, ſi lentes ita diſponantur, vt imago vltima ante oculum in O ſitnm in diſtantia iuſta, quam littera l deſignauimus, verſetur. Cum igitur in figuris locus oculi ante imaginem vltimam cadat haec diſtantia negatiue ſumta ipſi l aequalis eſt ponenda; vnde pro quouis lentium numero ſequentes habebimus determinationes.

I. Pro vnica lente

Cum hic ſit $O = o$, et $OF = \alpha = Aa$, oportet eſſe $Aa = -l$, ideoque $A = -\frac{l}{a}$: et $\alpha = -l$, vnde indoles huius lentis determinatur, ita vt eius diſtantia focalis eſſe debeat $= \frac{al}{l-a}$. Tab. III. Fig. 12.

II. Pro duabus lentibus

Ex inuenta diſtantia $bO = O$, erit $OG = \frac{O}{bN'}$. $G\eta$. Eſt vero $\frac{O}{bN'} = \frac{1}{\pi - \Phi}$; vnde fit $\frac{ABa\Phi}{\pi - \Phi} = -l$, hincque pro ſecunda lente $B = -\frac{(\pi-\Phi)l}{Aa\Phi}$. et $\mathfrak{B} = \frac{B}{B+1}$. Vel cum ſit $Aa\Phi = -\frac{(\pi-\Phi)l}{B}$, erit pro loco oculi Fig. 13.

$$O = \frac{-\mathfrak{B}\pi}{B(\mathfrak{B}\pi - \Phi)}\, l$$

III. Pro tribus lentibus

Hic eſt $OH = \frac{O}{cN''}$. $H\theta = \frac{H\theta}{\pi' - \pi + \Phi}$, vnde obtinetur: Fig. 14.

$$OH = \frac{ABC a\Phi}{\pi' - \pi + \Phi} = -l$$

ſicque pro vltima lente habebitur:

$$C = \frac{-(\pi' - \pi + \Phi)l}{AB a\Phi}$$

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At si pro determinatione primae lentis capiatur $A\alpha\Phi = \frac{-(\pi' - \pi + \Phi)l}{BC}$, erit distantia oculi

$$O = \frac{-\mathfrak{E}\pi'}{C(\mathfrak{E}\pi' - \pi + \Phi)}\, l$$

IV. Pro quatuor lentibus

Tab. III. Fig. 15.

Cum sit $OI = \frac{O}{a\mathfrak{N}'''}$ $I\mathfrak{i} = \frac{I\mathfrak{i}}{\pi'' - \pi' + \pi - \Phi}$, habebitur

$$OI = \frac{ABCD\alpha\Phi}{\pi'' - \pi' + \pi - \Phi} = -l$$

Vnde pro vltima lente

$$D = -\frac{(\pi'' - \pi' + \pi - \Phi)l}{ABC\alpha\Phi}$$

Cum autem sit $ABC\alpha\Phi = -\frac{(\pi'' - \pi' + \pi - \Phi)l}{D}$ erit in loco oculi hoc valore surrogando

$$O = \frac{-\mathfrak{D}\pi''}{D(\mathfrak{D}\pi'' - \pi' + \pi - \Phi)}\, l.$$

V. Pro quinque Lentibus

Simili modo pro quinque lentibus vltima ita comparata esse debet, vt sit

$$E = \frac{-(\pi''' - \pi'' + \pi' - \pi + \Phi)l}{ABCD\alpha\Phi}$$

Prima autem inde definita fit

$$O = \frac{-\mathfrak{E}\pi'''}{E(\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi)}\, l.$$

VI. Pro sex Lentibus

Eodem modo patet pro sex lentibus fore

$$F = \frac{-(\pi'''' - \pi''' + \pi'' - \pi' + \pi - \Phi)l}{ABCDE\alpha\Phi}$$

atque

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atque si hinc Aa definiatur:

$$O = \frac{-3\pi''''}{F(3\pi''''-\pi'''+\pi''-\pi'+\pi-\psi)}\, l$$

quas formulas quousque libuerit, continuare licet.

Coroll. 1.

276. Si distantia oculi iusta l fuerit infinita erit vt sequitur:

I. Pro vna lente $A = \infty$ et $\mathfrak{A} = 1$

II. Pro duabus lentibus $B = \infty$ et $\mathfrak{B} = 1$

III. Pro tribus lentibus $C = \infty$ et $\mathfrak{C} = 1$

IV. Pro quatuor lentibus $D = \infty$ et $\mathfrak{D} = 1$

etc.

Coroll. 2.

277. Casu ergo, quo distantia oculi iusta l est infinita, distantia oculi post lentem vltimam erit pro quouis lentium numero:

I. Pro vnica lente $O = \rho$

II. Pro duabus lentibus $O = \frac{A\alpha\pi\Phi}{(\pi-\Phi)^2}$

III. Pro tribus lentibus $O = \frac{AB\alpha\pi'\Phi}{(\pi'-\pi+\Phi)^2}$

IV. Pro quatuor lentibus $O = \frac{ABC\alpha\pi''\Phi}{(\pi''-\pi'+\pi-\Psi)^2}$

etc.

Scholion.

278. Hactenus campum apparentem Φ vt datum consideraui, ex eoque tam lentium indolem quam

quam earum disposition em determinaui, vt campus datae amplitudinis appareat, nihilque obstare deprehendimus, quominus huic conditioni satisfiat; cum numeri A, B, C, D, etc. penitus arbitrio relinquantur, aperturarum vero rationes π, π', π'', etc. infra $\frac{1}{3}$ vel $\frac{1}{4}$ accipi debeant. Verum hic multiplicationis ratio nondum in computum est ducta qua simul campus apparens ita adstringitur, vt certum limitem excedere nequeat. Quoniam igitur in omnibus instrumentis dioptricis multiplicatio imprimis proposita esse solet, quemadmodum per eam campus apparens definiatur in sequente problemate exponamus.

Problema 9.

279. Si instrumentum dioptricum ex quotcunque lentibus fuerit compositum, quarum quidem crassities vt nulla spectetur, simul vero multiplicationis ratio sit proposita, determinare campum apparentem.

Solutio.

Manentibus omnibus denominationibus, quibus hactenus sumus vsi, ita vt $a\Phi$ semidiametrum campi apparentis denotet, sit b distantia, ad quam multiplicationem referamus. Magnitudo igitur $a\Phi$ in distantia hac $=b$ nudo oculo cerneretur sub angulo, cuius tangens est $=\frac{a\Phi}{b}$. Quare si multiplicationis ratio statuatur $=m$, necesse est vt eadem magnitudo $a\Phi$ per lentes spectetur sub angulo, cuius tangens sit $=\frac{ma\Phi}{b}$.

Iam

Iam vero ex iis, quae in problematibus praecedentibus sunt tradita iste angulus facile assignatur, sicque obtinebitur angulus Φ, indeque semidiameter campi apparentis $a\Phi$. Cum autem haec multiplicatio non ad ipsos angulos sed eorum tangentes referatur euidens est tantum partes obiecti minimas circa centrum E sitas in ratione proposita multiplicari, remotiores vero in ratione minore. Quo notato hanc multiplicationis rationem m pro quouis lentium numero contemplemur.

I. Pro vnica Lente

Tangens anguli, quo imago $F\zeta$ ab oculo in O constituto conspicitur, est $\frac{F\zeta}{OF} = \frac{F\zeta}{aF}$ ob $aO = o$, Erit ergo $\frac{ma\Phi}{b} = \Phi$, seu $ma = b$. Hoc ergo casu campus apparens non determinatur sed multiplicationis ratio est $m = \frac{b}{a}$. Verum vt visio sit distincta, per superius problema debet esse $A = -\frac{1}{a}$ et distantia oculi post lentem $O = o$. At obiectum situ erecto cernetur: Tab. III. Fig. 12.

II. Pro duabus Lentibus

Tangens anguli, quo imago $G\eta$ ab oculo in O constituto cernetur, est $= \frac{bN'}{bO} = \pi - \Phi = \frac{ma\Phi}{b}$; vnde sequitur semidiameter campi apparentis Fig. 13.

$$\Phi = \frac{\pi b}{ma + b}$$ pro situ inuerso

Quo inuento, vt visio sit distincta oportet esse

$$B = \frac{-ml}{Ab}, \text{ et } \mathfrak{B} = \frac{-ml}{Ab - ml}$$

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hincque prodit distantia oculi post lentem oculatam

$$O = \frac{Abl(ma+b)}{mmal+Abb}$$

III. Pro tribus Lentibus

Tab. III. Fig. 14. Tangens anguli, quo imago $H\theta$ ab oculo in O constituto cernitur est $\frac{cN''}{cO} = \pi' - \pi + \Phi = \frac{ma\Phi}{b}$, vnde fit semidiameter campi apparentis:

$$\Phi = \frac{(\pi'-\pi)b}{ma-b} \text{ pro situ erecto}$$

Deinde vt visio sit distincta, oportet esse

$$C = \frac{-ml}{ABb} \text{ et } \mathfrak{C} = \frac{-ml}{ABb-ml}$$

Cum igitur sit $\pi' - \pi + \Phi = \frac{ma(\pi'-\pi)}{ma-b}$ erit

$$\mathfrak{C}\pi' - \pi + \Phi = \pi' - \pi + \Phi - \frac{ABb\pi'}{ABb-ml} = \frac{ma(\pi'-\pi)}{ma-b} - \frac{ABb\pi'}{ABb-ml}$$

hincque pro loco oculi

$$O = \frac{-ABbl\pi'}{(ABb-ml)(\mathfrak{C}\pi'-\pi+\Phi)} = \frac{ABbl(ma-b)\pi'}{(mmal-ABbb)\pi' + ma(ABb-ml)\pi}$$

IV. Pro quatuor lentibus

Tangens anguli, quo imago I: ab oculo in O Fig. 15. constituto cernitur est $= \frac{dN'''}{dO} = \pi'' - \pi' + \pi - \Phi = \frac{ma\Phi}{b}$;

vnde elicitur:

$$\Phi = \frac{(\pi''-\pi'+\pi)b}{ma+bb} \text{ pro situ inuerso}$$

Hinc porro pro visione distincta esse debet

$$D = \frac{-ml}{ABCb}, \text{ et } \mathfrak{D} = \frac{-ml}{ABCb-ml}$$

vnde fit

$$\mathfrak{D}\pi'' - \pi' + \pi - \Phi = \frac{ma(\pi''-\pi'+\pi)}{ma+b} - \frac{ABCb\pi''}{ABCb-ml}$$

et

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et pro loco oculi

$$O = \frac{ABCbl(ma+b)\pi''}{(mmal+ABCDb)\pi'' + ma(ABCb-ml)(\pi'-\pi)}.$$

V. Pro quinque lentibus

Eodem modo progrediendo pro campo apparente reperitur:

$$\Phi = \frac{(\pi''' - \pi'' + \pi' - \pi)b}{ma-b}$$ pro situ erecto

et vt visio euadat distincta

$$E = \frac{-ml}{ABCDb}$$ et $$\mathfrak{E} = \frac{-ml}{ABCDb-ml}$$

vnde pro loco oculi idoneo concluditur

$$O = \frac{ABCDbl(ma-b)\pi'''}{(mmal-ABCDbb)\pi''' + ma(ABCDb-ml)\pi'' - \pi' + \pi)}.$$

VI. Pro sex lentibus

Hic campus apparens ita definitur, vt sit:

$$\Phi = \frac{(\pi'''' - \pi''' + \pi'' - \pi' + \pi)b}{ma+b}$$ pro situ inuerso

visio vero distincta exigit

$$F = \frac{-ml}{ABCDEb}; \quad \mathfrak{F} = \frac{-ml}{ABCDEb-ml}$$

vnde pro loco oculi idoneo

$$O = \frac{ABCDEbl(ma+b)\pi''''}{(mmal+ABCDEbb)\pi'''' + ma(ABCDEb-ml)(\pi''' - \pi'' + \pi' - \pi)}$$

sicque progressio ad plures lentes est manifesta.

Coroll. 1.

280. Datis ergo rationibus aperturarum singularum lentium π, π', π'', etc. vna cum ratione multiplicationis m, distantia b, ad quam multiplicatio

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refertur, et distantia obiecti ante instrumentum a, determinatur campus apparens.

Coroll. 2.

281. Vt ergo campus apparens pro data multiplicatione maximus obtineatur litteris π, π', π'', etc. ita valores maximos tribui conueniet, vt alternatim sint positiui et negatiui.

Coroll. 3.

282. Si igitur valores π, π', π'', etc. vsque ad $\frac{1}{3}$ augeri liceat, maximus valor ipsius Φ pro quouis lentium numero erit vt sequitur:

Pro casu duarum lentium $\Phi = \frac{b}{3(ma+b)}$

Pro casu trium lentium $\Phi = \frac{2b}{3(ma-b)}$

Pro casu quatuor lentium $\Phi = \frac{3b}{3(ma+b)}$

Pro casu quinque lentium $\Phi = \frac{4b}{3(ma-b)}$

etc.

Coroll. 4.

283. Quo plures ergo lentes adhibentur eo magis campus apparens augeri potest simul vero patet, quo maior multiplicatio desideretur, eo minorem fieri campum apparentem.

Coroll.

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Coroll. 5.

284. Ratio multiplicationis m tam positiue quam negatiue capi potest. Si positiue accipitur pro lentium numero pari situm inuersum, pro impari autem situm erectum declarat. Contrarium vero euenit, si m fuerit numerus negatiuus.

Scholion 1.

285. Hic autem imprimis notandum est, valorem ipsius Φ tum solum angulum EAe praebere, quando fuerit tam exiguus, vt aliquot gradus non superet; si enim valor ipsius Φ prodeat multo maior, tum tangentem huius anguli EAe exprimit. Plerumque autem, si quidem multiplicatio sit modica, iste valor ipsius Φ tam paruus reperitur, vt sine errore pro ipso angulo EAe accipi possit. Hic igitur ob aliam causam amplitudo campi apparentis restringitur, vt certum limitem superare nequeat; cum enim angulus, quo radii in oculum incidentes in O ad axem inclinantur, nunquam possit esse rectus, neque fortasse vix 60° superare queat, quandoquidem ne nudo quidem oculo spatium in coelo maius quam 120° conspicere valeamus; si illum angulum maximum, quem oculus capere valeat, circiter 63° statuamus, vt eius tangens sit $= 2$ pro quouis lentium numero habebimus $\frac{ma\Phi}{b} = 2$, vnde fit $\Phi = \frac{2b}{ma}$, et $a\Phi = \frac{2b}{m}$. Data ergo multiplicatione m et distantia b ad quam refertur, semidiameter spatii in obiecto

conſpicui nunquam maior exiſtere poteſt quam $\frac{2b}{m}$ quotcunque etiam adhibeantur lentes, eaeque ita diſponantur vt maximum campum patefaciant. In Teleſcopiis ergo, vbi ſumitur $b = a$, et ſemidiameter campi ex ipſo angulo Φ aeſtimatur, eius tangens nunquam maior eſſe poteſt quam $\frac{2}{m}$: vnde ſequentem tabellam adiungo, quae pro quauis multiplicatione ſemidiametrum campi apparentis maximi oſtendit, quem nunquam ſuperare liceat.

Multiplicatio m	Semidiam: campi app: maximi	Multiplicatio m	Semidiam: campi app: maximi
5	$21^\circ, 48'$	60	$1^\circ, 54', 33''$
10	11, 18	70	1, 38, 13
15	7, 35	80	1, 25, 57
20	5, 42	90	1, 16, 24
25	4, 34	100	1, 8, 45
30	3, 49	150	0, 45, 50
35	$3, 16\frac{1}{2}$	200	0, 34, 22
40	2, 52	250	0, 27, 30
45	2, 33	300	0, 22, 55
50	$2, 17\frac{1}{2}$	400	0, 17, 11
		500	0, 13, 45

Quod ſi ergo numerum lentium multiplicando iam fere ad tantum campum apparentem pertigerimus, is vlterius nullo modo augeri poterit.

Scho-

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Scholion 2.

286. Quod ad locum oculi idoneum attinet, eum ideo in O constituimus, vt omnes radios per lentes transmissos accipiat, etiamsi pupilla maxime esset constricta: ex quo patet ob amplitudinem pupillae oculum de hoc loco sine vllo detrimento aliquantillum remoueri posse, ita vt superfluum foret hunc locum nimis sollicite obseruare, nisi forte apertura vltimae lentis fuerit admodum magna. Sin ea autem pupillam non superet, eaque adeo sit minor, manifestum est oculum ei immediate applicatum aeque omnes radios excipere, et eundem campum contueri, ac si in loco idoneo esset constitutus. His igitur casibus, si forte distantia O pro loco oculi prodeat negatiua, nihil de campo apparente perit, dummodo oculus lenti vltimae immediate applicetur. His itaque, quae ad visionem per instrumenta dioptrica in genere pertinent, expeditis, supereſt, vt inuestigemus, quantum visio ob diuersam radiorum refrangibilitatem turbetur, et quemadmodum hanc perturbationem eniture queamus.

CAPVT

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CAPVT VI.

DE CONFVSIONE A DIVERSA RADIORVM INDOLE ORIVNDA.

Problema 1.

287.

Si a puncto dato E radii per lentem PP transmittantur definire variationem in loco imaginis F, quae a diuerſa radiorum refrangibilitate oritur.

Solutio.

Sit diſtantia puncti E ante lentem $AE = a$, facierum autem lentis radius anterioris $= f$ poſterioris $= g$ et craſſities $Aa = v$, quae quantitates ſunt conſtantes. Poſita nunc refractionis ratione ex aere in vitrum $= n : 1$, ob diuerſam radiorum naturam numerus n erit variabilis, ideoque etiam locus imaginis F poſt lentem expreſſae, cuius diſtantia ſi ponatur $aF = \alpha$, erit ex ſupra inuentis

$$\frac{n-1}{f} = \frac{1}{a} + \frac{2n}{k+v} \text{ et } \frac{n-1}{g} = \frac{1}{\alpha} - \frac{2n}{k-v}$$

vbi

vbi quantitas k etiam pro variabili est habenda, quia tantum a, f, g et v sunt constantes. Quaestio ergo huc redit, vt si numerus n differentiali suo dn crescere sumatur, definiatur differentiale distantiae α. Quare differentientur ambae aequationes illae:

$$\frac{dn}{f} = \frac{2dn}{k+v} - \frac{2ndk}{(k+v)^2}$$
$$\frac{dn}{g} = \frac{-d\alpha}{\alpha\alpha} - \frac{2dn}{k-v} + \frac{2ndk}{(k-v)^2}.$$

indeque eliminato dk habebitur

$$\frac{dn(k+v)^2}{f} + \frac{dn(k-v)^2}{g} = 2\,dn(k+v) - 2dn(k-v) - \frac{d\alpha(k-v)^2}{\alpha\alpha}$$

Restituantur pro f et g valores initio positi, ac peruenietur ad hanc aequationem:

$$\frac{dn(k+v)^2}{a} + \frac{dn(k-v)^2}{\alpha} + 4v\,dn + \frac{(n-1)d\alpha}{\alpha\alpha}(k-v)^2 = 0$$

vnde reperitur:

$$\frac{d\alpha}{\alpha\alpha} = \frac{-dn}{(n-1)^2}\left(\frac{k+v}{k-v}\right)^2 - \frac{dn}{(n-1)\alpha} - \frac{4vdn}{(n-1)(k-v)^2} \text{ seu}$$
$$d\alpha = \frac{-\alpha dn}{n-1}\left(1 + \frac{\alpha}{a}\left(\frac{k+v}{k-v}\right)^2 + \frac{4\alpha v}{(k-v)^2}\right)$$

Tum vero cum etiam k sit quantitas variabilis erit

$$dk = \frac{-(k+v)dn}{n(n-1)}\left(1 + \frac{k+v}{2a}\right)$$

Cum ergo posuerimus $\frac{k-v}{k+v} = i$, ob $di = \frac{2vdk}{(k+v)^2}$ erit

$$di = \frac{-vdn}{n(n-1)}\left(\frac{1}{a} + \frac{2}{k+v}\right).$$

Coroll. 1.

288. Si $n:1$ denotet rationem refractionis radiorum mediae naturae, vt sit $n = \frac{31}{30} = 1,55$. erit

pro radiis rubris seu minime refractis $n = 1,54$, et violaceis $n = 1,56$. quorum valorum discrimen a medio, cum sit $= \frac{1}{100}$, pro differentiali dn haberi poterit.

Coroll. 2.

289. Quare si α denotet distantiam imaginis a radiis mediis formatae, pro ea quae a rubris formatur, erit $dn = -\frac{1}{100}$ et $\frac{dn}{n-1} = \frac{-1}{55}$. Hinc distantia imaginis rubrae post lentem erit $= \alpha + \frac{\alpha}{55}\left(1 + \frac{\alpha}{a}\left(\frac{k+v}{k-v}\right)^2 + \frac{4\alpha v}{(k-v)^2}\right)$ Distantia autem imaginis violaceae post lentem erit

$$\alpha - \frac{\alpha}{55}\left(1 + \frac{\alpha}{a}\left(\frac{k+v}{k-v}\right)^2 + \frac{4\alpha v}{(k-v)^2}\right).$$

Coroll. 3.

290. Si crassities lentis euanescat, vt sit $v = 0$, ob variabilitatem numeri n erit

$$d\alpha = \frac{-\alpha\, dn}{n-1}\left(1 + \frac{\alpha}{a}\right) = \frac{-\alpha\alpha\, dn}{n-1}\left(\frac{1}{a} + \frac{1}{\alpha}\right)$$

Ac si distantia focalis lentis ponatur $= p$, cum sit $\frac{1}{a} + \frac{1}{\alpha} = \frac{1}{p}$, erit $d\alpha = \frac{-\alpha\alpha\, dn}{(n-1)p} = \frac{-20\,\alpha\alpha dn}{11p}$.

Scholion.

291. Hoc ergo modo ob diuersam radiorum naturam valor distantiae α immutatur, vnde si ab ea tanquam ab obiecto radii porro ad lentem secundam emittantur, fiet etiam respectu huius lentis distantia obiecti variabilis. Quam ob causam in loco imaginis ab ea formatae duplex variatio orietur: id quod deinceps etiam in lentibus sequentibus multo magis

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magis eueniet. Hanc igitur variationem, quae pro quauis lente in loco imaginis naſcitur, in ſequente problemate determinemus.

Problema 2.

292. Si locus imaginis F, quae reſpectu lentis QQ vicem obiecti gerit, ob diuerſam radiorum naturam ipſe ſit variabilis, determinare variationem, quam ob eandem cauſam imago ſequens in G patietur.

Solutio.

Sit pro radiis mediae naturae, quibus reſpondet numerus n diſtantia obiecti F ante lentem $BF = b$; imaginisque inde proiectae diſtantia poſt lentem $bG = \mathfrak{b}$; dum autem n abit in $n+dn$, hae diſtantiae ambae b et $\mathfrak{b}$ capiant ſua incrementa differentialia db et $d\mathfrak{b}$. Ad quae inuenienda ſit lentis QQ radius faciei anterioris $= f$, poſterioris $= g$, et craſſities $Bb = v$, eritque vt ante:

$$\frac{n-1}{f} = \frac{1}{b} + \frac{2n}{k+v} \text{ et } \frac{n-1}{g} = \frac{1}{\mathfrak{b}} - \frac{2n}{k-v}$$

vbi k cum b et $\mathfrak{b}$ pro variabili eſt habenda. Differentiatione ergo inſtituta habebitur.

$$\frac{dn}{f} = \frac{-db}{bb} + \frac{2dn}{k+v} - \frac{2ndk}{(k+v)^2}$$

$$\frac{dn}{g} = -\frac{d\mathfrak{b}}{\mathfrak{b}\mathfrak{b}} - \frac{2dn}{k-v} + \frac{2ndk}{(k-v)^2}$$

vnde eliminato dk, fit

$$\frac{dn(k+v)^2}{f} + \frac{dn(k-v)^2}{g} = \frac{-db}{bb}(k+v)^2 - \frac{d\mathfrak{b}}{\mathfrak{b}\mathfrak{b}}(k-v)^2 + 4vdn$$

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quae multiplicata per $n-1$, si pro f et g valores dati substituantur prodit

$$\frac{dn(k+v)^2}{b}+2ndn(k+v)+\frac{dn(k-v)^2}{\mathfrak{c}}-2ndn(k-v)$$
$$=-\frac{(n-1)db}{bb}(k+v)^2-\frac{(n-1)d\mathfrak{c}}{\mathfrak{c}\mathfrak{c}}(k-v)^2+4(n-1)vdn$$

seu

$$\frac{dn(k+v)^2}{(n-1)b}+\frac{dn(k-v)^2}{(n-1)\mathfrak{c}}+\frac{4vdn}{n-1}+\frac{db}{bb}(k+v)^2+\frac{d\mathfrak{c}}{\mathfrak{c}\mathfrak{c}}(k-v)^2=0$$

Atque hinc elicitur:

$$d\mathfrak{c}=-\frac{\mathfrak{c}\mathfrak{c}db}{bb}\left(\frac{k+v}{k-v}\right)^2-\frac{\mathfrak{c}dn}{n-1}\left(1+\frac{\mathfrak{c}}{b}\left(\frac{k+v}{k-v}\right)^2+\frac{4\mathfrak{c}v}{(k-v)^2}\right)$$

Pro variabilitate autem ipsius k reperietur

$$\frac{dk}{(k+v)^2}=-\frac{db}{2nbb}-\frac{dn}{2n(n-1)b}-\frac{dn}{n(n-1)(k+v)}$$

Quare si ponatur $\frac{k-v}{k+v}=i$, ob $di=\frac{2vdk}{(k+v)^2}$ erit

$$di=\frac{-vdb}{nbb}-\frac{vdn}{n(n-1)b}-\frac{2vdn}{n(n-1)(k+v)}$$

ac si loco k numerus i introducatur, erit

$$d\mathfrak{c}=-\frac{\mathfrak{c}\mathfrak{c}db}{iibb}-\frac{bdn}{n-1}\left(1+\frac{\mathfrak{c}}{iib}+\frac{(1-ii)^2\mathfrak{c}}{iivv}\right)\text{ et}$$

$$di=\frac{-vdb}{nbb}-\frac{vdn}{n(n-1)}\left(\frac{1}{b}+\frac{1-ii}{vv}\right).$$

Coroll. 1.

293. Inuenta aequatio differentialis etiam hac forma repraesentari potest vt sit

$$\frac{id\mathfrak{c}}{\mathfrak{c}\mathfrak{c}}+\frac{db}{ibb}=\frac{-dn}{n-1}\left(\frac{i}{\mathfrak{c}}+\frac{1}{ib}+\frac{(1-ii)^2}{ivv}\right)$$

siue

siue restituendo k

$$\frac{dG}{GG}\left(\frac{k-v}{k+v}\right)+\frac{db}{bb}\left(\frac{k+v}{k-v}\right)=\frac{-dn}{n-1}\left(\frac{1}{G}\left(\frac{k-v}{k+v}\right)+\frac{1}{b}\left(\frac{k+v}{k-v}\right)+\frac{4v}{kk-vv}\right)$$

vbi haec obseruanda est analogia, vt quemadmodum ad b refertur $\frac{k+v}{k-v}$, ita ad G referatur $\frac{k-v}{k+v}$.

Coroll. 2.

294. Si lentis huius crassities euanescat, sit $v=0$, et $i=1$ vbi figura lentis non amplius in computum ingreditur sed sola distantia focalis, vnde variatio in loco imaginis G ita erit comparata, vt sit

$$\frac{dG}{GG}+\frac{db}{bb}=\frac{-dn}{n-1}\left(\frac{1}{G}+\frac{1}{b}\right) \text{ ideoque}$$

$$dG=\frac{-GG}{bb}db-\frac{GGdn}{n-1}\left(\frac{1}{G}+\frac{1}{b}\right).$$

Coroll. 3.

295. Si casum a radiis mediae naturae, ad quos formulae hactenus traditae sunt accommodatae, ad radios rubros transferre velimus, poni oportet $dn=-\frac{1}{100}$, sin autem ad radios violaceos $dn=+\frac{1}{100}$.

Problema 3.

296. Si radii ab obiecto E per lentes quotcunque transmittantur, determinare variationem in locis singularum imaginum, quae a diuersa radiorum refrangibilitate proficiscitur.

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Solutio.

Retineantur omnes denominationes, quibus in ſuperioribus capitibus ſumus vſi, ac ſint $a, \alpha, b, \mathfrak{C}, c, \gamma$ etc. diſtantiae determinatrices lentium pro radiis mediae naturae. Variata ergo ratione refractionis etiam hae diſtantiae variabuntur, quarum variationes differentialibus indicemus. Cum autem diſtantiae inter binas lentes maneant conſtantes, illae variationes ita erunt comparatae vt ſit

$$d\alpha + db = 0, \; d\mathfrak{C} + dc = 0, \; d\gamma + d.d = 0 \text{ etc.}$$

Cum iam diſtantia obiecti $AE = a$ ſit inuariabilis erit ex problemate primo ſi ponatur $\frac{k-v}{k+v} = i$

$$da = -db = \frac{-\alpha\alpha dn}{i(n-1)}\left(\frac{1}{\alpha} + \frac{1}{ia} + \frac{(1-i)^2}{iv}\right) = \frac{-\alpha\alpha dn}{n-1} \cdot \frac{k+v}{k-v}\left(\frac{k-v}{\alpha(k+v)} + \frac{k+v}{a(k-v)} + \frac{4v}{kk-vv}\right)$$

$$\text{et } di = \frac{-v\,dn}{n(n-1)}\left(\frac{1}{a} + \frac{1-i}{v}\right)$$

Deinde pro ſecunda lente, ad quam referuntur diſtantiae determinatrices b, et $\mathfrak{C}$ cum arbitraria k' et craſſitie v', vnde fecimus $\frac{k'-v'}{k'+v'} = i'$, habebimus

$$d\mathfrak{C} = -dc = \frac{-\mathfrak{C}\mathfrak{C}\,db}{i'i'bb} - \frac{\mathfrak{C}\mathfrak{C}\,dn}{i'(n-1)}\left(\frac{i'}{\mathfrak{C}} + \frac{1}{i'b} + \frac{(1-i')^2}{i'v'}\right)$$

$$\text{et } di' = \frac{-v'\,db}{nbb} - \frac{v'\,dn}{n(n-1)}\left(\frac{1}{b} + \frac{1-i'}{v'}\right)$$

Simili modo pro tertia lente, ad quam referuntur diſtantiae determinatrices c et γ cum arbitraria k'' et craſſitie v'' poſito $\frac{k''-v''}{k''+v''} = i''$ adipiſcemur:

$$d\gamma = -d.d = \frac{-\gamma\gamma\,dc}{i''i''c.c} - \frac{\gamma\gamma\,dn}{i''(n-1)}\left(\frac{i''}{\gamma} + \frac{1}{i''\mathfrak{C}} + \frac{(1-i'')^2}{i''v''}\right)$$

$$\text{et } di'' = \frac{-v''\,dc}{n\,c\,c} - \frac{v''\,dn}{n(n-1)}\left(\frac{1}{c} + \frac{1-i''}{v''}\right)$$

atque

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atque vlterius progrediendo obtinebimus sequentes formulas:

$$d\delta = -de = \frac{-\delta\delta\, dd}{ii''\, dd} - \frac{\delta\,\delta\, dn}{ii''(n-1)}\left(\frac{i''}{\delta} + \frac{1}{ii''d} + \frac{(1-ii'')^2}{ii''\, vv}\right)$$

$$\text{et } di''' = \frac{-v''\, dd}{n\, dd} - \frac{v''\, dn}{n(n-1)}\left(\frac{1}{d} + \frac{1-i'''}{v'''}\right)$$

vnde haec differentialia facile ad quotcunque lentes extenduntur. Atque si hic successiue valores differentialium dd, dc, db, iam ante definiti substituantur omnia haec differentialia tam distantiarum determinatricium, quam numerorum i, i', i'', i''' etc. per differentiale dn exprimentur.

Si ratio refractionis pro singulis lentibus sit diuersa pro iisque ordine exprimatur numeris, $n; n', n''$ etc. perspicuum est, differentialia hic inuenta sequenti modo expressum iri:

I. $$da = -db = \frac{-\alpha\alpha\, dn}{i(n-1)}\left(\frac{i}{\alpha} + \frac{1}{i\alpha} + \frac{(1-i)^2}{iv}\right)$$

$$di = \frac{-v\, dn}{n(n-1)}\left(\frac{1}{\alpha} + \frac{1-i}{v}\right)$$

II. $$d\mathfrak{B} = -dc = \frac{-\mathfrak{B}\mathfrak{B}\, db}{i'i'.\, bb} - \frac{\mathfrak{B}\mathfrak{B}\, dn'}{i'(n'-1)}\left(\frac{i'}{\mathfrak{B}} + \frac{1}{i'b} + \frac{(1-i')^2}{i'v'}\right)$$

$$di' = \frac{-v'\, db}{n'\, bb} - \frac{v'\, dn'}{n'(n'-1)}\left(\frac{1}{b} + \frac{1-i'}{v'}\right)$$

III. $$d\gamma = -dc = \frac{-\gamma\gamma\, dc}{i''.i'''.\, cc} - \frac{\gamma\gamma\, dn''}{i''(n''-1)}\left(\frac{i''}{\gamma} + \frac{1}{i''c} + \frac{(1-i'')^2}{i''.\, vv}\right)$$

$$di'' = \frac{-v''\, dc}{n''cc} - \frac{v''\, dn''}{n''(n''-1)}\left(\frac{1}{c} + \frac{1-i''}{v''}\right)$$

etc.

ex quibus formulis etiam mutationes singularum imaginum ac proinde etiam tandem vltimae imaginis facile

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facile definiri poterunt, seu potius loco imaginum angulos, sub quibus eae oculo ad iustam distantiam l positae sint adparituraе, considerемus.

Pro vna lente $\frac{1}{i} \cdot \frac{\alpha}{a} \cdot \frac{z}{l}$

Pro duabus lentibus $\frac{1}{i i'} \cdot \frac{\alpha\beta}{ab} \cdot \frac{z}{l}$

Pro tribus lentibus $\frac{1}{i i' i''} \cdot \frac{\alpha\beta\gamma}{abc} \cdot \frac{z}{l}$

etc.

quatenus scilicet praeter distantias a, α; b, β; c, γ etc. etiam litterae i, i', i'', i''' etc. sunt variabiles.

Coroll. 1.

297. Hinc igitur per differentiationem definire licet, quanta mutatio in loco vltimae imaginis, quae obiectum visionis constituit, ob diuersam radiorum refrangibilitatem oriri debeat.

Coroll. 2.

298. Deinde cum etiam magnitudinem cuiusque imaginis supra per distantias determinatrices et numeros i, i', i'' etc. definiuerimus, pro magnitudine imaginis habetur $\frac{1}{i i' i'' i'''} \cdot \frac{\alpha\beta\gamma\delta}{abcd} \cdot \frac{z}{l}$ (189). simili modo mutatio assignari potest, quam magnitudo vltimae imaginis ob diuersam refrangibilitatem radiorum patietur.

Coroll. 3.

299. Cognita autem vtraque mutatione, quam vltima imago tam respectu loci, quam magnitudinis subit,

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fubit, non difficulter colligetur, quanta confuſione ipſa viſio ob diuerſam radiorum refrangibilitatem perturbetur.

Coroll. 4.

300. Si craſſities lentium euaneſcat fiet

$$da = -db = \frac{-\alpha\alpha\, dn}{n-1}\left(\frac{1}{a}+\frac{1}{\alpha}\right)$$

$$d\beta = -dc = -\frac{\beta\beta\, db}{bb} - \frac{\beta\beta\, dn}{n-1}\left(\frac{1}{b}+\frac{1}{\beta}\right)$$

$$d\gamma = -dd = \frac{-\gamma\gamma\, dc}{cc} - \frac{\gamma\gamma\, dn}{n-1}\left(\frac{1}{c}+\frac{1}{\gamma}\right)$$

$$d\delta = -de = \frac{-\delta\delta\, dd}{dd} - \frac{\delta\delta\, dn}{n-1}\left(\frac{1}{d}+\frac{1}{\delta}\right)$$

etc.

numeri autem i, i', i'' etc. abeunt in vnitatem nullique mutationi amplius ſunt obnoxii.

Si ergo craſſities lentium euaneſcat, pro diuerſa refractione ſingularum lentium formulae ſuperiores abibunt in ſequentes:

I. $da = -db = \frac{-\alpha\alpha\, dn}{n-1}\left(\frac{1}{a}+\frac{1}{\alpha}\right)$

II. $d\beta = -dc = -\frac{\beta\beta\, db}{bb} - \frac{\beta\beta\, dn'}{n'-1}\left(\frac{1}{b}+\frac{1}{\beta}\right)$

III. $d\gamma = -dd = \frac{-\gamma\gamma\, dc}{cc} - \frac{\gamma\gamma\, dn''}{n''-1}\left(\frac{1}{c}+\frac{1}{\gamma}\right)$

etc.

Ceterum per ſe manifeſtum eſt, quando §. 288 dn vel $+\frac{1}{100}$ vel $-\frac{1}{100}$ ſignificare dicitur, id tantum de illa vitri ſpecie, pro qua eſt refractio radiorum me-

mediorum $n = \frac{31}{20}$ esse intelligendum; et pro aliis vitri speciebus differentialia dn', dn'', dn''', etc. haud mediocriter ab $\frac{1}{100}$ discrepare posse. Quanta autem futura sit haec diuersitas, optandum esset, vt ea potius experimentis, quam ex theoria quapiam definiretur.

Scholion

301. Cum igitur ob diuersam radiorum refrangibilitatem cuique imagini duplex alteratio inducatur, quarum altera eius magnitudinem, altera vero eius locum afficit, duplex inde confusio in visionem infertur. Si enim formulae in superioribus capitibus exhibitae ad radios mediae naturae restringantur, pro quibus est $n = \frac{31}{20}$, posito $dn = -\frac{1}{100}$, ex formulis hic traditis differentialibus locus et magnitudo imaginis a radiis rubris formatae definietur, posito autem $dn = +\frac{1}{100}$, locus et magnitudo imaginis violaceae declarabitur. Scilicet si $I\iota$ fuerit imago vltima visioni obiecta, quae a radiis mediae naturae formatur, per formulas modo inuentas, prout vel $dn = -\frac{1}{100}$ vel $dn = +\frac{1}{100}$ ponatur, definietur tam imago rubra $R\varrho$ quam violacea $V v$; atque ex natura differentialium manifestum est cum interualla IR et IV inter se aequalia esse debere, tum etiam differentias $I\iota - R\varrho$ et $V v - Ii$, ita vt oculo series innumerabilium imaginum inter extremas $R\varrho$ et $V v$ sitarum simul cernenda offeratur, vnde eo maior confusio oriatur necesse est, quo maior fuerit differentia tam ratione loci quam magnitudinis. Quare haec confusio penitus tolleretur, si eiusmodi lentium dispositio definiri posset, vt tam interuallum RV, quam

Tab. III. Fig. 16.

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quam differentia inter imagines Rϱ et Vv ad nihilum redigeretur, quod vtrumque niſi ſimul praeſtari queat, confuſionem perfecte tollere non licet Verumtamen etiamſi neutri harum conditionum ſatisfieri poſſit, tamen dabitur pro oculo eiusmodi locus O, vbi confuſio minime ſenſibilis percipiatur, qui erit in concurſu rectae vϱ productae cum axe: ibi enim omnes extremitates ϱv communibus radiis cernentur, neque propterea extremitas obiecti colore tincta apparebit. Quare ſi ſimul punctum O conueniat cum loco oculi idoneo alia confuſio non percipietur, niſi quae inde originem trahit, quod forte imagines extremae Rϱ et Vv nimis a diſtantia iuſta diſcrepent, ſiquidem media I ad diſtantiam iuſtam ob oculo fuerit remota. Neque tamen hinc ora obiecti coloribus iridis cincta apparebit, cui confuſionis ſpeciei maxime eſt occurrendum; ideoque ea quae adhuc adfuerit confuſio facile tolerari poterit, quae vero etiam omnino tolleretur ſi modo interuallum RV vel in nihilum redigi vel ſaltem ſatis paruum reddi poſſet. Hinc ergo intelligimus vitium illud, quo obiecta coloribus iridis circumdata ſaepe repraeſentantur, non tam neceſſario cum inſtrumentis dioptricis eſſe coniunctum, vt nullo pacto ab iis ſeparari queat, quamobrem eo magis operae erit pretium, vt inveſtigemus quomodo haec inſtrumenta ab iſto vitio liberari poſſint. Quae tota inveſtigatio huc redit, vt determinetur punctum O, vbi recta per terminos imaginum v, ι, ϱ ducta cum axe concurrit hocque punctum cum loco oculi iam

ſupra definito conueniens reddatur, ſi quidem fieri poteſt: vnde perſpicitur locum oculi O hac proprietate praeditum eſſe oportere, vt angulus, ſub quo vltima imago cernitur ob variabilitatem numero n tributam nullam mutationem patiatur. Tum vero inſuper videndum erit, num interualla IR et IV vel ad nihilum reduci, vel minima reddi queant.

Problema 4.

Tab. III. Fig. 12. **302. Propoſita vnica lente definire locum oculi, vnde obiectum ſine margine colorato cernatur.**

Solutio.

Sit obiecti Ee ante lentem diſtantia $EA=a$, imago vero per radios mediae naturae in $F\zeta$ repraeſentetur, ponaturque $aF=\alpha$. Pro lente vero ſit eius craſſities $Aa=v$, et quantitas arbitraria $=k$, vnde capiatur $\frac{k-v}{k+v}=i$. Hinc poſito $Ee=z$ erit $F\zeta=\frac{1}{i}\cdot\frac{\alpha}{a}z$ (85): quare ſi pro loco oculi ſtatuatur diſtantia $aO=O$, quae eſt fixa, erit $OF=\alpha-O$, et anguli $FO\zeta$ tangens $=\frac{1}{i}\frac{\alpha}{a}\frac{z}{\alpha-O}$, quae formula ob diuerſam radiorum refrangibilitatem nullam mutationem ſubire debet. Inde autem quantitates α et i tantum variantur, dum reliquae manent conſtantes. Quare iſtius formulae differentiale logarithmicum nihilo aequale poſitum praebet hanc aequationem

$$-\frac{di}{i}+\frac{d\alpha}{\alpha}-\frac{d\alpha}{\alpha-O}=0 \text{ ſeu } \frac{-di}{i}-\frac{O\,d\alpha}{\alpha(\alpha-O)}=0$$

vbi

vbi si valores supra inuenti substituantur, prodit

$$\frac{\alpha\,dn}{in(n-1)}\left(\frac{1}{\alpha}+\frac{1-i}{v}\right)+\frac{0\,\alpha\,dn}{i(n-1)(\alpha-0)}\left(\frac{i}{\alpha}+\frac{1}{i\alpha}+\frac{(1-i)^2}{iv}\right)=0$$

quae aequatio per $\frac{dn}{i(n-1)}$ diuisa praebet:

$$\frac{v}{n\alpha}+\frac{1-i}{n}+\frac{0\,\alpha}{\alpha-0}\left(\frac{i}{\alpha}+\frac{1}{i\alpha}+\frac{(1-i)^2}{iv}\right)=0$$

vnde locus oculi definiri poterit: qui si debeat conuenire cum supra inuento (238), vbi inuenimus $0=\frac{-i\alpha v}{n\alpha-iv}$, erit

$\alpha-0=\frac{n\alpha\alpha}{n\alpha-iv}$ et $\frac{0\,\alpha}{\alpha-0}=\frac{-iv}{n}$, hincque nostra aequatio per n multiplicata abit in:

$$\frac{v}{\alpha}+1-i-\frac{iiv}{\alpha}-\frac{v}{\alpha}-(1-i)^2=0$$

seu $i-ii-\frac{iiv}{\alpha}=0$, ideoque $i=\frac{\alpha}{\alpha+v}=\frac{k-v}{k+v}$. Quamobrem quantitatem arbitrariam k ita definiri conueniet vt fit $k=2\alpha+v$: et cum sit $i=\frac{\alpha}{\alpha+v}$ pro loco oculi habebimus $0=\frac{-\alpha v}{n\alpha+(n-1)v}=\frac{-10\,\alpha v}{31\alpha+11v}$ ob $n=\frac{31}{10}$.

Quod si porro hinc variationem in loco imaginis desideremus definiri oportet differentiale $d\alpha$, quod fiet:

$$d\alpha=\frac{-\alpha\alpha\,dn}{i(n-1)}\left(\frac{i}{\alpha}+\frac{1}{i\alpha}+\frac{(1-i)^2}{iv}\right)$$

et pro i posito valore $\frac{\alpha}{\alpha+v}$

$$d\alpha=\frac{-(\alpha+v)dn}{n-1}\left(1+\frac{\alpha+v}{\alpha}\right)$$

qui valor si ad nihilum redigi posset, confusio omnis a diuersa radiorum refrangibilitate oriunda perfecte tolleretur.

Coroll. 1.

303. Pro lentis ergo constructione quantitas arbitraria k ita accipi debet, vt sit $k = 2\alpha + v$; atque tum oculus in eo loco constitutus, vbi totum campum apparentem percipiat, simul nullam confusionem a diuersa radiorum indole sentiet.

Coroll. 2.

304. Vt autem oculus simul imaginem in distantia iusta aspiciat, oportet sit $\alpha - O = -l$, ideoque $l = \frac{-11\alpha(\alpha+v)}{31\alpha+11v}$. Vnde colligitur: $\alpha = \frac{-l}{2} - \frac{v}{2} - \sqrt{(\frac{1}{4}ll + \frac{9}{62}vl + \frac{1}{4}vv)}$, hincque $O = \frac{1}{2}l - \frac{1}{2}v - \sqrt{(\frac{1}{4}ll + \frac{9}{62}vl + \frac{1}{4}vv)}$ et $k = -l - 2\sqrt{(\frac{1}{4}ll + \frac{9}{62}vl + \frac{1}{4}vv)}$

Coroll. 3.

305. Potest vero insuper effici, vt etiam dq euanescat, quod euenit si $q + \alpha + v = 0$, hoc est $q = \frac{1}{2}l - \frac{1}{2}v + \sqrt{(\frac{1}{4}ll + \frac{9}{62}vl + \frac{1}{4}vv)}$. Verum cum hoc casu ob $\alpha = -q - v$ imago in ipsum obiectum cadet, ita, vt radii nullam refractionem pati sint censendi, visio per lentem perinde erit comparata atque nudis oculis.

Coroll. 4.

306. Si crassities lentis v plane euanescat, tam ob campum apparentem, quam diuersam radiorum refrangibilitatem fit $O = 0$ hoc ergo casu oculus lenti immediate applicatus nullam confusionem ob diuersam radiorum naturam percipiet. Dum ergo fuerit $\alpha = -l$ visio erit distincta. Scho-

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Scholion.

307. Hic ſcilicet penitus mentem abſtrahimus a confuſione iam ſupra determinata, quae a lentium apertura oritur, ideoque aperturam primae lentis vt euaneſcentem ſpectamus. Eam igitur hic tantum confuſionis ſpeciem contemplamur, quae a diuerſa radiorum refrangibilitate originem ducit; quam plerumque tolli obſeruauimus, ſi angulus ad O inuariabilis reddatur; tum enim ora obiecti ſatis bene terminata conſpicietur, neque coloribus iridis cincta. Interim tamen adhuc aliqua confuſio ſentiri poterit inde oriunda, quod ſi imago media iuſtam ab oculo diſtantiam teneat, imaginum extremarum altera ſit nimis propinqua alter animis remota, verum ſi earum interuallum non ſit admodum magnum, confuſio haec parum erit ſenſibilis. Ita hic inuenimus, quod experientia ſatis comprobatur, ſi obiecta per vnicam lentem ſpectemus, ea margine colorato deſtituta apparere dummodo oculus immediate applicetur quod ſi quando ſecus euenire videatur, cauſa aperturae lentis fine dubio erit tribuenda, cui conditioni rationes hic allegatae refragantur.

Problema 5.

308. Si inſtrumentum dioptricum duabus inſtructum ſit lentibus, definire locum oculi, vnde obiectum ſine margine colorato videatur. Tab. III. Fig. 3.

Solutio

Solutio.

Posita obiecti distantia $AE = a$, sint pro radiis mediae naturae reliquae distantiae determinatrices $aF = \alpha$, $Bl = b$ et $bG = \mathfrak{G}$ crassities vero lentium $Aa = v$, $Bb = v'$, et distantiae arbitrariae k et k', ponaturque $\frac{k-v}{k+v} = i$ et $\frac{k'-v'}{k'+v'} = i'$. His positis si magnitudo obiecti Ee vocetur $= z$, erit imago $G\eta = \frac{1}{ii'}\frac{\alpha\mathfrak{G}}{ab}z$, vnde si oculi distantia ponatur $bO = O$,

ob $GO = \mathfrak{G} - O$, erit anguli $GO\eta$ tangens $\frac{1}{ii'}\cdot\frac{\alpha\mathfrak{G}}{ab}\cdot\frac{z}{\mathfrak{G}-O}$ cuius differentiale logarithmicum nihilo aequatum praebet,

$$-\frac{di}{i} - \frac{di'}{i'} + \frac{d\alpha}{\alpha} - \frac{db}{b} + \frac{d\mathfrak{G}}{\mathfrak{G}} - \frac{d\mathfrak{G}}{\mathfrak{G}-O} = 0$$

quae valoribus supra (296) inuentis substitutis abit in:

$$\frac{v\,dn}{in(1-l)}\left(\frac{1}{a} + \frac{1-i}{v}\right) + \frac{v'\,db}{n'i'bb} + \frac{v'\,dn'}{i'n'(n'-1)}\left(\frac{1}{b} + \frac{1-i'}{v'}\right)$$
$$-\frac{db}{a} - \frac{db}{b} + \frac{O}{\mathfrak{G}-O}\left(\frac{\mathfrak{G}\,db}{i'i'bb} + \frac{\mathfrak{G}\,dn'}{i'(n'-1)}\left(\frac{i'}{\mathfrak{G}} + \frac{1}{i'b} + \frac{(1-i')^2}{i'v'}\right)\right) = 0$$

Verum conditio campi exigebat $O = \frac{\mathfrak{G}\mathfrak{G}}{\mathfrak{G} + \frac{1}{ii'}\cdot\frac{\alpha\mathfrak{G}}{ab}z}$, existente

$\mathfrak{G} = \left(\frac{i'}{i}\cdot\frac{\alpha+b}{a} - \frac{i'bv}{n'a\alpha} - \frac{1}{i}\cdot\frac{\alpha v'}{n'ab}\right)z$, (245), vnde fit

$$\frac{O}{\mathfrak{G}-O} = \frac{\mathfrak{G}\mathfrak{G}}{\frac{1}{ii'}\cdot\frac{\alpha\mathfrak{G}\mathfrak{G}}{ab}z} = \frac{ii'ab}{\alpha\mathfrak{G}}\left(\frac{i'}{i}\cdot\frac{\alpha+b}{a} - \frac{i'bv}{n\alpha\alpha} - \frac{1}{i}\cdot\frac{\alpha v'}{n'ab}\right)$$

Nunc

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Nunc vero est $db = \frac{\alpha\alpha\, dn}{\beta(n-1)}\left(\frac{i}{\alpha} + \frac{i'}{\beta\alpha} + \frac{(i-i')^2}{\varepsilon v}\right)$, quem valorem antequam substituamus, transformemus aequationem nostram in hanc formam

$$\frac{\varepsilon n}{n-1}\left(\frac{v}{i\alpha\alpha} + \frac{v-v'}{i'\alpha\beta} + \frac{v'}{i'\beta\beta} + \frac{v-v'}{i'\alpha\varepsilon} + \frac{\varepsilon 0}{i'(\varepsilon-0)}\left(\frac{i'}{\varepsilon} + \frac{\alpha}{i'\beta} + \frac{(i-i')^2}{i'v}\right)\right.$$

$$\left. + db\left(\frac{v'}{\alpha i'\beta\beta} - \frac{i}{\alpha\alpha} - \frac{i'}{\beta\beta} + \frac{\varepsilon 0}{i'i'\beta\beta(\varepsilon-0)}\right)\right) = 0$$

vbi posterius membrum abit in $-\frac{iv}{\alpha\alpha\alpha}\,db$, tum vero erit

$$0 = \frac{dn}{n-1}\left\{\begin{array}{l} 1 + \frac{b}{\alpha} + \frac{i'b}{\varepsilon}\left(1 + \frac{b}{\alpha}\right) + \frac{(1-i')^2 b(\alpha\varepsilon - b)}{\alpha v} + \frac{\alpha - i - i'}{\alpha} \\ \frac{i-iv}{\alpha\alpha} - \frac{i'v'}{\alpha\varepsilon} - \frac{i'bv}{\alpha\alpha\varepsilon} - \frac{ii'i'bbv}{\alpha\alpha\varepsilon\varepsilon} - \frac{i(1-i')^2 bbv}{\alpha\alpha\alpha v} \end{array}\right\}$$

distinguendo n' ab n erit

$$\frac{dn}{n-1}\left(\frac{iv}{\alpha\alpha} - \frac{(1-i)}{\alpha}\right) = \frac{dn'}{n'-1}\left\{\begin{array}{l} 1 + \frac{b}{\alpha} + \frac{i'b}{\varepsilon}\left(1 + \frac{b}{\alpha}\right) - \frac{ii'i'bbv}{\alpha\alpha\alpha\varepsilon} - \frac{i'v'}{\alpha v} + \frac{1-i'}{\alpha'} \\ -\frac{ibv}{\alpha\alpha\alpha} + \frac{b(1-i')^2}{v}\left(1 + \frac{b}{\alpha}\right) - \frac{i(1-i')^2 bbv}{\alpha\alpha\alpha v} \end{array}\right\}$$

cuius aequationis complicatio obstat, quominus quicquam commode inde concludi possit.

Coroll. 1.

309. Si ambae lentes crassitie careant, vt sit $v = 0$, $v' = 0$, et $i = i' = 1$, aequatio differentialis prima est

$$\frac{d\alpha}{\alpha} - \frac{db}{b} - \frac{0\, d\varepsilon}{\varepsilon(\varepsilon-0)} = 0$$

tum vero:

$$d\alpha = -db = \frac{-\alpha\alpha\, dn}{n-1}\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) \text{ et}$$

$$d\varepsilon = \frac{-\varepsilon\varepsilon db}{bb} - \frac{\varepsilon\varepsilon dn}{n-1}\left(\frac{1}{\varepsilon} + \frac{1}{b}\right) = \frac{-dn}{n-1}\left(\frac{\alpha\alpha\varepsilon\varepsilon}{bb}\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + \varepsilon\varepsilon\left(\frac{1}{\varepsilon} + \frac{1}{b}\right)\right.$$

qui-

quibus valoribus substitutis et per $\frac{dn}{n-1}$ diuisione facta fit

$$-\alpha\alpha\left(\frac{1}{\alpha}+\frac{1}{a}\right)\left(\frac{1}{\alpha}+\frac{1}{b}\right)+\frac{O\,\varepsilon}{\varepsilon-O}\left(\frac{\alpha\alpha}{bb}\left(\frac{1}{\alpha}+\frac{1}{a}\right)+\frac{1}{\varepsilon}+\frac{1}{b}\right)=0$$

Vnde si oculus lenti posteriori immediate applicaretur vt esset

$O=o$, deberet esse $\left(\frac{1}{\alpha}+\frac{1}{a}\right)\left(\frac{1}{\alpha}+\frac{1}{b}\right)=o$.

Coroll. 2.

310. Verum in eadem hypothesi, vt locus oculi congruat cum eo, quem visio campi exigit, debet esse $O=\frac{b\,\varepsilon(\alpha+b)}{b(\alpha+b)+\alpha\varepsilon}$, vnde fit $\varepsilon-O=\frac{\alpha\,\varepsilon\,\varepsilon}{b(\alpha+b)+\alpha\varepsilon}$, ideoque $\frac{O}{\varepsilon-O}=\frac{b(\alpha+b)}{\alpha\,\beta}=\frac{bb}{\varepsilon}\left(\frac{1}{\alpha}+\frac{1}{b}\right)$

quo valore substituto nostra aequatio erit

$$o=\left(\frac{1}{\alpha}+\frac{1}{b}\right)\left(-\alpha\alpha\left(\frac{1}{\alpha}+\frac{1}{a}\right)+bb\left(\frac{\alpha\alpha}{bb}\left(\frac{1}{\alpha}+\frac{1}{a}\right)+\frac{1}{\varepsilon}+\frac{1}{b}\right)\right)$$

quae reducitur ad hanc formam

$$o=bb\left(\frac{1}{\alpha}+\frac{1}{b}\right)\left(\frac{1}{\varepsilon}+\frac{1}{b}\right)$$

distinguendo n' ab n, membra a dn pendentia se destruunt et oritur

$$o=\frac{d\,n'}{n'-1}bb\left(\frac{1}{\alpha}+\frac{1}{b}\right)\left(\frac{1}{\varepsilon}+\frac{1}{b}\right)$$

quod ita ostenditur:

Cum aequatio prima differentialis praebeat:

$$\frac{d\alpha}{\alpha}-\frac{db}{b}-\frac{O\,d\varepsilon}{\varepsilon(\varepsilon-O)}=o \text{ siue}$$

$$d\alpha\left(\frac{1}{\alpha}+\frac{1}{b}\right)-\frac{O\,d\varepsilon}{\varepsilon(\varepsilon-O)}=o$$

Cum

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Cum igitur ex conditione campi apparentis sit

$\frac{0}{\mathfrak{E}-0} = \frac{b\,b'}{\mathfrak{E}}\left(\frac{1}{a}+\frac{1}{b}\right)$ erit

$$d\,\alpha\left(\tfrac{1}{a}+\tfrac{1}{b}\right) - \tfrac{b\,b}{\mathfrak{E}^2}\left(\tfrac{1}{a}+\tfrac{1}{b}\right)d\mathfrak{E} = 0$$

ideoque loco $d\mathfrak{E}$ suum valorem sustituendo

$$d\alpha\left(\tfrac{1}{a}+\tfrac{1}{b}\right) - \tfrac{b\,b}{\mathfrak{E}^2}\left(\tfrac{1}{a}+\tfrac{1}{b}\right)\left(\tfrac{\mathfrak{E}\mathfrak{E}\,d\,\alpha}{b\,b} - \tfrac{\mathfrak{E}^2\,d\,n'}{n'-1}\left(\tfrac{1}{b}+\tfrac{1}{\mathfrak{E}}\right)\right) = 0.$$

vbi membra, quae $d\alpha$ continent, manifesto se destruunt, et tota quaestio ad hanc aequationem perducitur

$$\tfrac{d\,n'}{n'-1}\cdot b\,b\left(\tfrac{1}{a}+\tfrac{1}{b}\right)\left(\tfrac{1}{\mathfrak{E}}+\tfrac{1}{b}\right) = 0.$$

Coroll. 3.

311. Quod si ergo huic conditioni satisfieri possit, obiectum sine margine colorato apparebit: praeterea vero confusio penitus tolleretur, si reddi liceret $d\mathfrak{E} = 0$, quod fit per hanc aequationem

$$\tfrac{a\,a}{b\,b}\left(\tfrac{1}{a}+\tfrac{1}{a}\right)+\tfrac{1}{\mathfrak{E}}+\tfrac{1}{b} = 0 \text{ siue}$$

$$+\tfrac{d\,n}{n-1}\cdot\tfrac{a\,a}{b\,b}\left(\tfrac{1}{a}+\tfrac{1}{a}\right)$$

$$+\tfrac{d\,n'}{n'-1}\left(\tfrac{1}{\mathfrak{E}}+\tfrac{1}{b}\right) = 0$$

$$a\,a\left(\tfrac{1}{a}+\tfrac{1}{a}\right)+b\,b\left(\tfrac{1}{\mathfrak{E}}+\tfrac{1}{b}\right) = 0.$$

Coroll. 4.

312. Priori autem aequationi satisfieri nequit nisi fuerit vel $\alpha+b=0$ vel $\frac{1}{\mathfrak{E}}+\frac{1}{b}=0$. Illo casu ambae lentes coniungerentur, vt vnicam constituerent, hoc vero posterioris distantia focalis fieret infinita, qui

casus iterum ad casum vnicae lentis rediret; foret enim $O = -a - b$, ob $\mathfrak{C} = -b$, et oculus priori lenti immediate applicari deberet.

Scholion.

313. Si simili modo has inuestigationes ad plures lentes extendere vellemus, non neglecta earum crassitie in formulas plane inextricabiles delaberemur vnde vix quicquam concludi posset. Verum quia in omnibus fere instrumentis dioptricis, praecipue quae pluribus lentibus constant, iis tam exigua crassities tribui solet, vt sine notabili errore pro nihilo haberi possit, tam taediosae indagationi facile supersedere poterimus. Ad quod accedit, quod hic non de summo rigore geometrico agatur, sed contenti esse queamus, dummodo hanc confusionem satis prope cognouerimus: ex quo sufficiet in consideratione plurium lentium earum crassitiem prorsus neglexisse.

Supplementum IV.

Si ratio refractionis in singulis lentibus sit diuersa solutio sequenti modo absoluetur:

I. Prima aequatio differentialis prorsus se habebit, vt in problemate, ita, vt sit

$$d\alpha\left(\frac{1}{a}+\frac{1}{b}\right)+d\mathfrak{C}\left(\frac{1}{\mathfrak{c}}+\frac{1}{c}\right)-\frac{O\,d\gamma}{\gamma(\gamma-O)}=0$$

et quia etiam, vt ante, est

$$\frac{O}{\gamma-O}=\frac{bbcc}{\alpha\mathfrak{C}\gamma}\left(1+\frac{\alpha}{b}\right)+\frac{cc}{\mathfrak{C}\gamma}\left(1+\frac{\mathfrak{C}}{c}\right)$$

erit

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erit noſtra aequatio

$$(\tfrac{1}{\alpha}+\tfrac{1}{\beta})(d\alpha-\tfrac{bbcc}{\mathfrak{C}^2\gamma^2}d\gamma)+(\tfrac{1}{\mathfrak{C}}+\tfrac{1}{c})(d\mathfrak{C}-\tfrac{cc}{\gamma\gamma}d\gamma)=0.$$

II. Nunc autem ratio diuerſae refractionis eſt habenda; vnde in ſuperioribus additamentis inuenimus eſſe

$$d\alpha=-\tfrac{\alpha\alpha dn}{n-1}(\tfrac{1}{\alpha}+\tfrac{1}{\alpha})$$
$$d\mathfrak{C}=\tfrac{\mathfrak{C}\mathfrak{C}d\alpha}{bb}-\tfrac{\mathfrak{C}\mathfrak{C}dn'}{n'-1}(\tfrac{1}{b}+\tfrac{1}{\mathfrak{C}})$$
$$d\gamma=\tfrac{\gamma\gamma d\mathfrak{C}}{cc}-\tfrac{\gamma\gamma dn''}{n''-1}(\tfrac{1}{c}+\tfrac{1}{\gamma})$$

Hincque ergo fiet

$$d\alpha-\tfrac{bbcc}{\mathfrak{C}^2\gamma^2}d\gamma=d\alpha-\tfrac{bbd\mathfrak{C}}{\mathfrak{C}^2}+\tfrac{bbccdn''}{\mathfrak{C}^2(n''-1)}(\tfrac{1}{c}+\tfrac{1}{\gamma})$$
$$=\tfrac{bb.dn'}{n'-1}(\tfrac{1}{b}+\tfrac{1}{\mathfrak{C}})+\tfrac{bbccdn''}{\mathfrak{C}^2(n''-1)}(\tfrac{1}{c}+\tfrac{1}{\gamma})$$

Deinde

$$d\mathfrak{C}-\tfrac{cc}{\gamma\gamma}d\gamma=\tfrac{ccdn''}{n''-1}(\tfrac{1}{c}+\tfrac{1}{\gamma})$$

III. Ex his ergo noſtra aequatio differentialis abibit in hanc formam:

$$(\tfrac{1}{\alpha}+\tfrac{1}{\beta})(\tfrac{bbdn'}{n'-1}[\tfrac{1}{b}+\tfrac{1}{\mathfrak{C}}]+\tfrac{bbcc dn''}{\mathfrak{C}^2(n''-1)}[\tfrac{1}{\mathfrak{C}}+\tfrac{1}{\gamma}])$$
$$+(\tfrac{1}{\mathfrak{C}}+\tfrac{1}{c})(\tfrac{ccdn''}{n''-1}[\tfrac{1}{c}+\tfrac{1}{\gamma}])$$

ſiue

$$\tfrac{dn'}{n'-1}.bb.(\tfrac{1}{\alpha}+\tfrac{1}{\beta})(\tfrac{1}{\mathfrak{C}}+\tfrac{1}{b})$$
$$+\tfrac{dn''}{n''-1}.cc(\tfrac{1}{c}+\tfrac{1}{\gamma})(\tfrac{bb}{\mathfrak{C}\mathfrak{C}}(\tfrac{1}{\alpha}+\tfrac{1}{\beta})+\tfrac{1}{\mathfrak{C}}+\tfrac{1}{c})=0$$

IV. Pro caſu autem illo ſingulari, quo oculus lenti vltimae immediate, debet adplicari, ob $O=o$, habebitur ſimpliciter haec aequatio

$$d\alpha(\tfrac{1}{\alpha}+\tfrac{1}{\beta})+d\beta(\tfrac{1}{\mathfrak{C}}+\tfrac{1}{c})=0$$

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ſiue, ſubſtituto valore $d\mathfrak{E}$

$$d\alpha\left(\tfrac{1}{\alpha}+\tfrac{1}{b}\right)+\frac{\mathfrak{E}\mathfrak{E}\,d\alpha}{bb}\left(\tfrac{1}{\mathfrak{E}}+\tfrac{1}{c}\right)$$
$$-\frac{\mathfrak{E}\mathfrak{E}\,dn'}{n'-1}\left(\tfrac{1}{b}+\tfrac{1}{\mathfrak{E}}\right)=0.$$
$$d\alpha\left(\tfrac{1}{\alpha}+\tfrac{1}{b}+\frac{\mathfrak{E}\mathfrak{E}}{bb}\left[\tfrac{1}{\mathfrak{E}}+\tfrac{1}{c}\right]\right)$$
$$-\frac{\mathfrak{E}^2\,dn'}{n'-1}\left(\tfrac{1}{b}+\tfrac{1}{\mathfrak{E}}\right)=0,$$

ſeu tandem

$$+\frac{dn}{n-1}\alpha\alpha\left(\tfrac{1}{b}+\tfrac{1}{\alpha}\right)\left(\tfrac{1}{\alpha}+\tfrac{1}{b}+\frac{\mathfrak{E}^2}{b^2}\left[\tfrac{1}{\mathfrak{E}}+\tfrac{1}{c}\right]\right)$$
$$+\frac{dn'}{n'-1}\mathfrak{E}\mathfrak{E}\left(\tfrac{1}{b}+\tfrac{1}{\mathfrak{E}}\right)=0.$$

V. Hoc modo tantum margo coloratus tollitur; vt autem tota confuſio tollatur, quod fit ſi $d\gamma=0$, inſuper ſatisfieri debet huic aequationi

$$0=\frac{dn}{n-1}\cdot\frac{\alpha\alpha\mathfrak{E}\mathfrak{E}\gamma\gamma}{bbcc}\left(\tfrac{1}{\alpha}+\tfrac{1}{\alpha}\right)$$
$$+\frac{dn'}{n'-1}\cdot\frac{\mathfrak{E}\mathfrak{E}\gamma\gamma}{ff}\left(\tfrac{1}{b}+\tfrac{1}{\mathfrak{E}}\right)$$
$$+\frac{dn''}{n''-1}\cdot\gamma\gamma\cdot\left(\tfrac{1}{c}+\tfrac{1}{\gamma}\right).$$

Problema 6.

Tab. III. Fig. 14.

314. Si inſtrumentum dioptricum tribus conſtet lentibus, quarum craſſities euaneſcat, eam definire diſpoſitionem, vt oculus in eo loco, quem campus poſtulat, conſtitutus obiectum ſine margine colorato conſpiciat.

Solutio.

Poſita ergo diſtantia obiecti ante lentem obiectiuam $AE=a$, eiusque magnitudine $E\varepsilon=z$, vocentur

centur distantiae imaginum a radiis mediae naturae formatarum, vt hactenus

$aF = \alpha$, $BF = b$; $bG = \mathfrak{C}$, $CG = c$, et $cH = \gamma$.

eritque imago vltima $H\theta = \frac{\alpha\mathfrak{C}\gamma}{abc}z$, et posita oculi post lentem vltimam distantia $cO = O$, erit $OH = \gamma - O$ et anguli $HO\theta$ tangens $= \frac{\alpha\mathfrak{C}\gamma}{abc}\cdot\frac{z}{\gamma - O}$, quae debet esse invariabilis. Posito ergo eius differentiali logarithmico $= 0$ habebimus hanc aequationem:

$$\frac{d\alpha}{\alpha} - \frac{db}{b} + \frac{d\mathfrak{C}}{\mathfrak{C}} - \frac{dc}{c} + \frac{d\gamma}{\gamma} - \frac{d\gamma}{\gamma - O} = 0$$

At et formulis supra (300) erutis habemus

$$d\alpha = -db = \frac{-dn}{n-1}\cdot\alpha\alpha\left(\tfrac{1}{a} + \tfrac{1}{\alpha}\right)$$

$$d\mathfrak{C} = -dc = -\frac{\mathfrak{C}\mathfrak{C}dn}{n-1}\left(\frac{\alpha\alpha}{bb}\left(\tfrac{1}{a} + \tfrac{1}{\alpha}\right) + \tfrac{1}{b} + \tfrac{1}{\mathfrak{C}}\right)$$

$$d\gamma = \frac{\gamma\gamma dn}{n-1}\left(\frac{\alpha\alpha\mathfrak{C}\mathfrak{C}}{bbcc}\left(\tfrac{1}{a} + \tfrac{1}{\alpha}\right) + \frac{\mathfrak{C}\mathfrak{C}}{cc}\left(\tfrac{1}{b} + \tfrac{1}{\mathfrak{C}}\right) + \tfrac{1}{c} + \tfrac{1}{\gamma}\right);$$

vnde nostra aequatio erit

$$d\alpha\left(\tfrac{1}{\alpha} + \tfrac{1}{b}\right) + d\mathfrak{C}\left(\tfrac{1}{\mathfrak{C}} + \tfrac{1}{c}\right) - \frac{O\,d\gamma}{\gamma(\gamma - O)} = 0$$

Sed ob campum apparentem supra 254 inuenimus:

$$\mathfrak{B} = \mathfrak{C} = b\left(1 + \tfrac{\alpha}{b}\right)\tfrac{z}{a}$$

$$\mathfrak{C} = c = \tfrac{bc}{\mathfrak{C}}\left(1 + \tfrac{\alpha}{b}\right)\tfrac{\mathfrak{C}}{a} + \tfrac{\alpha c}{b}\left(1 + \tfrac{\mathfrak{C}}{c}\right)\tfrac{z}{a}$$

hincque

$O = \frac{\gamma c}{c + \alpha\theta}$ et $\frac{\Phi}{\gamma - O} = \frac{c}{H\theta}$ vnde fit ob

$H\theta = \frac{\alpha\mathfrak{C}\gamma}{bc}\cdot\frac{z}{a}$

$$\frac{O}{\gamma - O} = \frac{bbcc}{\alpha\mathfrak{C}\mathfrak{C}\gamma}\left(1 + \frac{\alpha}{b}\right) + \frac{cc}{\mathfrak{C}\gamma}\left(1 + \frac{\mathfrak{C}}{c}\right) \text{ seu}$$

$$\frac{O}{\gamma(\gamma - O)} = \frac{bbcc}{\mathfrak{C}\mathfrak{C}\gamma\gamma}\left(\tfrac{1}{\alpha} + \tfrac{1}{b}\right) + \frac{cc}{\gamma\gamma}\left(\tfrac{1}{\mathfrak{C}} + \tfrac{1}{c}\right)$$

Valo-

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Valoribus iam his substitutis habebimus:

$$\left(\tfrac{1}{a}+\tfrac{1}{b}\right)\left(d\alpha-\tfrac{bbcc}{\varepsilon\varepsilon\gamma\gamma}d\gamma\right)+\left(\tfrac{1}{\varepsilon}+\tfrac{1}{c}\right)\left(d\mathfrak{C}-\tfrac{cc}{\gamma\gamma}d\gamma\right)=0$$

At est

$$d\alpha-\tfrac{bbcc}{\varepsilon\varepsilon\gamma\gamma}d\gamma=\tfrac{dn}{n-1}\left(bb\left(\tfrac{1}{b}+\tfrac{1}{\varepsilon}\right)+\tfrac{bbcc}{\varepsilon\varepsilon}\left(\tfrac{1}{c}+\tfrac{1}{\gamma}\right)\right)$$

$$d\mathfrak{C}-\tfrac{cc}{\gamma\gamma}d\gamma=\tfrac{dn}{n-1}\left(cc\left(\tfrac{1}{c}+\tfrac{1}{\gamma}\right)\right)$$

Quare facta diuisione per $\frac{dn}{n-1}$ nanciscemur hanc aequationem:

$$bb\left(\tfrac{1}{a}+\tfrac{1}{b}\right)\left(\tfrac{1}{b}+\tfrac{1}{\varepsilon}+\tfrac{cc}{\varepsilon\varepsilon}\left(\tfrac{1}{c}+\tfrac{1}{\gamma}\right)\right)+cc\left(\tfrac{1}{\varepsilon}+\tfrac{1}{c}\right)\left(\tfrac{1}{c}+\tfrac{1}{\gamma}\right)=0$$

Quodsi vero oculus lenti postremae immediate applicetur, seu sit $O=o$, conditio praescripta hanc postulat aequationem:

$$aa\left(\tfrac{1}{a}+\tfrac{1}{b}\right)\left(\tfrac{1}{b}+\tfrac{1}{\alpha}\right)+\mathfrak{C}\mathfrak{C}\left(\tfrac{1}{\varepsilon}+\tfrac{1}{c}\right)\left(\tfrac{aa}{bb}\left(\tfrac{1}{a}+\tfrac{1}{\alpha}\right)+\tfrac{1}{b}+\tfrac{1}{\varepsilon}\right)=0$$

Confusio vero a diuersa radiorum refrangibilitate orinunda perfecte tolletur si praeterea fuerit $d\gamma=0$ seu

$$\tfrac{aa\mathfrak{C}\mathfrak{C}}{bbcc}\left(\tfrac{1}{a}+\tfrac{1}{\alpha}\right)+\tfrac{\mathfrak{C}\mathfrak{C}}{cc}\left(\tfrac{1}{b}+\tfrac{1}{\varepsilon}\right)+\tfrac{1}{c}+\tfrac{1}{\gamma}=0.$$

Coroll. 1.

315. Si rationes aperturarum lentium in computum ducantur, eaque pro lente secunda ponatur $=\pi$, pro tertia $=\pi'$ erit $bN'=\frac{\pi b\varepsilon}{b+\varepsilon}$ et $CM''=cN''=\frac{\pi' c\gamma}{c+\gamma}$. Tum vero posito $\frac{\alpha}{a}=\Phi$, erit $G\eta=\frac{a\varepsilon}{ab}a\Phi$, et $H\theta=\frac{\alpha\beta\gamma}{abc}\cdot a\Phi$. Hinc fiet $\frac{O}{\gamma-O}=\frac{cN''}{H\theta}=\frac{\pi' abcc}{\alpha\varepsilon(c+\gamma)a\Phi}$ et $\frac{O}{\gamma(\gamma-O)}=\frac{\pi' abcc}{\alpha\varepsilon\gamma(c+\gamma)}\cdot\frac{1}{a\Phi}$.

Coroll.

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Coroll. 2.

316. Quodsi porro vt supra ponatur

$\alpha = Aa, \mathfrak{B} = Bb, \gamma = Cc$ erit $\frac{\Omega}{\gamma(\gamma-1)} = \frac{\pi'}{ABC(C+1)a\Phi}$;

tum vero $bN' = \frac{\pi Bb}{B+1}$ et $CM'' = \frac{\pi' Cc}{C+1}$: ac $G\eta = AB\alpha\Phi_\beta$.

Cum iam sit

$bN' + G\eta : bG = CM'' - G\eta : CG$ erit $\frac{bN'+G\eta}{bG} = \frac{CM''-G\eta}{CG}$

ideoque

$\frac{1}{bG} + \frac{1}{CG} = \frac{CM''}{CG.G\eta} - \frac{bN'}{bG.G\eta}$ hoc est $\frac{1}{\mathfrak{B}} + \frac{1}{\alpha} = \frac{\pi' C}{AB(C+1)a\Phi} - \frac{\pi}{AB(B+1)a\Phi}$

simili vero modo est $\frac{1}{\alpha} + \frac{1}{b} = \frac{\pi B}{A(B+1)a\Phi}$.

Coroll. 3.

317. Per easdem substitutiones fit

$\frac{1}{\mathfrak{a}} + \frac{1}{\alpha} = \frac{A+1}{Aa}$; $\frac{1}{b} + \frac{1}{\mathfrak{B}} = \frac{B+1}{Bb}$; $\frac{1}{c} + \frac{1}{\gamma} = \frac{C+1}{Cc}$ etc.

hinc que:

$aa\,(\frac{1}{a} + \frac{1}{\alpha}) = A(A+1)a$

$\mathfrak{B}\mathfrak{B}\,(\frac{1}{b} + \frac{1}{\mathfrak{B}}) = B(B+1)b$

$\gamma\gamma\,(\frac{1}{c} + \frac{1}{\gamma}) = C(C+1)c$ sicque porro.

Coroll. 4.

318. His ergo nouis denominationibus introductis differentialia ex §. 300 ita exprimentur

$da = -db = -\frac{dn}{n-1}.A(A+1)a$

$d\mathfrak{B} = -dc = -BBdb - \frac{dn}{n-1}.B(B+1)b$

$d\gamma = -dd = -CCdc - \frac{dn}{n-1}.C(C+1)c$

quae formulae commodius in calculum introducentur.

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Quemadmodum hic nouae formae adhibentur in sequentibus vsurpandae; ita et pro casu diuersae refractionis sequentibus formulis in posterum vti licebit:

$$\frac{0}{\gamma(\gamma-0)} = \frac{\pi'}{ABC(C+1)a\Phi},$$

$$\frac{1}{\alpha}+\frac{1}{b} = \frac{\pi B}{A(B+1)a\Phi}$$

$$\frac{1}{\beta}+\frac{1}{c} = \frac{\pi' C}{AB(C+1)a\Phi} - \frac{\pi}{AB(B+1)a\Phi}$$

$$\frac{1}{\gamma}+\frac{1}{d} = \frac{\pi'' D}{ABC(D+1)a\Phi} - \frac{\pi'}{ABC(C+1)a\Phi}$$

etc.

tum vero formulae differentiales erunt

$$d\alpha = -db = \frac{dn}{n-1}.A(A+1)a$$

$$d\beta = -dc = B^2 d\alpha \frac{dn'}{n'-1}.B(B+1)b$$

$$d\gamma = -dd = C^2 d\beta \frac{dn''}{n''-1}C(C+1)c$$

$$d\delta = -de = D^2 d\gamma \frac{dn'''}{n'''-1}.D(D+1)d$$

etc.

Atque ex his formulis vt margo coloratus euanescat, satisfieri debet huic aequationi

$$0 = \frac{dn'}{n'-1}.\frac{\pi b}{Aa\Phi} + \frac{dn''}{n''-1}.\frac{\pi' c}{ABa\Phi}$$

Vt autem haec confusio penitus tollatur, fieri debet

$$d\gamma = -\frac{dn}{n-1}.A(A+1)B^2C^2a$$
$$-\frac{dn'}{n'-1}.B(B+1)C^2.b$$
$$-\frac{dn''}{n''-1}.C(C+1)c = 0.$$

siue

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siue

$$\frac{dn}{n-1}\cdot\frac{(\mathfrak{A}+1)\alpha}{A}$$
$$+\frac{dn'}{n'-1}\cdot\frac{(\mathfrak{B}+1)b}{AAB}=0$$
$$+\frac{dn''}{n''-1}\cdot\frac{(\mathfrak{C}+1)\varepsilon}{A^2BC}.$$

Problema 7.

319. Si instrumentum dioptricum quatuor constet lentibus quarum crassities negligi queat, eam definire dispositionem vt oculus in eo loco quem campus apparens postulat, constitutus obiectum sine confusione a diuersa radiorum indole oriunda conspiciat. Tab. III. Fig. 15.

Solutio.

Posita distantia obiecti ante instrumentum $AE=\mathfrak{a}$, eiusque magnitudine $E\varepsilon=\mathfrak{z}$, vocentur distantiae imaginum a radiis mediae naturae formatarum vt hactenus.

$aF=\mathfrak{a}$, $BF=\mathfrak{b}$; $bG=\mathfrak{c}$, $CG=\mathfrak{c}$; $cH=\gamma$; $DG=\mathfrak{d}$; $dI=\delta$

Tum vero ponamus praeterea

$$\alpha=Aa,\ \mathfrak{B}=Bb;\ \gamma=Cc;\ \delta=Dd$$

atque introducantur rationes aperturarum pro singulis lentibus post primam, quae sint π pro secunda QQ, π' pro tertia RR et π'' pro quarta SS, sumto pro campo $\frac{\mathfrak{z}}{\mathfrak{a}}=\Phi$. His positis erunt imagines $F\zeta=Aa\Phi$; $G\eta=ABa\Phi$; $H\theta=ABCa\Phi$ et $I\iota=ABCDa\Phi=\frac{\alpha\mathfrak{B}\gamma\delta}{\mathfrak{a}bcd}\mathfrak{z}$.

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Iam posita oculi distantia post instrumentum $dO=0$, vt sit $OI=\delta-O$ erit anguli IOi tangens $\frac{\alpha\mathfrak{B}\gamma\delta}{abcd}\cdot\frac{z}{\delta-O}$, quae cum ob diuersam radiorum refrangibilitatem immutata manere debeat, differentiata dabit, hanc aequationem

$$\frac{d\alpha}{\alpha}-\frac{db}{b}+\frac{d\mathfrak{B}}{\mathfrak{B}}-\frac{dc}{c}+\frac{d\gamma}{\gamma}-\frac{dd}{d}-\frac{O\,d\,\delta}{\delta(\delta-O)}=0$$

quae ob $db=-d\alpha$, $dc=-d\mathfrak{B}$ et $dd=-d\gamma$ abit in hanc

$$d\alpha(\tfrac{1}{\alpha}+\tfrac{1}{b})+d\mathfrak{B}(\tfrac{1}{\mathfrak{B}}+\tfrac{1}{c})+d\gamma(\tfrac{1}{\gamma}+\tfrac{1}{d})-\frac{O\,d\,\delta i}{\delta(\delta-O)}=0$$

Verum modo ante notauimus fore (318)

$$d\alpha=\tfrac{-dn}{n-1}\cdot A(A+1)a$$
$$d\mathfrak{B}=BBd\alpha-\tfrac{d\,n}{n-1}\cdot B(B+1)b$$
$$d\gamma=CCd\mathfrak{B}-\tfrac{d\,n}{n-1}\cdot C(C+1)c$$
$$d\delta=DDd\gamma-\tfrac{d\,n}{n-1}\cdot D(D+1)d$$

vbi pro b, c, d, valores §. 266 assignati substitui debent. Porro vero iam animaduertimus esse (316)

$$\tfrac{1}{\alpha}+\tfrac{1}{b}=\tfrac{1}{A\alpha\Phi}\cdot\tfrac{\pi\,B}{B+1}$$
$$\tfrac{1}{\mathfrak{B}}+\tfrac{1}{c}=\tfrac{1}{AB\alpha\Phi}(\tfrac{\pi'C}{C+1}-\tfrac{\pi}{B+1})$$
$$\tfrac{1}{\gamma}+\tfrac{1}{d}=\tfrac{1}{ABC\alpha\Phi}(\tfrac{\pi''D}{D+1}-\tfrac{\pi'}{C+1})$$

atque

$$\frac{O}{\delta(\delta-O)}=\frac{\pi''}{ABCD(D+1)\alpha\Phi}.$$

Quod

Quodsi iam priores valores in posterioribus successiue substituantur, habebimus

$$da = \frac{-dn}{n-1}.\ A(A+1)a$$

$$d\mathfrak{b} = \frac{-dn}{n-1}.\ B(AB(A+1)a+(B+1)b$$

$$d\gamma = \frac{-dn}{n-1}.\ C(AB^2C(A+1)a+BC(B+1)b+(C+1)c)$$

$$d\delta = \frac{-dn}{n-1}.D(AB^2C^2D(A+1)a+BC^2D(B+1)b+CD(C+1)c+(D+1)d)$$

His igitur valoribus in aequatione differentiali substitutis, et diuisione per $\frac{dn}{(n-1)\Phi}$ facta aequatio nostra secundum singula membra distributa erit

$$-\frac{\pi B}{(B+1)}.(A+1)a$$

$$-\frac{1}{A}\left(\frac{\pi' C}{C+1}-\frac{\pi}{B+1}\right)(AB(A+1)a+(B+1)b)$$

$$-\frac{1}{AB}\left(\frac{\pi'' D}{D+1}-\frac{\pi'}{C+1}\right)(AB^2C(A+1)a+BC(B+1)b+(C+1)c)$$

$$+\frac{1}{ABC}\cdot\frac{\pi''}{D+1}(AB^2C^2D(A+1)a+BC^2D(B+1)b+CD(C+1)c+(D+1)d)=0$$

Terna autem priora membra negatiua sola collecta praebent

$$-\frac{BCD(A+1)\pi''}{D+1}a-\frac{CD(B+1)\pi''}{A(D+1)}b-\frac{D(C+1)\pi''}{AB(D+1)}c$$

$$+\frac{\pi}{A}b+\frac{\pi'}{AB}c$$

quibus si quartum addatur prodit

$$\frac{\pi}{A}b+\frac{\pi'}{AB}c+\frac{\pi''}{ABC}d$$

Iam duo hic casus considerari oportet, alterum quo punctum O post lentem vltimam cadit, alterum vero

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vero, quo ob distantiam O negatiuam oculus lenti vltimae immediate applicatur. Pro priori casu, quo distantia $dO=O$ prodit positiua habetur ista aequatio si quidem in aequatione modo inuenta pro b, c, d valores §. 266 inuenti substituantur

$$\frac{(B+1)\pi}{B\pi-(B+1)\Phi}+\frac{(C+1)\pi'}{C\pi'-(C+1)(\pi-\Phi)}+\frac{(D+1)\pi''}{D\pi''-(D+1)(\pi'-\pi+\Phi)}=0$$

Pro casu posteriori, quo distantia O euanescens assumitur facta multiplicatione per $\frac{D+1}{a}\Phi$.

$$\frac{BCD(A+1)\pi''}{\Phi}+\frac{CD(B+1)^2\pi''}{B\pi-(B+1)\Phi}+\frac{D(C+1)^2\pi''}{C\pi'-(C+1)(\pi-\Phi)}$$
$$=\frac{(B+1)(D+1)\pi}{B\pi-(B+1)\Phi}+\frac{(C+1)(D+1)\pi'}{C\pi'-(C+1)(\pi-\Phi)}.$$

Hoc modo efficitur, vt obiectum sine margine colorato appareat at omnis confusio tollatur si praeterea fuerit

$$AB^2C^2D(A+1)+\frac{ABC^2D(B+1)^2\Phi}{B\pi-(B+1)\Phi}+\frac{ABCD(C+1)^2\Phi}{C\pi'-(C+1)(\pi-\Phi)}+\frac{ABC(D+1)^2\Phi}{D\pi''-(D+1)(\pi'-\pi+\Phi)}=0$$

seu per ABC diuidendo:

$$BCD(A+1)+\frac{CD(B+1)^2\Phi}{B\pi-(B+1)\Phi}+\frac{D(C+1)^2\Phi}{C\pi'-(C+1)(\pi-\Phi)}+\frac{(D+1)^2\Phi}{D\pi''-(D+1)(\pi'-\pi+\Phi)}=0$$
$$BCD(A+1)+\frac{CD(B+1)b}{Aa}+\frac{D(C+1)c}{ABa}+\frac{(D+1)d}{ABCa}=0.$$

Coroll. 1.

320. Si vtrique conditioni satisfieri potest, vt nec locus nec magnitudo imaginis vllam mutationem patiatur locus oculi non amplius in computum ingreditur sed imago, vndecunque cernatur, ab omni confusione prorsus libera erit.

Coroll.

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Coroll. 2.

321. Vt ergo hunc summum perfectionis gradum consequamur, binis his aequationibus satisfieri oportet.

$$\frac{(B+1)\pi}{B\pi-(B+1)\Phi}+\frac{(C+1)\pi'}{C\pi'-(C+1)(\pi-\Phi)}+\frac{(D+1)\pi''}{D\pi''-(D+1)(\pi'-\pi+\Phi)}=0$$

et

$$\mathfrak{A}+1+\frac{(B+1)^2\Phi}{B(B\pi-(B+1)\Phi)}+\frac{(C+1)^2\Phi}{BC(C\pi'-(C+1)(\pi-\Phi))}+\frac{(D+1)^2\Phi}{BCD(D\pi''-(D+1)(\pi'-\pi+\Phi))}=0.$$

Coroll. 3.

322. Si porro vt supra fecimus ponamus

$$\frac{A}{A+1}=\mathfrak{A},\ \frac{B}{B+1}=\mathfrak{B},\ \frac{C}{C+1}=\mathfrak{C}\text{ et }\frac{D}{D+1}=\mathfrak{D}$$

istae aequationes sequenti modo simplicius exprimentur:

$$+\frac{\pi}{\mathfrak{B}\pi-\Phi}+\frac{\pi'}{\mathfrak{C}\pi'-\pi+\Phi}+\frac{\pi''}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}=0\text{ et}$$

$$\frac{1}{\mathfrak{A}}+\frac{\Phi}{\mathfrak{A}\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\Phi}{\mathfrak{A}\mathfrak{B}\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}+\frac{\Phi}{\mathfrak{A}\mathfrak{B}\mathfrak{C}\mathfrak{D}(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}=0$$

hic prima membra multiplicari debent per $\frac{d\,n}{n-1}$

secunda per $\frac{d\,n'}{n'-1}$

tertia per $\frac{d\,n''}{n''-1}$

etc.

Coroll. 4.

323. Sin autem non liceat has ambas aequationes adimplere, curandum est, vt saltem priori, qua magnitudo apparens a confusione liberatur, satisfiat

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fiat hoc enim modo obiectum sine margine colorato apparebit vbi quidem ad duos casus respici conuenit prout distantia oculi dO prodierit positiua vel negatiua.

Scholion.

324. Introducendis ergo rationibus aperturarum quibus supra iam commode sumus vsi ad campum apparentem definiendum aequationes etiam istae confusionem a diuersa radiorum refrangibilitate oriundam tollentes satis fiunt simplices, vt sine molestia tractari queant; si quidem crassities lentium negligatur. Hoc ergo modo problema generale, quicunque fuerit lentium numerus, expediri conueniet.

Supplementum V.

Si ratio refractionis in singulis lentibus discrepet, prodit primo quidem eadem aequatio differentialis

$$d\alpha\left(\tfrac{1}{a}+\tfrac{1}{b}\right)+d\beta\left(\tfrac{1}{c}+\tfrac{1}{c}\right)+d\gamma\left(\tfrac{1}{\gamma}+\tfrac{1}{d}\right)\frac{-\Omega d\delta}{\delta(\delta-O)}=0$$

in qua ergo casu $O=0$ vltimum membrum abiici debet.

I) Si autem O habeat valorem positiuum, erit, vt ante,

$$\frac{\Omega}{\delta(\delta-O)}=\frac{\pi^{m}}{ABCD(D+1)\alpha\beta}$$

atque etiam valores $\frac{1}{a}+\frac{1}{b}$; $\frac{1}{c}+\frac{1}{c}$; $\frac{1}{\gamma}+\frac{1}{d}$ manent iidem, vt ante.

At

At vero ob diuersitatem refractionis habebimus

$$d\alpha = \frac{-dn}{n-1}.\,A(A+1)a$$

$$d\mathfrak{B} = BB\,d\alpha - \frac{dn'}{n'-1}.\,B(B+1)b$$

$$d\gamma = CC\,d\mathfrak{B} - \frac{dn''}{n''-1}C.\,(C+1)c$$

$$d\delta = DD\,d\gamma - \frac{dn'''}{n'''-1}.\,D(D+1)d$$

Hinc ergo aequatio nostra successiue ita formetur:

$$\frac{-O\,d\delta}{\delta(\delta-O)} = \frac{-\pi''\,d\delta}{ABCD(D+1)a\Phi}$$

$$= \frac{-\pi''\,D\,d\gamma}{ABC(D+1)a\Phi} + \frac{dn'''}{n'''-1}\cdot\frac{\pi''.d}{ABC.a\Phi}$$

Addatur $d\gamma\left(\frac{1}{\gamma}+\frac{1}{d}\right) = \frac{d\gamma.\pi''D}{ABC(D+1)a\Phi} - \frac{\pi'\,d\gamma}{ABC(C+1)a\Phi}$

et prodibit $\frac{-\pi'\,d\gamma}{ABC(C+1)a\Phi} + \frac{dn'''}{n'''-1}\cdot\frac{\pi''.d}{ABC.a\Phi}$

et pro $d\gamma$ substituto valore

$$\frac{-\pi'\,C\,d\mathfrak{B}}{AB(C+1)a\Phi} + \frac{dn''}{n''-1}\cdot\frac{\pi'\,c}{AB\,a\Phi} + \frac{dn'''}{n'''-1}\cdot\frac{\pi''.d}{ABC.a\Phi}$$

Iam addatur

$$d\mathfrak{B}\left(\frac{1}{\mathfrak{B}}+\frac{1}{c}\right) = \frac{\pi'\,C\,d\mathfrak{B}}{AB(C+1)a\Phi} - \frac{\pi\,d\mathfrak{B}}{AB(B+1)a\Phi}$$

proditque

$$\frac{-\pi\,d\mathfrak{B}}{AB(B+1)a\Phi} + \frac{dn''}{n''-1}\cdot\frac{\pi'\,c}{AB.a\Phi} + \frac{dn'''}{n'''-1}\cdot\frac{\pi''\,d}{ABC\,a\Phi}$$

et substituto valore ipsius $d\mathfrak{B}$

$$\frac{-\pi B\,d\alpha}{A(B+1)a\Phi} + \frac{dn'}{n'-1}\cdot\frac{\pi.b}{A.a\Phi} + \frac{dn''}{n''-1}\cdot\frac{\pi'\,c}{AB\,a\Phi} + \frac{dn'''}{n'''-1}\cdot\frac{\pi''\,d}{ABC.a\Phi}$$

denique addatur $d\alpha\left(\frac{1}{a}+\frac{1}{b}\right) = \frac{\pi B\,d\alpha}{A(B+1)a\Phi}$

ac aequatio quaesita, qua margo coloratus euanescit, erit

$$\frac{dn'}{n'-1}\cdot\frac{\pi\,b}{A\,a\,\Phi} + \frac{dn''}{n''-1}\cdot\frac{\pi'\,c}{AB\,a\,\Phi} + \frac{dn'''}{n'''-1}\cdot\frac{\pi''\,d}{ABC.a\Phi} = 0$$

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II) Sin autem O habeat valorem negativum: tunc ſumi debet $O=o$ et pro eodem ſcopo aequatio ita formabitur:

Cum ſit

$$d\gamma\left(\frac{1}{\gamma}+\frac{1}{a}\right)=\frac{\pi''\,D\,d\gamma}{ABC(D+1)a\Phi}-\frac{\pi'\,d\gamma}{ABC(C+1)a\Phi}$$

ſubſtituto valore $d\gamma$ fiet

$$CC\,d\mathfrak{C}\left(\frac{\pi''\,D}{ABC(D+1)a\Phi}-\frac{\pi'}{ABC(C+1)a\Phi}\right)$$
$$-\frac{d\,n''}{n''-1}\cdot C(C+1)c\left(\frac{\pi''\,D}{ABC(D+1)a\Phi}-\frac{\pi'}{ABC(C+1)a\Phi}\right)$$

Addatur

$$d\mathfrak{C}\left(\frac{1}{\mathfrak{C}}+\frac{1}{c}\right)=\frac{\pi'C\,d\mathfrak{C}}{AB(C+1)a\Phi}-\frac{\pi\,d\mathfrak{C}}{AB(B+1)a\Phi}$$

eritque

$$d\mathfrak{C}\left(\frac{CD\pi''}{AB(D+1)a\Phi}-\frac{\pi}{AB(B+1)a\Phi}\right)-\frac{d\,n''}{n''-1}(C+1)c\left(\frac{\pi''\,D}{AB(D+1)a\Phi}-\frac{\pi'}{AB(C+1)a\Phi}\right)$$

et ſubſtituto valore ipſius $d\mathfrak{C}$

$$d\alpha\left(\frac{BCD\pi''}{A(D+1)a\Phi}-\frac{B\pi}{A(B+1)a\Phi}\right)$$
$$-\frac{d\,n'}{n'-1}\left(\frac{(B+1)CD\,b\pi''}{A(D+1)a\Phi}-\frac{\pi\,b}{A\,a\Phi}\right)-\frac{d\,n''}{n''-1}(C+1)c\left(\frac{\pi''\,D}{AB(D+1)a\Phi}-\frac{\pi'}{AB(C+1)a\Phi}\right)$$

Addatur

$$d\alpha\left(\frac{1}{\alpha}+\frac{1}{b}\right)=\frac{\pi B\,d\alpha}{A(B+1)a\Phi}$$

eritque

$$-\frac{dn}{n-1}\cdot\frac{(A+1)BCDa\pi''}{(D+1)a\Phi}$$
$$-\frac{dn'}{n'-1}\left(\frac{(B+1)CDb\pi''-(D+1)b\pi}{A(D+1)a\Phi}\right)$$
$$-\frac{dn''}{n''-1}\left(\frac{(C+1)Dc\pi''-(D+1)c\pi'}{AB(D+1)a\Phi}\right)$$

Vnde

Vnde pro casu $O = o$ aequatio, qua margo coloratus destruitur, erit

$$o = \frac{a dn}{n-1}.(A+1)BCD\pi''$$
$$+\frac{b dn'}{n'-1}.\left(\frac{(B+1)CD\pi''-(D+1)\pi}{A}\right)$$
$$+\frac{c d n''}{n''-1}\left(\frac{(C+1)D\pi''-(D+1)\pi'}{AB}\right)$$

III) Vt autem praeterea omnis confusio tollatur, insuper reddi oportet $d\delta = o$; vnde haec aequatio nascitur

$$o = \begin{cases} \frac{a d n}{n-1}.A(A+1)B^2C^2D^2 \\ +\frac{b d n'}{n'-1}.B(B+1)C^2D^2 \\ +\frac{c d n''}{n''-1}.C(C+1)D^2 \\ +\frac{d d n'''}{n'''-1}.D(D+1) \end{cases}$$

quae per $A^2B^2C^2D^2$ diuisa dat.

$$o = \begin{cases} \frac{a dn}{n-1}.\frac{A+1}{A} \\ +\frac{b dn'}{n'-1}.\frac{B+1}{A^2B} \\ +\frac{c dn''}{n''-1}.\frac{C+1}{A^2B^2C} \\ +\frac{d dn'''}{n'''-1}.\frac{D+1}{A^2B^2C^2D} \end{cases}$$

Circa hanc autem aequationem inprimis notandum est, si omnes lentes pari facultate refringente sint praeditae, ei satisfieri haud posse; ex quo haec aequatio proprie pertinet ad casum, quo diuersae refractiones locum habent.

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Problema 8.

325. Si inſtrumentum dioptricum ex quotcunque lentibus ſit compoſitum, quarum craſſitiem negligere liceat, eam determinare diſpoſitionem, vt oculus in eo loco, quem campus apparens poſtulat poſitus nullam confuſionem ſentiat.

Solutio.

Sit obiecti ante lentem primam diſtantia $AE=a$, eiusque magnitudo $E\varepsilon=z$, quae quidem conſpici queat, et ſtatuatur $\frac{z}{a}=\Phi$. Deinde ſint diſtantiae imaginum a radiis mediae naturae formatarum vt ſupra:

$$AE=a;\ BF=b;\ CG=c;\ DH=d;\ EI=e$$
$$a\,F=\alpha;\ b\,G=\mathfrak{B};\ c\,H=\gamma;\ d\,I=\delta;\ e\,K=\varepsilon$$

etc.

ac vocemus breuitatis gratia

$$\alpha=Aa,\ \mathfrak{B}=Bb,\ \gamma=Cc,\ \delta=Dd,\ \varepsilon=Ee \text{ etc.}$$

tum vero etiam

$$\tfrac{A}{A+1}=\mathfrak{A};\ \tfrac{B}{B+1}=\mathfrak{B};\ \tfrac{C}{C+1}=\mathfrak{C};\ \tfrac{D}{D+1}=\mathfrak{D},\ \tfrac{E}{E+1}=\mathfrak{E} \text{ etc.}$$

Iam apertura primae lentis PP vt euaneſcente conſiderata ſit ratio aperturae pro reliquis lentibus

$$QQ=\pi;\ RR=\pi';\ SS=\pi'';\ TT=\pi''' \text{ etc.}$$

His

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His positis supra vidimus (266) fore praeter $\alpha = Aa$

$$b = \frac{Aa\Phi}{\mathfrak{B}\pi - \Phi} \qquad \mathfrak{C} = \frac{ABa\Phi}{\mathfrak{B}\pi - \Phi}$$

$$c = \frac{ABa\Phi}{\mathfrak{C}\pi' - \pi + \Phi} \qquad \gamma = \frac{ABCa\Phi}{\mathfrak{C}\pi' - \pi + \Phi}$$

$$d = \frac{ABCa\Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi} \qquad \delta = \frac{ABCDa\Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi}$$

$$e = \frac{ABCDa\Phi}{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi} \qquad \varepsilon = \frac{ABCDEa\Phi}{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi}$$

etc. etc.

Deinde singularum imaginum magnitudo ita se habebit

$$F\zeta = Aa\Phi;\ G\eta = ABa\Phi;\ H\theta = ABCa\Phi;\ I\iota = ABCDa\Phi$$

etc.

et ipsi aperturarum semidiametri:

Lentis secundae $QQ = \frac{AB\Phi}{\mathfrak{B}\pi - \Phi}\pi$

Lentis tertiae $RR = \frac{ABCa\Phi}{\mathfrak{C}\pi' - \pi + \Phi}\pi'$

Lentis quartae $SS = \frac{ABCDa\Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi}\pi''$

Lentis quintae $TT = \frac{ABCDEa\Phi}{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi}\pi'''$

Mutata iam refractionis lege $n : 1$ infinite parum, distantiae $a, b, \mathfrak{C}, c; \gamma, d, \delta$ etc. tales mutationes recipiunt.

$$da = -\frac{dn}{n-1}\cdot Aa(A+1)$$

$$d\mathfrak{C} = \frac{-dn}{n-1}\cdot ABa\left((A+1)B + \frac{(B+1)\Phi}{\mathfrak{B}\pi - \Phi}\right)$$

$$d\gamma = \frac{-dn}{n-1}\cdot ABCa\left((A+1)BC + \frac{(B+1)C\Phi}{\mathfrak{B}\pi - \Phi} + \frac{(C+1)\Phi}{\mathfrak{C}\pi' - \pi + \Phi}\right)$$

$$d\delta = \frac{-dn}{n-1}\cdot ABCDa\left((A+1)BCD + \frac{(B+1)CD\Phi}{\mathfrak{B}\pi - \Phi} + \frac{(C+1)D\Phi}{\mathfrak{C}\pi' - \pi + \Phi} + \frac{(D+1)\Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi}\right)$$

etc.

Porro vero habetur:

$$\tfrac{1}{\alpha}+\tfrac{1}{b}=\tfrac{1}{A\alpha\Phi}\mathfrak{B}\pi$$

$$\tfrac{1}{\epsilon}+\tfrac{1}{c}=\tfrac{1}{AB\alpha\Phi}\left(\mathfrak{C}\pi'-\tfrac{\mathfrak{B}\pi}{B}\right)$$

$$\tfrac{1}{\gamma}+\tfrac{1}{d}=\tfrac{1}{ABC\alpha\Phi}\left(\mathfrak{D}\pi''-\tfrac{\mathfrak{C}\pi'}{C}\right)$$

$$\tfrac{1}{\delta}+\tfrac{1}{e}=\tfrac{1}{ABCD\alpha\Phi}\left(\mathfrak{E}\pi'''-\tfrac{\mathfrak{D}\pi''}{D}\right)$$

etc.

His positis pro quolibet lentium numero seorsim formulas quaesito satisfacientes expediamus: posita distantia oculi post lentem vltimam $=0$.

I. Pro vnica Lente

Habetur haec aequatio differentialis $\frac{-0\,d\alpha}{\alpha(\alpha-0)}=0$, pro quo casu tam magnitudo imaginis quam eius locus manebit inuariatus si fuerit $d\alpha=0$, hoc est $A(A+1)=0$, vnde deberet esse vel $A=0$ vel $A=-1$: quorum prius visio non admittit posterius autem lentem tollit. Tum vero ob campum esse debet $O=0$.

II. Pro duabus lentibus

Habetur haec aequatio differentialis, qua margo coloratus tollitur:

$$d\alpha\left(\tfrac{1}{\alpha}+\tfrac{1}{b}\right)-\tfrac{0\,d\epsilon}{\epsilon(\epsilon-0)}=0$$

at ob campum apparentem est $\frac{0}{\epsilon(\epsilon-0)}=\frac{1}{BB}\left(\frac{1}{\alpha}+\frac{1}{b}\right)$, ita vt habeamus:

$$d\alpha\left(\tfrac{1}{\alpha}+\tfrac{1}{b}\right)-\tfrac{d\epsilon}{BB}\left(\tfrac{1}{\alpha}+\tfrac{1}{b}\right)=0$$

Verum

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Verum eſt $d\mathfrak{G} = BB\mathfrak{B}d\alpha - \frac{d\pi}{\pi - 1} \cdot AB\mathfrak{B}\alpha \cdot \frac{(B+1)\Phi}{\mathfrak{B}\pi - \Phi}$

Quare ſi O habeat valorem poſitiuum erit ob

$$\mathfrak{B}(B+1) = B, \frac{\pi}{\mathfrak{B}\pi - \Phi} = 0$$

Sin autem valor O prodeat negatiuus, quo caſu capitur $O = \alpha$ erit $\frac{(A+1)\mathfrak{B}\pi}{\Phi} = \alpha$.

Omnis autem confuſio penitus tolletur, ſi inſuper fuerit

$$\frac{1}{\alpha} + \frac{\Phi}{AB(\mathfrak{B}\pi - \Phi)} = 0$$

III. Pro tribus Lentibus

Si calculum eodem modo proſequamur, obiectum ſine margine colorato conſpicietur:

1. Si ex campo apparente diſtantia O prodeat poſitiua hanc aequationem adimplendo:

$$\frac{\pi}{\mathfrak{B}\pi - \Phi} + \frac{\pi'}{\mathfrak{C}\pi' - \pi + \Phi} = 0.$$

2. Si ob diſtantiam O prodeuntem negatiuam capiatur $O = 0$ huic aequationi erit ſatisfaciendum:

$$\frac{\pi'}{\alpha\Phi} + \frac{\pi'}{AB(\mathfrak{B}\pi - \Phi)} = \frac{\pi}{AB\mathfrak{C}(\mathfrak{B}\pi - \Phi)}$$

Omnis autem confuſio penitus tolletur ſi fuerit inſuper

$$\frac{1}{\alpha} + \frac{\Phi}{AB(\mathfrak{B}\pi - \Phi)} + \frac{\Phi}{AB\mathfrak{C}(\mathfrak{C}\pi' - \pi + \Phi)} = \alpha.$$

IV. Pro quatuor Lentibus

Vt obiectum ſine margine colorato ſpectetur:

1. Si

1. Si ex campo apparente distantia O prodeat positiua huic aequationi erit satisfaciendum:

$$\frac{\pi}{\mathfrak{B}\pi-\Phi}+\frac{\pi'}{\mathfrak{C}\pi'-\pi+\Phi}+\frac{\pi''}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}=0$$

2. Sin autem capiatur $O=0$, huic

$$\frac{\pi''}{\mathfrak{A}\Phi}+\frac{\pi''}{\mathfrak{AB}(\mathfrak{B}\pi-\Phi)}+\frac{\pi''}{\mathfrak{ABC}(\mathfrak{C}\pi'-\pi+\Phi)}=\frac{\pi}{\mathfrak{ABCD}(\mathfrak{B}\pi-\Phi)}+\frac{\pi'}{\mathfrak{ABCD}(\mathfrak{C}\pi'-\pi+\Phi)}$$

Omnis vero confusio penitus tolletur si fuerit praeterea

$$\frac{1}{\mathfrak{A}}+\frac{\Phi}{\mathfrak{AB}(\mathfrak{B}\pi-\Phi)}+\frac{\Phi}{\mathfrak{ABC}(\mathfrak{C}\pi'-\pi+\Phi)}+\frac{\Phi}{\mathfrak{ABCD}(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}=0$$

V. Pro quinque Lentibus

Vt obiectum tantum sine margine colorato conspiciatur

1. Si ex campo apparente distantia O prodeat positiua, huic aequationi erit satisfaciendum:

$$\frac{\pi}{\mathfrak{B}\pi-\Phi}+\frac{\pi'}{\mathfrak{C}\pi'-\pi+\Phi}+\frac{\pi''}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}+\frac{\pi'''}{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi}=0$$

2. Sin autem capiatur $O=0$ huic

$$\frac{\pi'''}{\mathfrak{A}\Phi}+\frac{\pi'''}{\mathfrak{AB}(\mathfrak{B}\pi-\Phi)}+\frac{\pi'''}{\mathfrak{ABC}(\mathfrak{C}\pi'-\pi+\Phi)}+\frac{\pi'''}{\mathfrak{ABCD}(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}$$
$$\frac{1}{\mathfrak{ABCDE}}\left(\frac{\pi}{\mathfrak{B}\pi-\Phi}+\frac{\pi'}{\mathfrak{C}\pi'-\pi+\Phi}+\frac{\pi''}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}\right)$$

Omnis autem confusio penitus tolletur, si praeterea satisfiat huic aequationi:

$$\frac{1}{\mathfrak{A}}+\frac{\Phi}{\mathfrak{AB}(\mathfrak{B}\pi-\Phi)}+\frac{\Phi}{\mathfrak{ABC}(\mathfrak{C}\pi'-\pi+\Phi)}+\frac{\Phi}{\mathfrak{ABCD}(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}+\frac{\Phi}{\mathfrak{ABCDE}(\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi)}=0$$

Atque hinc manifesta est progressio ad maiorem lentium numerum.

Coroll.

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Coroll. 1.

326. In casu ergo vnicae lentis licet quidem obiectum a margine colorato liberare neque vero confusionem penitus tollere. Casu autem duarum lentium ne margo quidem coloratus tolli potest, siquidem oculus in eo loco, quem campi apparentis conditio postulat, teneatur.

Coroll. 2.

327. Quodsi vero plures duabus habeantur lentes sufficiens quantitatum numerus adest, quarum determinatione non solum margo coloratus deleri, sed etiam forte omnis confusio penitus auferri posse videtur praecipue si lentium numerus ternarium superet.

Scholion.

328. Quod ergo incommodum a diuersa radiorum natura oriundum adeo graue vel summo Neutono est visum, vt instrumenta dioptrica nullo modo ab eo liberari posse sit arbitratus, id quidem saltem quod ad marginem coloratum attinet ad quem Neutonus inprimis spectabat, iam satis feliciter tolli posse certum est; ita vt saltem ob hanc causam non opus sit ad Telescopia Catoptrica confugere. Hoc autem vitio sublato si praeterea alterum confusionis fontem obstruamus lentes scilicet nullam confusionem parientes adhibendo, nullum est dubium quin instrumenta dioptrica ad summum perfectionis gradum euehi queant.

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Quae igitur hactenus particulatim circa singulas horum instrumentorum affectiones proposuimus, ea colligi conueniet, vnde in capite sequente praecepta generalia pro omnium instrumentorum dioptricorum constructione tradere est visum.

Supplementum VI.

Ex iis, quae ante sunt adiecta, poterimus etiam problematis solutionem pro casu exhibere, quo singulae lentes peculiari refractione sunt praeditae, vbi quidem tantum postremae aequationes pro confusione vitanda mutationem quandam postulant; interim tamen etiam priores formulas, quibus locus oculi, quem campus apparens requirit, distinctius repraesentemus.

I. *Distantia Oculi post vltimam lentem* pro quouis lentium numero se habebit, vt sequitur.

Num. lentium	O id est, distantia oculi post lentem vltimam
I.	o
II.	$\frac{A\mathfrak{B}a\pi\Phi}{(\pi-\Phi)(\mathfrak{B}\pi-\Phi)}$ seu $\frac{\mathfrak{B}b\pi}{\pi-\Phi}$
III.	$\frac{AB.\mathfrak{C}a\pi'\Phi}{(\pi'-\pi+\Phi)(\mathfrak{C}\pi'-\pi+\Phi)}$ seu $\frac{\mathfrak{C}c\pi'}{\pi'-\pi+\Phi}$
IV.	$\frac{ABC\mathfrak{D}.a\pi''\Phi}{(\pi''-\pi'+\pi-\Phi)(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}$ seu $\frac{\mathfrak{D}d\pi''}{\pi''-\pi'+\pi-\Phi}$
V.	$\frac{ABCD\mathfrak{E}.a\pi'''\Phi}{(\pi'''-\pi''+\pi'-\pi+\Phi)(\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi)}$ seu $\frac{\mathfrak{E}e\pi'''}{\pi'''-\pi''+\pi'-\pi+\Phi}$
	etc.

II.

II. Si valor ipsius O sit positiuus, ad marginem coloratum tollendum sequentes aequationes sunt adimplendae:

Num. lentium	
I.	$0 = 0$
II.	$0 = \frac{b d n'}{n'-1} \cdot \frac{\pi}{A a \Phi}$
III.	$0 = \frac{b d n'}{n'-1} \cdot \frac{\pi}{A a \Phi} + \frac{c d n''}{n''-1} \cdot \frac{\pi'}{AB a \Phi}$
IV.	$0 = \frac{b d n'}{n'-1} \cdot \frac{\pi}{A a \Phi} + \frac{c d n''}{n''-1} \cdot \frac{\pi'}{AB a \Phi} + \frac{d d n'''}{n'''-1} \cdot \frac{\pi''}{ABC a \Phi}$

etc.

III. Si valor ipsius O prodeat negatiuus, quo casu capi debet $O = o$, ad marginem coloratum tollendum sequentes aequationes sunt adimplendae.

Num. lentium	
I.	$0 = 0$
II.	$0 = \frac{a d n}{n-1} \cdot (A+1) B \pi$
III.	$0 = \frac{a d n}{n-1} (A+1) B C \pi' + \frac{b d n'}{n'-1} \cdot \frac{(B+1) C \pi' - (C+1) \pi}{A}$
IV.	$0 = \frac{a d n}{n-1} \cdot (A+1) BCD \pi'' + \frac{b d n'}{n'-1} \left(\frac{(B+1) CD \pi'' - (D+1) \pi}{A} \right) + \frac{c d n''}{n''-1} \left(\frac{(C+1) D \pi'' - (D+1) \pi'}{AB} \right)$

etc.

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IV. Vt autem insuper omnis confusio huius generis tollatur, sequentes aequationes sunt adimplendae:

Num. lentium	
I.	$0 = \frac{a\,dn}{n-1} \cdot \frac{A+1}{A}$
II.	$0 = \frac{a\,dn}{n-1} \cdot \frac{A+1}{A} + \frac{b\,d\,n'}{n'-1} \cdot \frac{B+1}{A\,B}$
III.	$0 = \frac{a\,dn}{n-1} \cdot \frac{A+1}{A} + \frac{b\,dn'}{n'-1} \cdot \frac{B+1}{A^2\,B} + \frac{c\,dn''}{n''-1} \cdot \frac{C+1}{A\,B\,C}$
IV.	$0 = \frac{a\,dn}{n-1} \cdot \frac{A+1}{A} + \frac{b\,d\,n'}{n'-1} \cdot \frac{B+1}{A\,B} + \frac{c\,dn''}{n''-1} \cdot \frac{C+1}{A^2\,B\,C} + \frac{d\,dn'''}{n'''-1} \cdot \frac{D+1}{A^2\,B^2\,C^2\,D}$

etc.

Quarum formularum ordo hinc distinctius perspicitur, quam in problemate.

CAPVT

CAPVT VII.

DE CONSTRVCTIONE INSTRVMENTORVM DIOPTRICORVM IN GENERE.

Problema 1.

329.

Si instrumentum dioptricum vnica constet lente crassitiei cuiuscunque, PP, definire omnia momenta ad visionem pertinentia. Tab. III. Fig. 12.

Solutio.

Quod primo ad ipsius lentis structuram attinet ponatur obiecti Eε ante eam distantia $AE = a$, imaginisque Fζ post eam proiectae $aF = \alpha$; tum vero lentis crassities $Aa = v$ et quantitas arbitraria $= k$, vnde capiatur $\frac{k-v}{k+v} = i$. Hinc facies lentis ita erunt formatae, vt sit existente $n = \frac{31}{20}$ Conf. §. 68.

radius faciei anterioris $= \frac{(n-1)a(k+v)}{k+v+2na}$

radius faciei posterioris $= \frac{(n-1)\alpha(k-v)}{k-v-2n\alpha}$.

Ponatur porro semidiameter obiecti conspicui $Eε = z$, et semidiameter aperturae in facie anteriori $AM = x$, in facie autem posteriori semidiameter aperturae non

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sit minor quam ix. Denique oculi post lentem distantia vocetur $aO = O$, cuius distantia iusta sit $= l$. His positis sequentia momenta perpendi oportet.

1. Debet esse $O = a + l$ vt distantia imaginis visae naturae oculi conueniat.

2. Consideranda venit multiplicatio, quae ex ratione definitur, quam diameter obiecti visus tenet ad eiusdem diametrum visum si nudo oculo in distantia data $= b$ spectaretur. Quodsi ergo hic exponens multiplicationis ponatur $= m$ erit

$m = \frac{1}{i} \cdot \frac{ab}{al}$ pro situ inuerso

3. Gradus claritatis determinatur semidiametro coni luminosi, qui a quouis obiecti puncto in oculum immittitur, qui si ponatur $= y$ erit

$y = i\,l \cdot \frac{x}{a} = \frac{bx}{ma}$

4. Confusio inquinans visionem mensuratur semidiametro apparente circuli, qui nudo oculo aeque magnus cernitur, ac singula obiecti puncta per lentem spectata. Hanc mensuram vocaui (194) semidiametrum confusionis, ad quem definiendum si ponatur:

$$P = \frac{n}{2(n-1)^2}\left(\left(\frac{n}{a} + \frac{1}{k+v}\right)\left(\frac{1}{ia} + \frac{2}{k-v}\right)^2 + \left(\frac{n}{a} - \frac{2}{k-v}\right)\left(\frac{i}{a} - \frac{2}{k+v}\right)^2\right)$$

erit semidiameter confusionis $= \frac{1}{4} i \cdot \frac{a}{l} x^3 P = \frac{1}{4} i^2 \cdot \frac{ma}{b} x^3 . P.$

5. Vt oculus maximum campum apparentem percipiat, debet esse $O = \frac{-iav}{na - iv}$

6. Quae

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6. Quae distantia si fuerit positiua, semidiameter campi seu obiecti conspicui $E\varepsilon = z$, ita pendet ab apertura faciei posterioris, vt posito huius semidiametro $= a$ sit $z = \frac{na}{v}.a$.

7. Sin autem distantia illa pro O assignata prodierit negatiua, oculum lenti immediate applicari conueniet, vt sit $O = o$; tum vero pro a posito semidiametro pupillae ω erit pro campo apparente $z = \frac{na}{v}\omega$.

8. Vt obiectum sine margine colorato appareat, existente distantia O ante inuenta positiua, debet esse $i = \frac{\alpha}{\alpha+v}$. seu $k = 2\alpha + v$.

9. Sin autem ob illam distantiam negatiuam prodeuntem capiatur $O = o$, vt margo coloratus euitetur, oportet esse $\frac{v}{\alpha} + 1 - i = o$, vnde fit $k = -2\alpha - v$.

10. Omnis denique confusio a diuersa radiorum refrangibilitate oriunda prorsus tolletur, si insuper fieri posset $(\alpha+v)(\alpha+\alpha+v) = o$

Coroll. 1.

330. Si exponens multiplicationis m cum gradu claritatis y proponatur, erit $my = \frac{bx}{a}$. quae proprietas ad lentium numerum quantumuis magnum patet. Cum ergo $\frac{b}{a}$ sit quantitas data, erit x vt my; tum vero y vt $\frac{x}{m}$, ac m vt $\frac{x}{y}$

Coroll. 2.

331. Tam maior ergo multiplicatio quam maior claritatis gradus postulat maiorem aperturam. Verum

Verum aucto x confusio augetur in ratione triplicata, siquidem quantitas P maneat eadem: quare si x ex confusione etiamnum tolerabili determinetur, simul quantitas my determinatur.

Coroll. 3.

332. Vt igitur tam multiplicationem m quam claritatem y salua confusione maxime augere liceat, quantitatem arbitrariam k ita definiri conueniet, vt litterae P minimus valor concilietur.

Cum autem posuerimus $\frac{k-v}{k+v} = i$, valor ipsius P fiet minimus, huic aequationi satisfaciendo:

$$2i^4\left(\tfrac{n}{\alpha}+\tfrac{1}{v}\right)\left(\tfrac{1}{\alpha}+\tfrac{1}{v}\right)^2-\tfrac{2ni^3}{\alpha v}\left(\tfrac{1}{\alpha}+\tfrac{1}{v}\right)-\tfrac{i^3}{v}\left(\tfrac{1}{\alpha}+\tfrac{2}{v}\right)^2$$
$$+\tfrac{2ni}{\alpha v}\left(\tfrac{1}{\alpha}+\tfrac{1}{v}\right)+\tfrac{i}{v}\left(\tfrac{1}{\alpha}+\tfrac{2}{v}\right)^2-2\left(\tfrac{n}{\alpha}+\tfrac{1}{v}\right)\left(\tfrac{1}{\alpha}+\tfrac{1}{v}\right)^2=0$$

vti §. 45 inuenimus.

Coroll. 4.

333. Si loco i valor sustituatur suus $\frac{k-v}{k+v}$, aequatio transibit in hanc formam:

$$+\tfrac{2n}{\alpha^3}(k+v)^4+\tfrac{(2n+1)}{\alpha\alpha v}(k+v)^3(k+3v)+\tfrac{4(n+2)}{\alpha v}(k+v)^2$$
$$-\tfrac{2n}{\alpha^3}(k-v)^4-\tfrac{(2n+1)}{\alpha\alpha v}(k-v)^3(k-3v)+\tfrac{4(n+2)}{\alpha v}(k-v)^3+32k=0$$

quae per v multiplicata etiam ad casum accommodari potest quo est $v=0$: tum autem reperitur

$$k=\tfrac{4(n+2)\alpha\alpha}{(2n+1)(\alpha-\alpha)} \text{ et}$$
$$P=\tfrac{n}{8(n-1)^2(n+2)}\left(\tfrac{1}{\alpha}+\tfrac{1}{\alpha}\right)\left((4n-1)\left(\tfrac{1}{\alpha}+\tfrac{1}{\alpha}\right)^2+\tfrac{4(n-1)^2}{\alpha\alpha}\right)$$

Scholion

Scholion

334. Hic non generatim omnem visionem, quae fit per vnicam lentem, considero, sed tantum eam, qua maximus campus apparens conspicitur; quamobrem locum oculi ita definiui, vt ipsi maximus campus adferatur. Sin autem minore campo velimus esse contenti oculus quoque alibi post lentem constitutus obiecta distincte cernere poterit, quemadmodum per lentes satis amplas, quae tabularum vitrearum nomine sunt notae, fieri solet. Hunc autem casum quoniam per se facile expeditur, atque in instrumentis dioptricis magnitudo campi potissimum spectatur, hic non attingo.

Problema 2.

335. Si instrumentum dioptricum duabus constet lentibus cuiuscunque crassitiei definire omnia momenta ad visionem pertinentia. Tab. III. Fig. 13.

Solutio.

Obiecto constituto in $E\varepsilon$, eius imagines proiiciantur per istas lentes in $F\zeta$ et $G\eta$, ac ponantur quantitates vtramque lentem determinantes

$AE = a$, $aF = \alpha$, crassities $A\alpha = v$ et distantia arbitraria $= k$

$BF = b$, $bG = \beta$, crassities $B\beta = v'$ et distantia arbitraria $= k'$

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ponaturque breuitatis gratia $\frac{k-v}{k+v} = i$ et $\frac{k'-v'}{k'+v'} = i'$. Hinc existente $n = \frac{31}{20}$, constructio vtriusque lentis ita se habebit,

	Radius faciei anterioris	posterioris
Pro Lente PP	$\frac{(n-1)\alpha(k+v)}{k+v+2n\alpha}$	$\frac{(n-1)\alpha(k-v)}{k-v-2n\alpha}$
Pro Lente QQ	$\frac{(n-1)\beta(k'+v')}{k'+v'+2n\beta}$	$\frac{(n-1)\beta(k'-v')}{k'-v'-2n\beta}$

Sit porro semidiameter aperturae lentis primae PP in facie anteriori $= x$, in facie autem posteriori maior sit quam ix. Tum vero lentis secundae QQ semidiameter aperturae in facie anteriori maior esse debet quam $i\frac{bx}{a}$, in posteriori vero maior quam $i'i\frac{bx}{a}$. Deinde sit semidiameter obiecti conspicui $Ee = z$, distantia oculi $bO = O$, eiusque distantia iusta $= l$. His positis ad sequentia momenta erit attendendum.

1. Vt oculus imaginem $Gγ$ in distantia iusta conspiciat, oportet esse $O = \xi + l$.

2. Posita distantia $= b$ ad quam multiplicatio refratur, sit exponens multiplicationis $= m$, ex supra inuenimus esse oportere $m = \frac{1}{in} \frac{\alpha\xi b}{\alpha b l}$ pro situ erecto.

3. Denotante y gradum claritatis fiet

$$y = ii'l.\frac{bx}{\alpha\beta} \text{ ideoque } my = \frac{bx}{\alpha}$$

4. Pro confusione posito breuitatis ergo

$$\mathfrak{P} = \frac{n}{2(n-1)}\left(\left(\frac{n}{\alpha}+\frac{2}{k+v}\right)\left(\frac{i}{\alpha}+\frac{2}{k-v}\right)^2+\left(\frac{n}{\alpha}+\frac{2}{k-v}\right)\left(\frac{i}{\alpha}-\frac{2}{k+v}\right)^2\right)$$

$$\mathfrak{Q} = \frac{n}{2(n-1)}\left(\left(\frac{n}{\beta}+\frac{2}{k'+v'}\right)\left(\frac{i'}{\beta}+\frac{2}{k'-v'}\right)^2+\left(\frac{n}{\beta}-\frac{2}{k'-v'}\right)\left(\frac{i'}{\beta}-\frac{2}{k'+v'}\right)^2\right)$$

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atque

semid. confusionis $= \frac{1}{4} ii'' . \frac{\alpha}{\beta} . \frac{b}{a} x^3 \left(\frac{1}{i'i'} . \frac{\alpha\alpha}{bb} \mathfrak{P} + ii . \frac{\mathfrak{B}\mathfrak{b}}{aa} \mathfrak{Q} \right)$.

5. Pro campo apparente definiendo, qui pendet ab apertura singularum facierum lentium, si ponamus

Semidiametrum aperturae

Pro lente PP $\begin{cases} \text{faciei anterioris} = \mathfrak{A} = x \\ \text{faciei posterioris} = \mathfrak{a} \end{cases}$

Pro lente QQ $\begin{cases} \text{faciei anterioris} = \mathfrak{B} \\ \text{faciei posterioris} = \mathfrak{b} \end{cases}$

habebimus sequentes aequationes

$$\mathfrak{A} = \frac{v}{n\alpha} x$$

$$\mathfrak{B} = \left(\frac{1}{i} . \frac{\alpha + b}{a} - \frac{bv}{n\alpha a} \right) x$$

$$\mathfrak{b} = \left(\frac{i'}{i} . \frac{\alpha + b}{a} - \frac{i'bv}{n\alpha a} - \frac{1}{i} . \frac{\alpha v'}{nab} \right) x$$

ex quibus tribus aequationibus valor ipsius x minimus praebet semidiametrum campi apparentis.

6. Quem campum vt oculus reuera perspiciat, eius locus ita debet sumi, vt sit

$$O = \frac{\frac{i'}{i} \frac{\alpha + b}{a} - \frac{i'bv}{n\alpha a} - \frac{1}{i} \frac{\alpha v'}{nab}}{\frac{i'}{i} \frac{\alpha + b}{a} - \frac{i'bv}{n\alpha a} - \frac{1}{ii'} \frac{\alpha v'}{nab} + \frac{1}{ii'} . \frac{\alpha\alpha}{ab}} . \frac{v}{n}$$

si quidem haec distantia fuerit positiua.

7. Sin autem haec distantia prodierit negatiua, oculum lenti QQ immediate applicari conuenit, vt sit $O = 0$; tum vero pro campo apparente inue-

niendo in aequationibus n°. 5 allatis loco $\mathfrak{C}$ scribatur semidiameter pupillae ω, atque ex iisdem valor ipsius z minimus erutus dabit semidiametrum campi apparentis.

8. Obiectum porro sine margine colorato cernetur, existente distantia O n° 6 inuenta positiua, si huic aequationi satisfiat:

$$0 = 1 + \frac{b}{a} + \frac{i'i'b}{\mathfrak{C}}\left(1 + \frac{b}{a}\right) + \frac{(1-i')^2 b(a+b)}{a v'} + \frac{2-i-i'}{n} - \frac{iv}{na} - \frac{i'v'}{n\mathfrak{C}} - \frac{ibv}{naa} - \frac{ii'i'bbv}{naa\mathfrak{C}} - \frac{i(1-i')^2 bbv}{naav'}$$

9. Sin autem capiatur distantia O$=0$, obiectum margine colorato carebit, si fuerit

$$0 = \frac{1-i}{in} + \frac{1-i'}{i'n} + \frac{v}{ina} + \frac{v'}{inb} + \alpha\left(1 + \frac{\alpha}{ia} + \frac{(1-i)^2\alpha}{i\,i\,v}\right)\left(\frac{v'}{ni'bb} - \frac{1}{a} - \frac{1}{b}\right)$$

10. Omnis autem confusio penitus tolletur, quae quidem a diuersa radiorum refrangibilitate proficiscitur, si huic aequationi satisfacere licuerit $d\mathfrak{C} = 0$, seu huic

$$0 = \frac{aa}{i'bb}\left(\frac{i}{a} + \frac{1}{ia} + \frac{(1-i)^2}{iv}\right) + \frac{i'}{\mathfrak{C}} + \frac{1}{i'b} + \frac{(1-i')^2}{i'v'}.$$

Coroll. 1.

336. Quia ambarum lentium distantia aB necessario est positiua, oportet sit $\alpha + b > 0$. Tum vero aperturae ita debent esse comparatae vt sit $\mathfrak{A} > ix$; $\mathfrak{B} > i.\frac{bx}{a}$ et $\mathfrak{C} > ii'.\frac{bx}{a}$.

Coroll. 2.

337. Casu igitur, quo ob valorem ipsius O prodeuntem negatiuum distantia O$=0$. assumitur, quia

quia tum $b = \omega$ statuitur, etiam esse debet $\omega > ii'\frac{bx}{a}$ seu $x < \frac{a\omega}{ii'b}$. Scilicet hoc casu inutile esset primam aperturam maiorem sumere, quia radii non in oculum ingrederentur.

Scholion.

338. Si simili modo instrumenta dioptrica pluribus lentibus instructa prosequi vellemus, in calculos tantopere intricatos delaberemur, vt ex iis nihil fere concludere liceret. Oritur autem haec calculi complicatio a crassitie lentium, qua neglecta omnia fiunt multo concinniora. Quare si pluribus lentibus vti velimus, crassitiem tam paruam assumamus, vt sine errore pro nihilo haberi possit, id quod in praxi etiam sedulo obseruari solet. Praeterea etiam campi apparentis locique oculi determinatio non capit rigorem geometricum, parumque refert si in ea aliquantum aberretur. Similis quoque ratio est conditionum, quibus effectus a diuersa radiorum refrangibilitate oriundus tollitur; sufficiet enim iis proxime satisfeciſſe, cum perfecta huius confusionis destructione sperari quidem possit. Quocirca in sequentibus, vbi instrumenta pluribus lentibus instructa euoluemus, crassitiem lentium in calculo penitus praetermittamus: in quo negotio ne eadem toties repetere opus habeamus, problema generale praemittamus in quo omnia momenta generatim tantum exponamus eaque deinceps pro quouis lentium numero accurate describamus.

Problema 3.

339. Si inſtrumentum dioptricum compoſitum ſit ex lentibus quotcunque quarum craſſitiem negligere liceat, elementa eius conſtructionem continentia exponere, ex quibus deinceps regulae dirigentes conſtructionem ipſam ſtabiliri poſſint.

Soluțio.

Obiecto in E conſtituto ponantur diſtantiae determinatrices ſingularum lentium vna cum numeris arbitrariis ad ſingulas pertinentibus vt ſequitur.

Pro Lente prima $EA=a$; $aF=\alpha$; num: arb: $=\lambda$

Pro Lente ſecunda $FB=b$; $bG=\mathfrak{G}$; num: arb: $=\lambda'$

Pro Lente tertia $GC=c$; $cH=\gamma$; num: arb: $=\lambda''$

Pro Lente quarta $HD=d$; $dI=\delta$; num: arb: $=\lambda'''$

Pro Lente quinta $IE=e$; $eK=\varepsilon$; num: arb: $=\lambda''''$

Pro Lente ſexta $KF=f$; $fL=\zeta$; num: arb: $=\lambda'''''$

etc.

ex quibus elementis quomodo ſingulae lentes debeant formari ſupra eſt expoſitum.

Nunc autem porro ponatur breuitatis gratia:

$\alpha=Aa$; $\mathfrak{G}=Bb$; $\gamma=Cc$; $\delta=Dd$; $\varepsilon=Ee$; $\zeta=Ff$

etc.

tum

tum vero statuatur etiam

$$\frac{A}{\mathfrak{A}a} = \mathfrak{A}; \quad \frac{B}{\mathfrak{B}b} = \mathfrak{B}; \quad \frac{C}{\mathfrak{C}c} = \mathfrak{C}; \quad \frac{D}{\mathfrak{D}d} = \mathfrak{D}; \quad \frac{E}{\mathfrak{E}e} = \mathfrak{E}; \quad \frac{F}{\mathfrak{F}f} = \mathfrak{F}$$

etc.

ita vt sint distantiae focales lentium:

$$\mathfrak{A}a; \ \mathfrak{B}b; \ \mathfrak{C}c; \ \mathfrak{D}d; \ \mathfrak{E}e; \ \mathfrak{F}f \quad \text{etc.}$$

Nunc sit semidiameter aperturae primae lentis obiectiuae $= x$, pro reliquis vero lentibus ratio aperturarum litteris π, π', π'', π''', π'''' etc. exponatur; ita vt semidiameter aperturae, cuiusque maior accipi debeat quam secundum has rationes: scilicet capi oportebit:

Semid. apert.
Lentis Secundae $> \pi \mathfrak{B}b$
Lentis tertiae $> \pi' \mathfrak{C}c$
Lentis quartae $> \pi'' \mathfrak{D}d$
Lentis quintae $> \pi''' \mathfrak{E}e$
Lentis sextae $> \pi'''' \mathfrak{F}f$ etc.

Quod si iam semidiameter spatii in obiecto conspicui vocetur $= z$, fiatque $\frac{z}{a} = \Phi$, vt sit $z = a\Phi$, atque hinc distantiae determinatrices ita exprimentur, vt sit primo $a = Aa$ tum vero:

$$b = \frac{Aa\Phi}{\mathfrak{B}\pi - \Phi}; \quad \beta = \frac{ABa\Phi}{\mathfrak{B}\pi - \Phi}$$

$$c = \frac{ABa\Phi}{\mathfrak{C}\pi - \pi'\Phi}; \quad \gamma = \frac{ABCa\Phi}{\mathfrak{C}\pi - \pi'\Phi}$$

$$d = \frac{ABCa\Phi}{\mathfrak{D}\pi - \pi' + \pi''\Phi}; \quad \delta = \frac{ABCDa\Phi}{\mathfrak{D}\pi - \pi' + \pi''\Phi}$$

$$e = \frac{ABCDa\Phi}{\mathfrak{E}\pi - \pi'' + \pi' - \pi'''\Phi}; \quad \varepsilon = \frac{ABCDEa\Phi}{\mathfrak{E}\pi - \pi'' + \pi' - \pi'''\Phi}$$

$$f = \frac{ABCDEa\Phi}{\mathfrak{F}\pi - \pi'' + \pi' - \pi'' + \pi'''\Phi}; \quad \zeta = \frac{ABCDEFa\Phi}{\mathfrak{F}\pi - \pi'' + \pi' - \pi'' + \pi'''\Phi} \quad \text{etc.}$$

Circa

Circa has expreſſiones primum notandum eſt, aggregata $\alpha+b$; $\mathfrak{C}+c$; $\gamma+d$; $\delta+e$; $\varepsilon+f$ etc. eſſe oportere poſitiua quippe quibus lentium interualla exprimuntur. Erit nempe

Interuallum
Lentium

I et II $= \frac{AB\alpha\pi}{\mathfrak{B}\pi-\Phi} > 0$

II et III $= \frac{AB\alpha\Phi(\mathfrak{C}\pi'-(1-\mathfrak{B})\pi)}{(\mathfrak{B}\pi-\Phi)(\mathfrak{C}\pi'-\pi+\Phi)} > 0$

III et IV $= \frac{ABC\alpha\Phi(\mathfrak{D}\pi''-(1-\mathfrak{C})\pi')}{(\mathfrak{C}\pi'-\pi+\Phi)(\mathfrak{D}\pi''-\pi'+\pi-\Phi)} > 0$

IV et V $= \frac{ABCD\alpha\Phi(\mathfrak{C}\pi'''-(1-\mathfrak{D})\pi'')}{(\mathfrak{D}\pi''-\pi'+\pi-\Phi)(\mathfrak{C}\pi'''-\pi''+\pi'-\pi+\Phi)} > 0$

V et VI $= \frac{ABCDE\alpha\Phi(\mathfrak{B}\pi''''-(1-\mathfrak{C})\pi''')}{(\mathfrak{C}\pi'''-\pi''+\pi'-\pi+\Phi)(\mathfrak{B}\pi''''-\pi'''+\pi''-\pi'+\pi-\Phi)} > 0$

etc.

Praeterea vt lentes ſequentes omnes radios a prima exceptos transmittant, debet eſſe

Semid: aperturae

Lentis ſecundae $> \frac{\Phi}{\mathfrak{B}\pi-\Phi}x$

Lentis tertiae $> \frac{\Phi}{\mathfrak{C}\pi'-\pi+\Phi}x$

Lentis quartae $> \frac{\Phi}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}x$

Lentis quintae $> \frac{\Phi}{\mathfrak{C}\pi'''-\pi''+\pi'-\pi+\Phi}x$

Lentis ſextae $> \frac{\Phi}{\mathfrak{B}\pi''''-\pi'''+\pi''-\pi'+\pi-\Phi}x$

etc.

Denique

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Denique ex §. 214 posito $\mu = 0,938191$ et $\nu = 0,232692$ erunt litterarum maiuscularum P, Q, R, etc. valores

$$P = \frac{\mu}{A^3 a^3}(A+1)(\lambda(A+1)^2+\nu A)$$

$$Q = \frac{\mu}{B^3 b^3}(B+1)(\lambda'(B+1)^2+\nu B)$$

$$R = \frac{\mu}{C^3 c^3}(C+1)(\lambda''(C+1)^2+\nu C)$$

$$S = \frac{\mu}{D^3 d^3}(D+1)(\lambda'''(D+1)^2+\nu D)$$

$$T = \frac{\mu}{E^3 e^3}(E+1)(\lambda''''(E+1)^2+\nu E)$$

$$V = \frac{\mu}{F^3 f^3}(F+1)(\lambda'''''(F+1)^2+\nu F)$$

etc.

His ita constitutis denominationibus, sit O distantia oculi post lentem vltimam, cuius distantia iusta ponatur $= l$. Deinde statuatur exponens multiplicationis $= m$, relatus ad distantiam b, ita vt diameter obiecti per instrumentum visi m vicibus maior cernatur, quam si idem obiectum a nudo oculo in distantia $= b$ aspiceretur: multiplicationi autem adiungi conuenit situm indicando, vtrum obiectum situ erecto an inuerso sit appariturum. Porro gradus claritatis y denotet semidiametrum coni luminosi, qui a quouis obiecti puncto in oculum transmittitur posito semidiametro pupillae $= \omega$. Semidiametrum denique confusionis voco semidiametrum apparentem circulorum qui in nudo oculo aeque magni depinguntur, atque singula obiecti puncta per instrumentum in oculo.

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Coroll. 1.

340. Quia pro apertura singularum lentium geminos limites inuenimus, semidiameter aperturae cuiusque conuenientissime aggregato amborum limitum aequalis assumitur, vel saltem non minor. Vterque autem limes etsi forte alter prodeat negatiuus, affirmatiue accipi debet.

Coroll. 2.

341. Hinc ergo sequentes consequimur formulas pro singularum lentium aperturis.

Semidiam: apert:

Lentis secundae $= \frac{AB\mathfrak{a}\pi + x}{\mathfrak{B}\pi - \Phi} \cdot \Phi$

Lentis tertiae $= \frac{AB\mathfrak{C}a\pi' + x}{\mathfrak{C}\pi' - \pi + \Phi} \cdot \Phi$

Lentis quartae $= \frac{ABC\mathfrak{D}a\pi'' + x}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi} \cdot \Phi$

Lentis quintae $= \frac{ABCD\mathfrak{E}a\pi''' + x}{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi} \cdot \Phi$

Lentis sextae $= \frac{ABCDE\mathfrak{F}a\pi'''' + x}{\mathfrak{F}\pi'''' - \pi''' + \pi'' - \pi' + \pi - \Phi} \cdot \Phi$

etc.

Coroll. 3.

342. Ex distantiis binis determinatricibus cum numero arbitrario quaelibet lens facile construitur; id quod pro lente prima ex superioribus repetamus. Nempe si sit $\lambda > 1$ lens simplex satisfaciet cuius constructio posito breuitatis gratia:

$\varrho = 0,190781$; $\sigma = 1,627401$ et $\tau = 0,905130$

ita

ita se habet:

anterioris posterioris

radius faciei .. $\frac{\alpha\alpha}{\varrho\alpha+\sigma a\pm\tau(a+\alpha)\sqrt{(\lambda-1)}}$; $\frac{\alpha\alpha}{\varrho\alpha+\sigma a\mp\tau(a+\alpha)\sqrt{(\lambda-1)}}$

seu numeris substitutis si sit $\lambda = 1+v$

radius faciei $\begin{cases} \text{anterioris} = \frac{\alpha\alpha}{+0,190781\alpha+1,627401a\pm0,905133(a+\alpha)\sqrt{v}} \\ \text{posterioris} = \frac{\alpha\alpha}{+1,627401\alpha+0,190781a\mp0,905133(a+\alpha)\sqrt{v}} \end{cases}$

Coroll. 4.

343. Sin autem sit $\lambda < 1$, sed tamen $> 0,191827$ lens est duplicanda seu ex duabus simplicibus componenda, quarum constructio, si ponatur $\lambda = 0,191827+v$ ita se habet

Pro Lente radius faciei

priori $\begin{cases} \text{anterioris} = \frac{\alpha\alpha}{-0,622915\alpha+0,813700a\pm0,905133(a+\alpha)\sqrt{v}} \\ \text{posterioris} = \frac{\alpha\alpha}{+1,532010\alpha+0,055390a\mp0,905133(a+\alpha)\sqrt{v}} \end{cases}$

posteriori $\begin{cases} \text{anterioris} = \frac{\alpha\alpha}{+0,055390\alpha+1,532010a\pm0,905133(a+\alpha)\sqrt{v}} \\ \text{posterioris} = \frac{\alpha\alpha}{+0,813700\alpha-0,622519a\mp0,905133(a+\alpha)\sqrt{v}} \end{cases}$

Coroll. 5.

344. At si sit $\lambda = 0,042165+v$, lente vtendum est triplicata ita construenda:

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Pro lente radius

Priori { anterioris $= \frac{aa}{-0,854153a+0,542467a\pm 0,905133(a+x)\sqrt{v}}$
posterioris $= \frac{aa}{+1,500214a+0,063594a\pm 0,905133(a+\alpha)\sqrt{v}}$

Media { anterioris $= \frac{aa}{-0,415280a+1,021340a\pm 0,905133(a+x)\sqrt{v}}$
posterioris $= \frac{aa}{+1,021340a-0,415280a\pm 0,905133(a+\alpha)\sqrt{v}}$

Posteriori { anterioris $= \frac{aa}{+0,063594a+1,500214a\pm 0,905133(a+\alpha)\sqrt{v}}$
posterioris $= \frac{aa}{+0,542467a-0,854153a\pm 0,905133(a+\alpha)\sqrt{v}}$

Coroll. 6.

345. Si denique fit $\lambda = -0,010216+v$ lens facienda est quadruplicata ita construenda:

Pro lente radius faciei

Prima { anterioris $= \frac{aa}{-1,029770a+0,406850a\pm 0,905133(a+x)\sqrt{v}}$
posterioris $= \frac{aa}{+1,484315a+0,047655a\pm 0,905133(a+x)\sqrt{v}}$

Secunda { anterioris $= \frac{aa}{-0,670615a+0,766005a\pm 0,905133(a+\alpha)\sqrt{v}}$
posterioris $= \frac{aa}{+1,125160a-0,311460a\pm 0,905133(a+x)\sqrt{v}}$

Tertia { anterioris $= \frac{aa}{-0,311460a+1,125160a\pm 0,905133(a+x)\sqrt{v}}$
posterioris $= \frac{aa}{+0,766005a-0,670615a\pm 0,905133(a+x)\sqrt{v}}$

Quarta { anterioris $= \frac{aa}{+0,047655a+1,484315a\pm 0,905133(a+x)\sqrt{v}}$
posterioris $= \frac{aa}{+0,406850a-1,029770a\pm 0,905133(a+x)\sqrt{v}}$

Scholion

Scholion.

346. Quae in his corollariis de conſtructione lentis primae ſunt allata, mutatis litteris a et α ad reliquas lentes facile accommodari manifeſtum eſt quarum loco, ſi vſus requirat, vt numeri λ', λ'' etc. ſint vnitate minores, etiam lentes ſiue duplicatae ſiue triplicatae ſiue adeo quadruplicatae adhiberi debebunt. Ceterum hae formulae ad lentes quotcunque patent ita vt propoſito lentium numero quocunque tantum literae, quae ad lentes ſequentes pertinerent, ſint omittendae. In his autem denominationibus ſtatim introduximus campum apparentem numero Φ contentum, cum ſit ſemidiameter ſpatii in obiecto conſpicui $z = a\Phi$. Interim tamen campum apparentem non ad lubitum augere licet, ſiquidem is multiplicationis ratione et numeris π, π', π'' etc. determinatur. Hi autem numeri ſemper infra $\frac{1}{3}$ imo $\frac{1}{2}$ ſubſiſtunt, ac plerumque $\frac{1}{4}$ vel adeo $\frac{1}{5}$ ſuperare nequeunt; quandoque etiam minores accipi debent, id quod formulae coroll. 2. quouis caſu declarabunt, quibus ſemidiameter aperturae cuiusque lentis exprimitur. Quare retentis his denominationibus momenta conſtructionis pro quouis lentium numero determinato expendamus.

Problema 4.

347. Si inſtrumentum dioptricum duabus conſtet lentibus, quarum craſſitiem negligere liceat, definire momenta quibus conſtructio continetur.

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Solutio.

Cum hic lens secunda sit vltima, momenta constructionis ita se habebunt.

1. Vt oculus imaginem in distantia iusta conspiciat, debet esse $O = \mathfrak{E} + l$ ideoque

$O = l + \frac{AB\alpha\Phi}{\mathfrak{B}\pi - \Phi}$. et ex conditione campi apparentis

$O = \frac{\mathfrak{B}b\pi}{\pi - \Phi}$.

2. Exponens multiplicationis m ad distantiam b relatus praebet hanc aequationem

$m = AB\frac{b}{l}$ pro situ erecto

sin autem situs inuersus desideretur, numerus m negatiue est capiendus.

3. Pro gradu claritatis semper est $y = \frac{bx}{ma}$.

4. Pro campo apparente habebitur haec aequatio:

$\pi - \Phi = \frac{-ma\Phi}{b}$ vnde fit $\Phi = \frac{-\pi b}{ma - b}$.

5. Si distantia O hinc prodeat negatiua, capi debet $O = o$ seu $\frac{AB\alpha\Phi}{\mathfrak{B}\pi - \Phi} = -l$ tum vero π eiusmodi valorem induit, vt apertura lentis ocularis non superet aperturam pupillae; fiet nempe $\pi\mathfrak{B}b = \omega$, hincque

$\Phi = \frac{\mathfrak{B}\pi\omega}{AB\mathfrak{B}2\pi + \omega}$. vel $(B+1)\omega = -\pi l$.

6. Semidiameter confusionis sequenti formula exprimetur.

$\frac{Bbb}{4Aal}x^3\left(\frac{AAaa}{bb}P + \frac{bb}{AAaa}Q\right)$

quae

quae factis ſubſtitutionibus abit in hanc:

$$\frac{\mu m x^3}{4aab}\left(\frac{(A+1)(\lambda(A+1)^2+\nu A)}{A^3}+\frac{\Phi(B+1)(\lambda'(B+1)^2+\nu B)}{A^3B^3(\mathfrak{B}\pi-\Phi)}\right)$$

7. Vt obiectum ſine margine colorato appareat, ſi diſtantia O fuerit poſitiua, neceſſe eſt ſit:

$$\frac{\pi}{\mathfrak{B}\pi-\Phi}=0$$

8. Sin autem capiatur $O=o$, fieri debet $\frac{(A+1)\mathfrak{B}\pi}{\Phi}=0$ ſeu $\frac{\pi}{\mathfrak{a}\Phi}=0$.

9. Omnis autem confuſio a diuerſa radiorum refrangibilitate oriunda penitus tolletur ſi inſuper fuerit

$$\frac{1}{\mathfrak{A}}+\frac{\Phi}{\mathfrak{AB}(\mathfrak{B}\pi-\Phi)}=0$$

10. Praeterea autem erit diſtantia lentium

$\frac{A\mathfrak{B}a\pi}{\mathfrak{B}\pi-\Phi}$ quae debet eſſe poſitiua.

11. Denique Semidiameter aperturae lentis ocularis eſt;

$$\frac{A\mathfrak{B}a\pi\pm x}{\mathfrak{B}\pi-\Phi}.\Phi$$

vbi ſignorum $\pm$ id capi debet, quod valorem praebet maximum ſiue poſitiuum ſiue negatiuum.

Coroll. 1.

348. Cum ſit (n°. 4) $\Phi=\frac{-\pi b}{ma-b}$, et ob multiplicationem $B=\frac{ml}{Ab}$, hincque $\mathfrak{B}=\frac{ml}{ml+Ab}$; habebitur pro loco oculi $O=\frac{-Abl(ma-b)}{mmal+Abb}$.

Tum

Tum vero est $\mathfrak{B}\pi - \Phi = \frac{ml\pi}{ml+Ab} + \frac{\pi b}{ma-b} = \frac{\pi(mmal+Abb)}{(ma-b)(ml+Ab)}$, vnde fit lentium distantia $\frac{A\mathfrak{B}a\pi}{\mathfrak{B}\pi-\Phi} = \frac{mAa(ma-b)}{mmal+Abb}l$.

Coroll. 2.

349. Cum sit $\frac{A}{A+} = \mathfrak{A}$ et $A = \frac{\mathfrak{A}}{1-\mathfrak{A}}$ semidiameter confusionis etiam ita exprimi potest vt sit

$$\frac{\mu m x^3}{4aab}\left(\frac{\lambda+\nu\mathfrak{A}(1-\mathfrak{A})}{\mathfrak{A}^3} + \frac{\Phi(\lambda'+\nu\mathfrak{B}(1-\mathfrak{B}))}{\Delta\cdot\mathfrak{B}(\mathfrak{B}\pi-\Phi)}\right).$$

Coroll. 3.

350. Cum sit lentium distantia $= \frac{A\mathfrak{B}a\pi}{\mathfrak{B}\pi-\Phi}$, obiectum sine coloribus cerni nequit, nisi lentium distantia euanescat; siquidem oculus in eo loco, quem campus apparens postulat, teneatur. Et quoniam huic conditioni satisfieri nequit, multo minus tota confusio ex n°. 9 tolli poterit.

Scholion.

351. Hic statim ab instrumentis duabus lentibus instructis incepi, quoniam lens vnica facillime expeditur. Primum enim necesse est vt sit $O = l+\alpha = l+Aa$. Tum vero multiplicatio praebet $m = \frac{b}{a}$ pro situ erecto, et campus apparens non terminatur sumendo $Aa+l=0$, seu $O=0$, ita vt oculus lenti immediate debeat applicari. Semidiameter vero confusionis erit

$= \frac{\alpha}{4l}x^3 . P = \frac{Aax^3}{4l} . P = -\frac{1}{4}x^3 P = \frac{-max^3}{4b} . P$ ob $l = -Aa$ et $ma = b$

Quare semidiameter confusionis erit

$$\frac{\mu m x^3}{4aab}\left(\frac{(A+1)(\lambda(A+1)^2+\nu A)}{A^3}\right) = \frac{\mu m x^3}{4aab} \cdot \frac{\lambda+\nu\mathfrak{A}(1-\mathfrak{A})}{\mathfrak{A}^3}.$$

Pro

Pro gradu claritatis autem habetur vt ſemper $y = \frac{bx}{ma}$.

Obiectum porro hoc caſu ſine margine colorato cernetur, tota autem confuſio tolli nequit, niſi ſit $\frac{1}{a} = 0$ ſeu $A = -1$, id quod ſcopo lentium repugnat. Quare ad conſiderationem plurium lentium progrediamur.

Problema 5.

352. Si inſtrumentum dioptricum tribus inſtructum ſit lentibus, quarum craſſities tam ſit parva, vt negligi queat, definire cuncta momenta ad conſtructionem dirigendam neceſſaria.

Solutio.

Hic igitur lens tertia erit vltima ſeu ocularis, ideoque momenta ſequenti modo ſe habebunt:

1. Vt oculo imago in diſtantia iuſta ſpectanda offeratur debet eſſe $O = \gamma + l$, ac loco γ valore ſubſtituto

$O = l + \frac{ABC\,a\,\Phi}{\mathfrak{C}\pi' - \pi + \Phi}$ et ex conditione campi apparentis

$O = \frac{\mathfrak{C}\,c\,\pi'}{\pi' - \pi + \Phi}$.

2. Exponens multiplicationis m ad diſtantiam b relatus praebet hanc aequationem:

$m = -\frac{ABCb}{l}$ pro ſitu erecto.

vnde ſi ſitus inuerſus deſideretur, numerum m negatiue accipi conuenit.

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3. Pro gradu claritatis habemus, vt semper, $y = \frac{b\,x}{m\,a}$.

4. Pro campo autem apparente definiendo habemus hanc aequationem

$$\Phi = \frac{-\pi + \pi'}{m\,a - b}\,.\,b.$$

5. At si distantia O hinc prodeat negatiua, vt capi oporteat $O = o$, ideoque $\frac{A\,B\,C\,a\,\Phi}{\mathfrak{C}\pi' - \pi + \Phi} = -l$ campus apparens definiri debet ex hac aequatione $\pi' \mathfrak{C} c = \omega$ seu $\frac{A\,B\,\mathfrak{C}\,a\,\Phi\,\pi'}{\mathfrak{C}\pi' - \pi + \Phi} = \omega$ vel $(C+1)\omega = -\pi' l$.

6. Semidiameter confusionis vero ita exprimetur:

$$\frac{C\,c\,c'}{4\,A\,B\,a\,l}\,x^3\left(\frac{A\,A\,B\,B\,a\,a}{c\,c}\,P + \frac{B\,B\,b^4}{A\,A\,a\,a\,c\,c}\,Q + \frac{c\,c}{A\,A\,B\,B\,a\,a}\,R\right)$$

quae ob $\frac{1}{l} = \frac{-m}{A\,B\,C\,b}$ abit signo mutato in hanc:

$$\frac{\mu\,m\,x^3}{4\,a\,a\,b}\left\{\begin{array}{l} +\frac{(A+1)(\lambda(A+1)^2+\nu A)}{A^3} \\ +\frac{\Phi(B+1)(\lambda'(B+1)^2+\nu B)}{A^3 B^3(\mathfrak{D}\pi - \Phi)} \\ +\frac{\Phi(C+1)(\lambda''(C+1)^2+\nu C)}{A^3 B^3 C^3(\mathfrak{C}\pi' - \pi + \Phi)}. \end{array}\right.$$

7. Vt obiectum sine margine colorato appareat, si quidem distantia O prodierit positiua, esse oportet

$$\frac{\pi}{\mathfrak{D}\pi - \Phi} + \frac{\pi'}{\mathfrak{C}\pi' - \pi + \Phi} = o$$

8. Sin autem ob istam distantiam prodeuntem negatiuam capiatur $O = o$, margo coloratus euanescet faciendo:

$$\frac{\pi'}{\mathfrak{D}\Phi} + \frac{\pi'}{A\mathfrak{D}(\mathfrak{D}\pi - \Phi)} = \frac{\pi}{A\,B\,\mathfrak{C}(\mathfrak{D}\pi - \Phi)}$$

9. Quod

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9. Quod si marginem coloratum tollere liceat, praeterea visio ab omni confusione liberabitur, si huic aequationi satisfiat.

$$\frac{1}{\mathfrak{B}} + \frac{\Phi}{AB\mathfrak{B}(\mathfrak{B}\pi - \Phi)} + \frac{\Phi}{AB\mathfrak{C}(\mathfrak{C}\pi' - \pi + \Phi)} = 0.$$

10. Denique effici debet, vt distantia lentium fiat positiua vnde habebitur:

Interuallum
Lentium

I et II $= \frac{A\mathfrak{B}a\Phi}{\mathfrak{B}\pi - \Phi} > 0$

II et III $= \frac{AB a\Phi(\mathfrak{C}\pi' - (1-\mathfrak{B})\pi)}{(\mathfrak{B}\pi - \Phi)(\mathfrak{C}\pi' - \pi + \Phi)} > 0$

11. Tandem numeros π et π' ita accipi oportet, vt aperturae lentium non fiant nimis magnae: est vero

Semidiameter
aperturae

Lentis secundae $= \frac{A\mathfrak{B}a\pi \pm x}{\mathfrak{B}\pi - \Phi} \cdot \Phi$

Lentis tertiae $= \frac{AB\mathfrak{C}a\pi' \pm x}{\mathfrak{C}\pi' - \pi + \Phi} \Phi.$

Coroll. 1.

353. Cum sit $C = \frac{-ml}{ABb}$, ideoque $\mathfrak{C} = \frac{-ml}{ABb - ml}$; et $\Phi = \frac{-\pi + \pi'}{ma - b} \cdot b$, si hi valores substituantur erit pro loco oculi.

$$O = \frac{ABbl(ma-b)\pi'}{(mmal - ABbb)\pi' + ma(ABb - ml)\pi}.$$

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Coroll. 2.

354. Semidiameter confuſionis etiam ita exprimi poteſt vt ſit

$$\frac{\mu m x^3}{4aab}\left(\frac{\lambda+\nu\mathfrak{A}(1-\mathfrak{A})}{\mathfrak{A}^3}+\frac{\Phi(\lambda'+\nu\mathfrak{B}(1-\mathfrak{B}))}{A^3\mathfrak{B}^3(\mathfrak{B}\pi-\Phi)}+\frac{\Phi(\lambda''+\nu\mathfrak{C}(1-\mathfrak{C}))}{A^3B^3\mathfrak{C}\cdot(\mathfrak{C}\pi'-\pi+\Phi)}\right)$$

cuius formae cohaerentia cum praecedentibus iam ita eſt manifeſta, vt ad plures lentes facile extendi queat.

Problema 6.

355. Si inſtrumentum dioptricum quatuor lentibus ſit inſtructum, quarum craſſities in computum duci non mereatur, definire omnia momenta ad conſtructionem dirigendam neceſſaria.

Solutio.

Quia hic lens quarta eſt vltima.

1. Vt oculo imago poſtrema in diſtantia iuſta offeratur debet eſſe $O=\delta+l$, ideoque

$O=l+\frac{ABCD\alpha\Phi}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}$ et ex conditione campi apparentis

$O=\frac{\mathfrak{D}d\pi''}{\pi''-\pi'+\pi-\Phi}$.

2. Exponens multiplicationis m ad diſtantiam b relatus, ſi obiectum vt hactenus ſitu erecto exhiberi aſſumatur, erit

$m=+ABCD.\frac{b}{l}$ pro ſitu erecto.

ſin autem ſitus deſideretur inuerſus numerus m negatiue eſt accipiendus.

3. Pro

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3. Pro gradu claritatis perpetuo habetur $y = \frac{bx}{ma}$.

4. Campus autem apparens definiri debet ex hac aequatione

$$\Phi = \frac{-\pi + \pi' - \pi''}{ma - b} b$$

hinc enim $a\Phi$ praebebit semidiametrum spatii in obiecto conspicui, siquidem distantia O fuerit positiua.

5. Sin autem distantia O prodeat negatiua quo casu capi oportet $O = o$, seu $\frac{ABCDa\Phi}{D\pi' - \pi' + \pi - \Phi} = -l$ quantitas Φ determinari debet ex hac aequatione $\pi'' D d = \omega$ seu hac

$$\frac{ABCDa\pi''\Phi}{D\pi'' - \pi' + \pi - \Phi} = \omega \text{ siue } (D+1)\omega = -\pi'' l.$$

6. Semidiameter confusionis ita reperietur expressus:

$$\frac{\mu : m x^3}{4aab} \left\{ \begin{array}{l} + \frac{(A+1)(\lambda(A+1)^2 + \nu A)}{A^3} \\ + \frac{\Phi(B+1)(\lambda'(B+1)^2 + \nu B)}{A^3 B^3 (\mathfrak{B}\pi - \Phi)} \\ + \frac{\Phi(C+1)(\lambda''(C+1)^2 + \nu C)}{A^3 B^3 C^3 (\mathfrak{C}\pi' - \pi + \Phi)} \\ + \frac{\Phi(D+1)(\lambda'''(D+1)^2 + \nu D)}{A^3 B^3 C^3 D^3 (\mathfrak{D}\pi'' - \pi' + \pi - \Phi)} \end{array} \right.$$

7. Vt obiectum sine margine colorato appareat, siquidem distantia O sit positiua esse oportet:

$$\frac{\pi}{\mathfrak{B}\pi - \Phi} + \frac{\pi'}{\mathfrak{C}\pi' - \pi + \Phi} + \frac{\pi''}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi} = o.$$

8. Verum si acceperimus $O = o$, margo coloratus euanescet, huic aequationi satisfaciendo.

$$\frac{\pi''}{\mathfrak{A}\Phi} + \frac{\pi''}{A\mathfrak{B}(\mathfrak{B}\pi - \Phi)} + \frac{\pi''}{ABC(\mathfrak{C}\pi' - \pi + \Phi)} = \frac{\pi}{ABC\mathfrak{D}(\mathfrak{B}\pi - \Phi)} + \frac{\pi'}{ABC\mathfrak{D}(\mathfrak{C}\pi' - \pi + \Phi)}.$$

9. Omnis autem confusio a diuersa radiorum natura oriunda penitus tolletur, si satisfieri liceat huic aequationi:

$$\frac{1}{k}+\frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\Phi}{AB\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}+\frac{\Phi}{ABC\mathfrak{D}(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}=0.$$

10. Denique efficiendum est vt distantiae lentium sint positiuae vnde oriuntur formulae sequentes:

Interuallum
Lentium

I et II $=\frac{A\mathfrak{B}a\pi}{\mathfrak{B}\pi-\Phi}>0$

II et III $=\frac{AB\mathfrak{C}a\Phi(\mathfrak{C}\pi'-(1-\mathfrak{B})\pi)}{(\mathfrak{B}\pi-\Phi)(\mathfrak{C}\pi'-\pi+\Phi)}>0$

III et IV $=\frac{ABC\mathfrak{D}a\Phi(\mathfrak{D}\pi''-(1-\mathfrak{C})\pi')}{(\mathfrak{C}\pi'-\pi+\Phi)(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}>0.$

11. Aperturae autem singularum lentium ita sunt capiendae vt sit

Semid: apert:

Lentis secundae $=\frac{A\mathfrak{B}a\pi+x}{\mathfrak{B}\pi-\Phi}.\ \Phi$

Lentis tertiae $=\frac{AB\mathfrak{C}a\pi'+x}{\mathfrak{C}\pi'-\pi+\Phi}.\ \Phi$

Lentis quartae $=\frac{ABC\mathfrak{D}a\pi''+x}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}.\ \Phi.$

Coroll. 1.

356. Cum sit ob multiplicationem $\mathfrak{D}=\frac{\mathfrak{m}l}{ABCb}$ ideoque $\mathfrak{D}=\frac{\mathfrak{m}l}{ABCb+ml}$ et $\Phi=\frac{-\pi+\pi'-\pi''}{ma-b}.b$; pro loco oculi habebitur haec expressio:

$$0=\frac{-ABCbl(ma-\mathfrak{B})\pi''}{(\mathfrak{m}mal+ABCbb)\pi''-ma(ABCb+ml)(\pi'-\pi)}$$

Coroll.

Coroll. 2.

357. Quia hic tot occurrunt litterae determinandae, dubium nullum esse videtur, quin formulis sub n°. 7 vel 8 et 9 contentis satisfieri queat, et certum saltem est, id fieri posse, si plura adhibeantur mediorum refringentium genera.

Problema 7.

358. Si instrumentum dioptricum quinque lentibus sit instructum, quarum crassities ob paruitatem negligi queat, definire omnia momenta constructionem dirigentia.

Solutio.

Cum hic lens quinta vltimum locum teneat, habebimus.

1. Pro loco oculi, vt imago visa ipsi in iusta distantia offeratur debet esse $O = \varepsilon + l$ ideoque

$$O = l + \frac{ABCDE\varepsilon\Phi}{\varepsilon\pi''' - \pi'' + \pi' - \pi + \Phi}$$ et ex conditione campi apparentis

$$O = \frac{\varepsilon\varepsilon\pi'''}{\pi''' - \pi'' + \pi' - \pi + \Phi}.$$

2. Exponens multiplicationis m ad distantiam $= b$ relatus, si quidem obiectum situ erecto sit spectandum

$$m = -ABCDE.\frac{b}{l}.$$

pro situ autem inuerso numerus m negatiue capi debet.

3. Pro

3. Pro gradu claritatis est vt hactenus $y = \frac{bx}{ma}$.

4. Campus apparens definiri debet ex hac aequatione

$$\Phi = \frac{-\pi + \pi' - \pi'' + \pi'''}{ma - b} \cdot b$$

siquidem distantia illa O prodierit positiua.

5. Sin autem haec distantia prodeat negatiua vt capiatur $O = o$, seu $\frac{ABCDE\alpha\Phi}{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi} = -l$, campus apparens non ex formula n°. 4 sed ex hac $\pi''' \mathfrak{E} e = \omega$ definiri debet, vnde fit

$$\frac{ABCD\mathfrak{E}\alpha\pi'''\Phi}{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi} = \omega \text{ seu } (E+1)\omega = -\pi'''l$$

6. Semidiameter confusionis ab apertura lentium oriundae sequenti modo exprimitur,

$$\frac{\mu m x^{\alpha}}{4aab} \left\{ \begin{array}{l} + \frac{(A+1)(\lambda(A+1)^2 + \nu A)}{A^3} \\ + \frac{\Phi(B+1)(\lambda'(B+1)^2 + \nu B)}{A^6 B^3 (\mathfrak{D}\pi - \Phi)} \\ + \frac{\Phi(C+1)(\lambda''(C+1)^2 + \nu C)}{A^3 B^3 C^3 (\mathfrak{E}\pi' - \pi + \Phi)} \\ + \frac{\Phi(D+1)(\lambda'''(D+1)^2 + \nu D)}{A^3 B^3 C^3 D^3 (\mathfrak{D}\pi'' - \pi' + \pi - \Phi)} \\ + \frac{\Phi(E+1)(\lambda''''(E+1)^2 + \nu E)}{A^3 B^3 C^3 D^3 E^3 (\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi)} \end{array} \right.$$

7. Vt obiectum sine margine colorato appareat, siquidem distantia O fuerit positiua, huic aequationi erit satisfaciendum:

$$\frac{\pi}{\mathfrak{D}\pi - \Phi} + \frac{\pi'}{\mathfrak{E}\pi' - \pi + \Phi} + \frac{\pi''}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi} + \frac{\pi'''}{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi} = o.$$

8. At

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8. At si distantia O sit negativa, capiaturque $O = o$ pro hoc scopo obtinendo oportet sit

$$\frac{\pi'''}{\mathfrak{A}\Phi} + \frac{\pi'''}{\mathfrak{A}\mathfrak{B}(\mathfrak{B}\pi - \Phi)} + \frac{\pi'''}{\mathfrak{A}\mathfrak{B}\mathfrak{C}(\mathfrak{C}\pi' - \pi + \Phi)} + \frac{\pi'''}{\mathfrak{A}\mathfrak{B}\mathfrak{C}\mathfrak{D}(\mathfrak{D}\pi'' - \pi' + \pi - \Phi)}$$
$$= \frac{1}{\mathfrak{A}\mathfrak{B}\mathfrak{C}\mathfrak{D}\mathfrak{E}}\left(\frac{\pi}{\mathfrak{B}\pi - \Phi} + \frac{\pi'}{\mathfrak{C}\pi' - \pi + \Phi} + \frac{\pi''}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi}\right).$$

9. Omnis autem confusio a diuersa radiorum natura orlunda penitus tolletur, si fieri possit

$$\frac{1}{\mathfrak{A}} + \frac{\Phi}{\mathfrak{A}\mathfrak{B}(\mathfrak{B}\pi - \Phi)} + \frac{\Phi}{\mathfrak{A}\mathfrak{B}\mathfrak{C}(\mathfrak{C}\pi' - \pi + \Phi)} + \frac{\Phi}{\mathfrak{A}\mathfrak{B}\mathfrak{C}\mathfrak{D}(\mathfrak{D}\pi'' - \pi' + \pi - \Phi)} + \frac{\Phi}{\mathfrak{A}\mathfrak{B}\mathfrak{C}\mathfrak{D}\mathfrak{E}(\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi)} = 0$$

10. Tum vero efficiendum est, vt intervalla lentium omnia fiant positiva, vnde oriuntur formulae sequentes.

Interuallum
Lentium

I. et II. $= \frac{\mathfrak{A}\mathfrak{B}\alpha\pi}{\mathfrak{B}\pi - \Phi} > 0$

II. et III. $= \frac{\mathfrak{A}\mathfrak{B}\alpha\Phi(\mathfrak{C}\pi' - (1 - \mathfrak{B})\pi)}{(\mathfrak{B}\pi - \Phi)(\mathfrak{C}\pi' - \pi + \Phi)} > 0$

III. et IV. $= \frac{\mathfrak{A}\mathfrak{B}\mathfrak{C}\alpha\Phi(\mathfrak{D}\pi'' - (1 - \mathfrak{C})\pi')}{(\mathfrak{C}\pi' - \pi + \Phi)(\mathfrak{D}\pi'' - \pi' + \pi - \Phi)} > 0$

IV. et V. $= \frac{\mathfrak{A}\mathfrak{B}\mathfrak{C}\mathfrak{D}\alpha\Phi(\mathfrak{E}\pi''' - (1 - \mathfrak{D})\pi'')}{(\mathfrak{D}\pi'' - \pi' + \pi - \Phi)(\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi)} > 0$

11. Aperturae denique singularum lentium ita sunt capiendae, vt sit:

Semid. apert:

Lentis secundae $= \frac{\mathfrak{A}\mathfrak{B}\alpha\pi + x}{\mathfrak{B}\pi - \Phi}\Phi$

Lentis tertiae $= \frac{\mathfrak{A}\mathfrak{B}\mathfrak{C}\alpha\pi' + x}{\mathfrak{C}\pi' - \pi + \Phi}\Phi$

Lentis quartae $= \frac{\mathfrak{A}\mathfrak{B}\mathfrak{C}\mathfrak{D}\alpha\pi'' + x}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi}\Phi$

Lentis quintae $= \frac{\mathfrak{A}\mathfrak{B}\mathfrak{C}\mathfrak{D}\mathfrak{E}\alpha\pi''' + x}{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi}\Phi.$

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Corollarium.

359. Ob multiplicationem ergo erit:

$$E = \frac{-ml}{ABCDb} \text{ et } \mathfrak{E} = \frac{-ml}{ABCDb - mL}$$

vnde oritur:

$$O = \frac{ABCDbl(ma-b)\pi'''}{(mmal - ABCDbb)\pi''' + ma(ABCDb - ml)(\pi'' - \pi' + \pi)}.$$

Scholion.

360. Hactenus ſuppoſui oculum vel in eo loco teneri, quem viſio campi apparentis poſtulat, vel lenti poſtremae immediate applicari; quorum vtroque caſu efficiendum eſt, vt imago vltima oculo ad diſtantiam iuſtam offeratur. Hic quidem locum oculi, quem campus praebet ſtatim coniunxi cum iuſta imaginis ſpectandae diſtantia; ſed etiam ſine reſpectu ſiue ad hanc diſtantiam ſiue ad multiplicationem habito definiri poteſt. Ex ſuperioribus enim §. 271 habemus

Pro caſu

vnius lentis $O = o$

duarum lentium $O = \frac{A\mathfrak{B}a\pi\Phi}{(\pi - \Phi)(\mathfrak{B}\pi - \Phi)}$:

trium lentium $O = \frac{AB\mathfrak{C}a\pi'\Phi}{(\pi' - \pi + \Phi)(\mathfrak{C}\pi' - \pi + \Phi)}$

quatuor lentium $O = \frac{ABC\mathfrak{D}a\pi''\Phi}{(\pi'' - \pi' + \pi - \Phi)(\mathfrak{D}\pi'' - \pi' + \pi - \Phi)}$

quinque lentium $O = \frac{ABCD\mathfrak{E}.a\pi'''\Phi}{(\pi''' - \pi'' + \pi' - \pi + \Phi)(\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi)}$

etc.

Verum

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Verum si nolimus ad campum apparentem respicere semper eiusmodi locum oculo assignare licet, vnde obiectum sine margine colorato cernatur. Haud abs re autem fore arbitror hunc locum determinasse, quia ex eius distantia a loco oculi ob rationem campi assumto facilius iudicare licebit, quanto margine cinctum apparere debeat obiectum si oculus in loco isto posteriori teneatur. Si enim aequationi n°. 7 appositae satisfieri nequeat, nil aliud intelligimus, nisi obiectum non sine margine colorato esse appariturum at si insuper constet ille locus, vnde obiectum sine huiusmodi margine spectari posset, facilius quantitatem istius marginis colligere poterimus.

Problema 8.

361. Si instrumentum dioptricum ex lentibus quotcunque constet, quarum crassitiem negligere liceat, eum pro oculo locum assignare, vnde obiectum sine margine colorato conspiciatur.

Solutio.

Ponatur haec oculi post lentem vltimam distantia $= \Omega$, atque ex aequationibus §. 325 datis, si pro O scribatur. Ω haec ipsa distantia Ω, quam quaerimus, elici potest. Scilicet pro quouis lentium numero erit vt sequitur.

CAPVT VII.

I. Pro unica Lente.

Habetur haec aequatio $\frac{\Omega\, d\alpha}{\alpha(\alpha-\Omega)}=0$ vnde fit $\frac{\Omega}{\alpha(\alpha-\Omega)}=0$, seu $\frac{\Omega}{\alpha-\Omega}=0$. Verum etiam campus apparens exigit $\Omega=0$, vnde hoc casu ambo loca oculi O et Ω conveniunt, neque margo coloratus est pertimescendus.

II. Pro duabus Lentibus

Hoc casu reperta est ista aequatio

$$\frac{\Omega\, d\epsilon}{\epsilon(\epsilon-\Omega)}=d\alpha\left(\frac{1}{\alpha}+\frac{1}{b}\right)$$

vnde substitutis valoribus supra assignatis oritur

$$\frac{\Omega}{\epsilon(\epsilon-\Omega)}=\frac{\pi}{\alpha\Phi}:(B+1)AB\alpha\left(\frac{1}{\alpha}+\frac{\Phi}{A\alpha(\vartheta\pi-\phi)}\right)$$

seu si ponamus

$$Y=\frac{\pi}{\alpha\Phi} \text{ et } Z=\frac{1}{\alpha}+\frac{\Phi}{A\alpha(\vartheta\pi-\phi)}$$

erit

$$\frac{\Omega}{\epsilon-\Omega}=\frac{Y}{AB(B+1)\alpha Z}\cdot\epsilon=\frac{Y}{(B+1)Z}\cdot\frac{\Phi}{\vartheta\pi-\phi}$$

III. Pro tribus Lentibus.

Eodem modo si ponamus

$$Y=\frac{\pi}{\alpha\Phi}+\frac{\pi'}{AB(\vartheta\pi-\phi)}-\frac{\pi}{AB\epsilon(\vartheta\pi-\phi)} \quad \text{et}$$

$$Z=\frac{1}{\alpha}+\frac{\Phi}{AB(\vartheta\pi-\phi)}+\frac{\Phi'}{AB\epsilon(\vartheta\pi'-\pi+\phi)}$$

erit pro casu trium lentium

$$\frac{\Omega}{\gamma-\Omega}=\frac{Y}{ABC(C+1)\alpha Z}\cdot\gamma=\frac{Y}{(C+1)Z}\cdot\frac{\Phi'}{\epsilon\pi'-\pi+\Phi}$$

vbi

vbi notandum est $Y = \delta$ et $Z = o$ esse eas ipsas aequationes, quae in superioribus problematibus sub numeris 8 et 9 retulimus.

IV. Pro quatuor lentibus

Si iam ponatur:

$$Y = \frac{\pi''}{\mathfrak{A}\Phi} + \frac{\pi''}{AB(\mathfrak{B}\pi - \Phi)} + \frac{\pi''}{ABC(\mathfrak{C}\pi' - \pi + \Phi)} + \frac{\pi''}{ABCD}\left(\frac{\pi}{(\mathfrak{B}\pi - \Phi)} + \frac{\pi'}{\mathfrak{C}\pi' - \pi + \Phi}\right)$$

$$Z = \frac{1}{\mathfrak{A}} + \frac{\Phi}{AB(\mathfrak{B}\pi - \Phi)} + \frac{\Phi}{ABC(\mathfrak{C}\pi' - \pi + \Phi)} + \frac{\Phi}{ABCD(\mathfrak{D}\pi'' - \pi' + \pi - \Phi)}$$

erit:

$$\frac{\Omega}{\delta - \Omega} = \frac{Y}{ABCD(D+1)\mathfrak{a}Z}\,\delta$$

hincque $\frac{\Omega}{\delta - \Omega} = \frac{Y}{(D+1)Z} \cdot \frac{\Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi}$.

V. Pro quinque Lentibus.

Posito secundum aequationes n^{i}. 8 et 9 exhibitas

$$Y = \frac{\pi'''}{\mathfrak{A}\Phi} + \frac{\pi'''}{AB(\mathfrak{B}\pi - \Phi)} + \frac{\pi'''}{ABC(\mathfrak{C}\pi'' - \pi + \Phi)} + \frac{\pi'''}{ABCD(\mathfrak{D}\pi''' - \pi' + \pi - \Phi)}$$
$$- \frac{1}{ABCD\mathfrak{E}}\left(\frac{\pi}{\mathfrak{B}\pi - \Phi} + \frac{\pi'}{\mathfrak{C}\pi' - \pi + \Phi} + \frac{\pi''}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi}\right)$$

$$Z = \frac{1}{\mathfrak{A}} + \frac{\Phi}{AB(\mathfrak{B}\pi - \Phi)} + \frac{\Phi}{ABC(\mathfrak{C}\pi' - \pi + \Phi)} + \frac{\Phi}{ABCD(\mathfrak{D}\pi'' - \pi' + \pi - \Phi)}$$
$$+ \frac{\Phi}{ABCDE(\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi)}$$

habebimus

$$\frac{\Omega}{\varepsilon - \Omega} = \frac{Y}{ABCDE(E+1)\mathfrak{a}Z} \cdot \varepsilon \text{ vel}$$

$$\frac{\Omega}{\varepsilon - \Omega} = \frac{Y}{(E+1)Z} \cdot \frac{\Phi}{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi}.$$

Lex harum formularum pro quouis maiori lentium numero hinc satis est perspicua.

Coroll. 1.

362. Cum igitur supra pro quouis lentium numero exhibuerimus aequationes n°. 8. et n°. 9. formula n°. 8. dabit valorem Y et formula n°. 9. valorem ipsius Z, quibus cognitis locus oculi, vbi margo coloratus euanescit, facile innotescit.

Coroll. 2.

363. Si praeterea ex aequatione n°. 7. exposita ponatur

$$X=\frac{\pi}{\mathfrak{B}\pi-\Phi}+\frac{\pi'}{\mathfrak{C}\pi'-\pi+\Phi}+\frac{\pi''}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}+\frac{\pi'''}{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi}$$

erit quidem pro casu quinque lentium

$$Y=\frac{\pi'''}{\Phi}Z-\frac{1}{ABCD\mathfrak{E}}X$$

Pro quatuor lentibus $Y=\frac{\pi''}{\Phi}Z-\frac{1}{ABC\mathfrak{D}}X$

Pro tribus lentibus $Y=\frac{\pi'}{\Phi}Z-\frac{1}{AB\mathfrak{C}}X$

et pro duabus lentibus $Y=\frac{\pi}{\Phi}Z-\frac{1}{A\mathfrak{B}}X$.

Scholion.

364. Quo facilius haec oculi loca littera Ω designata cum praecedentibus littera O designatis comparare liceat, notandum est esse.

Pro casu

Vnius Lentis $\frac{O}{\alpha-O}=o$

Duarum Lentium $\frac{O}{\mathfrak{b}-O}=\frac{\pi}{(B+1)(\mathfrak{B}\pi-\Phi)}$

Trium Lentium $\frac{O}{\gamma-O}=\frac{\pi'}{(C+1)(\mathfrak{C}\pi'-\pi+\Phi)}$

Quatuor Lentium $\frac{O}{\delta-O}=\frac{\pi''}{(D+1)(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}$

Quinque Lentium $\frac{O}{\varepsilon-O}=\frac{\pi'''}{(E+1)(\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi)}$ etc.

Quare

Quare vt ambo loca congruant retentis pro quouis lentium numero litteris Y et Z modo adhibitis, oportet sit

Pro vna Lente $\delta = 0$

Pro duabus Lentibus $\frac{Y}{Z} = \frac{\pi}{\Phi}$

Pro tribus Lentibus $\frac{Y}{Z} = \frac{\pi'}{\Phi}$

Pro quatuor Lentibus $\frac{Y}{Z} = \frac{\pi''}{\Phi}$

Pro quinque Lentibus $\frac{Y}{Z} = \frac{\pi'''}{\Phi}$ etc.

vnde generatim fit $X = 0$, quae est ipsa aequatio n°. 7. allata. Haec igitur sunt principia generalia, ex quibus constructio instrumentorum dioptricorum erit perficienda, quae quidem instrumenta hic potissimum ad visionem accommodaui. Nihilo vero minus ad repraesentationem obiectorum in camera obscura super Tabula alba adhiberi possunt, vbi praeter iam data praecepta tenendum est, has effigies etiam sine margine colorato esse apparituras, si aequationibus n°. 7. satisfiat, omnem vero confusionem a diuersa radiorum natura oriundam penitus tolli, si insuper aequationibus n°. 9 satisfiat.

Supplementum VII.

Si ratio refractionis in singulis lentibus fuerit diuersa et secundum lentium ordinem litteris n, n', n'', n''' etc. exhibeatur, inde pro lentium constructione

I°. Litterae respondentes ϱ, σ, τ et pro confusione

finſione litterae μ et ν conuenienter definiri debent ſecundum formulas ſupra datas §. 55

$$\varrho = \frac{1}{2(n-1)} + \frac{1}{n+2} - 1 \; ; \; \sigma = \frac{1}{2(n-1)} - \frac{1}{n+2} + 1$$

$$\tau = \frac{1}{3}\left(\frac{1}{2(n-1)} + \frac{1}{n+2}\right)\sqrt{(4n-1)}$$

ita, vt ſit

$$\varrho + \sigma = \frac{1}{n-1} \text{ et } \sigma - \varrho = \frac{3}{n+2} + 2 = \frac{2(n+1)}{n+2}.$$

Porro autem

$$\mu = \frac{1}{4(n+2)} + \frac{1}{4(n-1)} + \frac{1}{8(n-1)^2}$$

$$\nu = \frac{4(n-1)^2}{4n-1}.$$

vnde intelligitur, quemadmodum ex ratione refractionis π' litterae reſpondentes $\varrho', \sigma', \tau', \mu', \nu'$ definiri debeant.

II°. Quod iam ad elementa occaſione campi apparentis introducta π, π', π'' etc. attinet, diſtantiae determinatrices lentium inde eodem modo manent determinatae, vt ſupra, ita, vt non opus ſit, eas formulas hic transcribere; interim tamen meminiſſe iuuabit; formulas primitiuas, vnde illae ſunt natae, quae ſunt

1°. $\frac{\mathfrak{D}\pi - \Phi}{\Phi} = \frac{Aa}{b}$

2°. $\frac{\mathfrak{e}\pi' - \pi + \Phi}{\Phi} = \frac{ABa}{c}$

3°. $\frac{\mathfrak{D}\pi'' - \pi' + \pi - \Phi}{\Phi} = \frac{ABCa}{d}$

4°. $\frac{\mathfrak{e}\pi''' - \pi'' + \pi' - \pi + \Phi}{\Phi} = \frac{ABCDa}{e}$

etc.

III.

III. Verum valores litterarum P, Q, R etc. ob diuerſas refractiones mutantur, vt ſequitur.

$$P = \frac{\mu}{A^3 a^3}(A+1)\{\lambda(A+1)^2+\nu A\}$$

$$Q = \frac{\mu'}{B^3 b^3}(B+1)(\lambda'(B+1)^2+\nu' B);$$

$$R = \frac{\mu''}{C^3 c^3}(C+1)(\lambda''(C+1)^2+\nu'' C);$$

$$S = \frac{\mu'''}{D^3 d^3}(D+1)(\lambda'''(D+1)^2+\nu''' D);$$

etc.

IV. Diſtantiae lentium etiam manent, vt ante, vna cum ſemidiametris aperturarum ſingularum lentium; at radii binarum facierum vtriusque lentis ita ſunt ad praeſentem caſum accommodandae, vt pro, lente cuius refractio eſt n', ei reſpondentes litterae ϱ', σ', et τ' vſurpari debeant, ſimilique modo etiam pro ſequentibus lentibus, quarum refractio litteris n'', n''' etc. indicatur.

His praemiſſis ſingula momenta, quae in conſtructione inſtrumentorum dioptricorum ſunt obſeruanda, ſequenti modo repraeſentabimus.

I. Pro loco oculi ſeu eius diſtantia poſt vltimam lentem $= O$

habebimus pro ſingulis lentium numeris ſequentes determinationes:

Num. lentium.	
I.	$O = o = \alpha + l$; ideoque $\alpha = -l = Aa$
II.	$O = \frac{\mathfrak{B} b \pi}{\pi - \Phi} = \mathfrak{C} + l$; ideoque $\mathfrak{C} = \frac{\mathfrak{B} b \pi}{\pi - \Phi} - l = Bb$; $l = \frac{-Bb(\mathfrak{B}\pi - \Phi)}{\pi - \Phi}$
III.	$O = \frac{\mathfrak{C} c \pi'}{\pi' - \pi + \Phi} = \gamma + l$; hinc $\gamma = \frac{\mathfrak{C} c \pi'}{\pi' - \pi + \Phi} - l = Cc$; hinc $l = -\frac{Cc(\mathfrak{C}\pi' - \pi + \Phi)}{\pi' - \pi + \Phi}$
IV.	$O = \frac{\mathfrak{D} d \pi''}{\pi'' - \pi' + \pi - \Phi} = \delta + l$; hinc $\delta = \frac{\mathfrak{D} d \pi''}{\pi'' - \pi' + \pi - \Phi} - l$; adeoque $l = \frac{-Dd(\mathfrak{D}\pi'' - \pi' + \pi - \Phi)}{\pi'' - \pi' + \pi - \Phi}$
V.	$O = \frac{\mathfrak{E} e \pi'''}{\pi''' - \pi'' + \pi' - \pi + \Phi} = \varepsilon + l$; hinc $\varepsilon = \frac{\mathfrak{E} e \pi'''}{\pi''' - \pi'' + \pi' - \pi + \Phi} - l$; adeoque $l = -\frac{Ee(\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi)}{\pi''' - \pi'' + \pi' - \pi + \Phi}$
	etc.

II. Pro ratione multiplicationis ad diſtantiam b relata

habebimus pro quolibet lentium numero ſequentes valores litterae m.

I.	$m = -A\frac{b}{l}$	Pro ſitu erecto, ſi m hinc prodierit poſitiuum; pro ſitu autem inuerſo, ſi m obtinuerit valorem negatiuum.
II.	$m = +AB\frac{b}{l}$	
III.	$m = -ABC\frac{b}{l}$	
IV.	$m = +ABCD\frac{b}{l}$	
V.	$m = -ABCDE\frac{b}{l}$	
	etc.	

III.

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III. Pro gradu claritatis y

ex ſuperioribus liquet, eum ſemper pari modo exprimi; quantuſcunque fuerit lentium numerus; perpetuo enim erit $y = \frac{bx}{ma}$

IV. Pro Campo apparente ſi quidem O habeat valorem poſitiuum,

eius ſemidiameter Φ pro quouis lentium numero ſequenti modo definietur:

Num. lentium.	
I.	$\Phi = \infty$ ſeu indefinitum
II.	$\Phi = \frac{-\pi b}{m a - b}$
III.	$\Phi = \frac{-\pi + \pi'}{m a - b} . b$
IV.	$\Phi = \frac{-\pi + \pi' - \pi''}{m a - b} . b$
V.	$\Phi = \frac{-\pi + \pi' - \pi'' + \pi'''}{m a - b} . b$ etc.

(IV.) Pro Campo apparente,

ſi diſtantia O fuerit negatiua, quo caſu oculus lenti vltimae immediate adplicari debet, Φ definietur ex aequationibus ſequentibus.

Num. lentium.	
I.	$\Phi = \infty$ indefinitum
II.	$\frac{A \mathfrak{B} a \Phi}{\mathfrak{B} \pi - \Phi} = \omega$
III.	$\frac{A B \mathfrak{C} a \Phi}{\mathfrak{C} \pi' - \pi + \Phi} = \omega$
IV.	$\frac{A B C \mathfrak{D} a \Phi}{\mathfrak{D} \pi'' - \pi' + \pi \Phi} = \omega$
V.	$\frac{A B C D \mathfrak{E} . a \Phi}{\mathfrak{E} \pi''' - \pi'' + \pi' - \pi + \Phi} = \omega$. etc.

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V. Pro ſemidiametro confuſionis habebuntur ſequentes expreſſiones

I. $\frac{m x^3}{4aab} \cdot \frac{\mu(A+1)(\lambda(A+1)^2+\nu A)}{A^3}$

II. $\frac{m x^3}{4aab}\left(\frac{\mu(A+1)(\lambda(A+1)^2+\nu A)}{A^3}\right)+\frac{\mu'(B+1)(\lambda'(B+1)^2+\nu' B)}{A^3 B(\mathfrak{D}\pi-\Phi)}\Phi\Big\}$

III. $\frac{m x^3}{4aab}\left\{\frac{\mu(A+1)(\lambda(A+1)^2+\nu A)}{A^3}+\frac{\mu'(B+1)(\lambda'(B+1)^2+\nu' B)}{A^3 B^3(\mathfrak{D}\pi-\Phi)}\Phi\right.$
$\left.+\frac{\mu''(C+1)(\lambda''(C+1)^2+\nu'' C)}{A^3 B^3 C^3(\mathfrak{C}\pi'-\pi+\Phi)}\Phi\right\}$

IV. $\frac{m x^3}{4aab}\left\{\frac{\mu(A+1)(\lambda(A+1)^2+\nu A)}{A^3}+\frac{\mu'(B+1)(\lambda'(B+1)^2+\nu' B)}{A^3 B^3(\mathfrak{D}\pi-\Phi)}\Phi\right.$
$\left.\frac{\mu''(C+1)(\lambda''(C+1)^2+\nu'' C)}{A^3 B^3 C^3(\mathfrak{C}\pi'-\pi+\Phi)}\Phi+\frac{\mu'''(D+1)(\lambda'''(D+1)^2+\nu''' D)}{A^3 B^3 C^3 D^3(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}\Phi\right\}$

V. $\frac{m x^3}{4aab}\left\{\frac{\mu(A+1)(\lambda(A+1)^2+\nu A)}{A^3}+\frac{\mu'(B+1)(\lambda'(B+1)^2+\nu' B)}{A^3 B^3(\mathfrak{D}\pi-\Phi)}\Phi\right.$
$+\frac{\mu''(C+1)(\lambda''(C+1)^2+\nu'' C)}{A^3 B^3 C^3(\mathfrak{C}\pi'-\pi+\Phi)}\Phi+\frac{\mu'''(D+1)(\lambda'''(D+1)^2+\nu''' D)}{A^3 B^3 C^3 D^3(\mathfrak{D}\pi''-\pi'+\pi+\Phi)}\Phi$
$\left.+\frac{\mu''''(F+1)(\lambda''''(E+1)^2+\nu'''' E)}{A^3 B^3 C^3 D^3 E^3(\mathfrak{C}\pi'''-\pi''+\pi'-\pi+\Phi)}\Phi\right\}$
etc.

VI. Pro tollendo margine colorato,

ſi diſtantia O prodierit poſitiva, obiectum ſine margine colorato apparebit, ſatisfaciendo ſequentibus aequationibus:

Num. lentium.	
I.	$o = o$
II.	$o = \frac{bdn'}{n'-1} \cdot \frac{\pi}{Aa\Phi}$
III.	$o = \frac{bdn'}{n'-1} \cdot \frac{\pi}{Aa\Phi} + \frac{cdn''}{n''-1} \cdot \frac{\pi'}{ABa\Phi}$
VI.	$o = \frac{bdn'}{n'-1} \cdot \frac{\pi}{Aa\Phi} + \frac{cdn''}{n''-1} \cdot \frac{\pi'}{ABa\Phi} + \frac{ddn'''}{n'''-1} \cdot \frac{\pi''}{ABCa\Phi}$ etc.

(VI.)

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(VI) Pro tollendo margine colorato.

Sin autem distantia O prodierit negatius, margo coloratus euanescet, si sequentibus aequationibus satisfiat

I. $0 = 0$

II. $0 = \frac{a\,dn}{n-1}(A+1)B\pi$

III. $0 = \frac{a\,dn}{n-1}(A+1)BC\pi' + \frac{b\,dn'}{n'-1}\frac{(B+1)C\pi' - (C+1)\pi}{A}$

IV. $0 = \frac{a\,dn}{n-1}(A+1)BCD\pi'' + \frac{b\,dn'}{n'-1}\left(\frac{(B+1)CD\pi'' - (D+1)\pi'}{A}\right) + \frac{c\,dn''}{n''-1}\left(\frac{(C+1)D\pi'' - (D+1)\pi'}{AB}\right)$

V. $0 = \frac{a\,dn}{n-1}(A+1)BCDE\pi''' + \frac{b\,dn'}{n'-1}\left(\frac{(B+1)CDE\pi''' - (E+1)\pi''}{A}\right) + \frac{c\,dn''}{n''-1}\left(\frac{(C+1)DE\pi''' - (E+1)\pi'}{AB}\right) + \frac{d\,dn'''}{n'''-1}\left(\frac{(D+1)E\pi''' - (E+1)\pi''}{ABC}\right)$

etc.

VII. Pro tollenda confusione omni insuper sequentibus aequationibus est satisfaciendum

I. $0 = \frac{a\,dn}{n-1}\cdot\frac{A+1}{A}$

II. $0 = \frac{a\,dn}{n-1}\cdot\frac{A+1}{A} + \frac{b\,dn'}{n'-1}\cdot\frac{B+1}{A^2B}$

III. $0 = \frac{a\,dn}{n-1}\cdot\frac{A+1}{A} + \frac{b\,dn'}{n'-1}\cdot\frac{B+1}{A^2B} + \frac{c\,dn''}{n''-1}\cdot\frac{C+1}{A^2B^2C}$

IV. $0 = \frac{a\,dn}{n-1}\cdot\frac{A+1}{A} + \frac{b\,dn'}{n'-1}\cdot\frac{B+1}{A^2B} + \frac{c\,dn''}{n''-1}\cdot\frac{C+1}{A^2B^2C} + \frac{d\,dn'''}{n'''-1}\cdot\frac{D+1}{A^2B^2C^2D}$

etc.

Diſtantiae autem lentium et ſemidiametri aperturarum perinde definiuntur, ac ſupra; tantum notetur inſuper, formulas pro ſemidiametro confuſionis exhibitas, niſi penitus ad nihilum redigi queant, aequales poni debere formulae $\frac{1}{4k^3}$, exiſtente circiter $k = 40$ (§. 193) vel adhuc minore, prout circumſtantiae poſtulauerint.

Supplementum VIII.

De Lentibus obiectiuis perfectis.

In Cap. III. nullas alias lentes obiectiuas commemorauimus, niſi quae ex eadem vitri ſpecie et ex principio minimi ſunt paratae, neque tales lentes ex diuerſis vitri ſpeciebus conficere conuenit, quoniam eae ad confuſionem ex diuerſa radiorum refrangibilitate oriundam, tollendam ſunt ineptae. Cum igitur iſta confuſio aliter tolli nequeat, niſi diuerſae materiae refringentes adhibeantur; hic accuratius perpendamus, quemadmodum ex diuerſis mediis diaphanis eiusmodi lentes conſtrui queant, in quibus non ſolum prior confuſio ex apertura lentium, ſed etiam poſterior ex diuerſa radiorum refrangibilitate oriunda ad nihilum redigi queat, cuiusmodi lentes compoſitas merito *perfectas* appellare licebit.

I. Primo igitur examinemus, quomodo poſteriori confuſioni ſit occurrendum, quando quotcunque lentes inuicem iungantur, ita, vt earum diſtantiae quaſi euaneſcant. Quem in finem conſiderentur formulae

ſupra

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ſupra datae tam pro margine colorato vitando, quam pro confuſione penitus tollenda N°. VI, VII. At ex aequatione N°. VI. patet, marginem coloratum euaneſcere, ſi litterae π, π', π'' etc. ſint $=0$, quod quidem ſponte euenit, ſi lentes immediate coniungantur; quare in genere nullus margo coloratus eſt metuendus, ſtatim atque interualla omnia lentium euaneſcunt.

II. Vt autem aequationibus N°. VII. ſatisfiat, pro caſu duarum lentium immediate iunctarum habemus

$$0=\frac{a\,dn}{n-1}\cdot\frac{A+1}{A}+\frac{b\,dn'}{n'-1}\cdot\frac{B+1}{A^2\,B}$$

$$0=\frac{a\,dn}{n-1}\cdot\frac{\alpha+\alpha}{\alpha}+\frac{b\,dn'}{n'-1}\cdot\frac{(b+\mathfrak{b})\alpha^2}{\alpha\cdot\mathfrak{b}}$$

$$0=\frac{d\,n}{n-1}\cdot\frac{\alpha+\alpha}{a\,\alpha}+\frac{b\,dn'}{n'-1}\cdot\frac{b+\mathfrak{b}}{\alpha^2\,\mathfrak{b}};\ \text{ſeu ob } \alpha=-b$$

$$0=\frac{d\,n}{n-1}\cdot\frac{\alpha+\alpha}{a\,\alpha}+\frac{d\,n'}{n'-1}\cdot\frac{b+\mathfrak{b}}{b\,\mathfrak{b}},$$

$$0=\frac{d\,n}{n-1}\cdot\left(\tfrac{1}{a}+\tfrac{1}{\alpha}\right)+\frac{d\,n'}{n'-1}\cdot\left(\tfrac{1}{b}+\tfrac{1}{\mathfrak{b}}\right)$$

vbi duo hi coëfficientes denotant reciproca diſtantiarum focalium primae et ſecundae lentis.

III. Nunc inſuper effici debet, vt prior confuſio ab apertura lentium oriunda ad nihilum redigatur; vbi obſeruare debemus, hoc caſu eſſe debere $\alpha+b=0$; $\mathfrak{b}+c=0$; $\gamma+d=0$ etc. pro numero lentium, quas coniungere velimus. Vnde ſemidiameter confu-

confusionis ex lentibus iunctis ortae euanescet, si reddatur

$$0 = \frac{\mu(A+1)(\lambda(A+1)^2+\nu A}{A^3} - \frac{\mu'(B+1)(\lambda'(B+1)^2+\nu' B)}{A^3 B^3}$$
$$+ \frac{\mu''(C+1)(\lambda''(C+1)^2+\nu'' C)}{A^3 B^3 C^3} \text{ etc.}$$

quoniam supra iam vidimus, fore $\pi = 0$, $\pi' = 0$ etc. atque hinc pro quouis numero lentium inuicem iungendarum constructionem lentium perfectarum petere debemus; quas inuestigationes hic suscipiamus.

De lentibus perfectis ex duabus lentibus compositis.

Quoniam hoc casu distantiae determinatrices ipsius lentis compositae sunt a et $\mathfrak{e}$; statuamus

$\frac{1}{a}+\frac{1}{\alpha} = f(\frac{1}{a}+\frac{1}{\mathfrak{e}})$ et $\frac{1}{b}+\frac{1}{\beta} = g(\frac{1}{a}+\frac{1}{\mathfrak{e}})$ siue

$\frac{a\alpha}{a+\alpha} = \frac{1}{f}(\frac{a\mathfrak{e}}{a+\mathfrak{e}})$; $\frac{b\mathfrak{e}}{b+\beta} = \frac{1}{g}\cdot\frac{a\mathfrak{e}}{a+\mathfrak{e}}$;

vnde pro prima conditione implenda sequitur:

$0 = \frac{dn}{n-1}\cdot f + \frac{dn'}{n'-1}\cdot g$. Porro ob $\frac{1}{\alpha} = \frac{f-1}{a} + \frac{f}{\mathfrak{e}}$ et $\frac{1}{b} = \frac{g}{a} + \frac{g-1}{\mathfrak{e}}$ et $\alpha + b = 0$, erit $\frac{f+g-1}{a} + \frac{f+g-1}{\mathfrak{e}} = 0$, seu $f+g = 1$.

Sit breuitatis gratia $\frac{dn}{n-1} = \zeta$ et $\frac{dn'}{n'-1} = \eta$, vt sit $\zeta f + \eta g = 0$;

ex qua aequatione cum altera $f+g = 1$ coniuncta vtraque littera f et g determinabitur, ita, vt sit

$$f = \frac{\eta}{\eta-\zeta} \text{ et } g = -\frac{\zeta}{\eta-\zeta}$$

deinde

deinde cum sit

$$A = \frac{\alpha}{a} = \frac{\varepsilon}{\varepsilon(f-1)+af} = \frac{\varepsilon}{af-\varepsilon g} = \frac{(\eta-\zeta)\varepsilon}{a\eta+\varepsilon\zeta} \text{ et}$$

$$B = \frac{\varepsilon}{b} = \frac{g\varepsilon+(g-1)a}{a} = \frac{g\varepsilon-fa}{a} = \frac{-\zeta\varepsilon-\eta a}{(\eta-\zeta)a};$$

$$\text{ergo } A+1 = \frac{\eta(a+\varepsilon)}{a\eta+\varepsilon\zeta} \text{ et } B+1 = \frac{-\zeta(a+\varepsilon)}{a(\eta-\zeta)};$$

vnde pro confusione priori tollenda habebimus hanc aequationem

$$0 = \frac{\mu\lambda.\eta^3(a+\varepsilon)^3}{(\eta a+\zeta\varepsilon)^3} + \frac{\mu\nu\eta(\eta-\zeta)(a+\varepsilon)\varepsilon}{(\eta a+\zeta\varepsilon)^2}$$
$$\frac{-\mu'\lambda'.\zeta^3(a+\varepsilon)^3}{(\eta a+\zeta\varepsilon)^3} + \frac{\mu'\nu'\zeta(\eta-\zeta)(a+\varepsilon)a}{(\eta a+\zeta\varepsilon)^2}$$

quae reducitur ad hanc

$$0 = \frac{(a+\varepsilon)^2}{\eta a+\zeta\varepsilon}(\mu\lambda\eta^3-\mu'\lambda'\zeta^3)+(\eta-\zeta)(\mu\nu\eta\varepsilon+\mu'\nu'\zeta a)$$

Breuius etiam ex §. 214 deducitur haec aequatio, cui satisfieri debet $P+Q=0$ ob $\alpha=-b$, existente pro hoc casu

$$P = \mu f\left(\tfrac{1}{a}+\tfrac{1}{\varepsilon}\right)\left(\lambda\left(\tfrac{1}{a}+\tfrac{1}{\varepsilon}\right)^2+\tfrac{\nu}{\varepsilon}\left(\tfrac{f-1}{a}+\tfrac{f}{\varepsilon}\right)\right) \text{ et}$$

$$Q = \mu' g\left(\tfrac{1}{a}+\tfrac{1}{\varepsilon}\right)\left(\lambda' g^2\left(\tfrac{1}{a}+\tfrac{1}{\varepsilon}\right)^2+\tfrac{\nu'}{\varepsilon}\left(\tfrac{g}{a}+\tfrac{g-1}{\varepsilon}\right)\right).$$

ex qua aequatione siue λ, siue λ' inueniri potest, dummodo hae litterae non minores prodeant vnitate; quamobrem quosdam eiusmodi casus euoluamus et ad praxin accommodemus.

IV. Tria autem mediorum diaphanorum genera hic potissimum contemplabimur, quorum primum sit aqua pluuia, pro qua est $n=\frac{4}{3}=1,3333$; secundum vitrum ordinarium, pro quo est $n=\frac{31}{20}=1,5500$ et

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tertium vitrum Cryſtallinum, anglice Flint-Glaſs, pro quo eſt $n = \frac{3}{2} = 1,6000$; factoque calculo inuenimus, pro litteris graecis inde pendentibus ſequentes valores:

	Aqua	Vitrum commune	Vitrum Cryſtall.
n	$= 1,3333$	$= 1,5500$	$= 1,6000$
ϱ	$= 0,8000$	$= 0,1907$	$= 0,1111$
σ	$= 2,2000$	$= 1,6274$	$= 1,5555$
τ	$= 1,2490$	$= 0,9051$	$= 0,8606$
μ	$= 1,9500$	$= 0,9381$	$= 0,8333$
ν	$= 0,1025$	$= 0,2326$	$= 0,2666$
$\mu\nu$	$= 0,1999$	$= 0,2184$	$= 0,2221.$

V. Quod autem ad valores differentialium dn attinet, optandum eſſet, vt ii per experimenta accuratiſſime definirentur; interim tamen duae ſequentes hypotheſes ex theoria deductae perpendi merentur, quandoquidem *Newtoniana*, ex qua dn ipſi $n-1$ foret proportionale, ad confuſionem tollendam eſſet inepta. Prima autem hypotheſis, quam dudum propoſueram, facit dn ipſi n. log. n proportionale; altera autem ex theoria attractionis deducta dat dn ipſi $\frac{nn-1}{n}$ proportionale; atque hinc valores formulae $\frac{dn}{n-1}$ pro vtraque hypotheſi euoluamus:

I^ma Hypotheſis $dn = n$ log. n

	Aqua	Vitrum commune	Vitrum Cryſtall.
dn	0,1665	0,2950	0,3265
$\frac{dn}{n-1}$	0,4995	0,5364	0,5442

II^da

II^da Hypotheſis $dn = \frac{nn-1}{n}$

	Aqua	Vitrum commune	Vitrum Cryſtall.
$\frac{dn}{n-1}$	1, 7500	1, 6452	1, 6250

VI. Verum neutra harum hypotheſium cum iis experimentis, quae Celeb. *Dollondus* circa prismata inſtituit, ſubſiſtere poteſt. Vſus nempe eſt duobus prismatibus, quorum alterum ex vitro communi, erat paratum, angulo refringente exiſtente triginta graduum; alterum autem ex vitro chryſtallino confectum continebat angulum 19°; hisque duobus prismatibus inuerſe iunctis obſeruauit, in ſpectro inde ad parietem proiecto iridis colores nullos adparere; vnde concluſit, (ſi n pro ratione refractionis vitri communis, n' vero pro refractione vitri chryſtallini aſſumatur) fore $dn : dn' = 2 : 3$, quam rationem vocat rationem diſperſionis; quo experimento admiſſo ſequitur, $\frac{dn}{n-1} : \frac{dn'}{n'-1} = 8 : 11$; ita, vt pro noſtro calculo foret $\zeta = 8$ et $\eta = 11$, quae ratio ab allatis hypotheſibus enormiter aberrat. Prima enim dat $\zeta : \eta = 0,5364 : 0,5442$ hoc eſt proxime, vt 68 : 69. Altera autem praebet $\zeta : \eta = 1,6452 : 1,6250$ vel proxime, vti 81 : 80, vnde patet, hypotheſin poſteriorem non poſſe ſubſiſtere, quia ex ea ſequeretur $\zeta > \eta$; prior vero tantum adhuc diſcrepat, vt cum experimento neutiquam conciliari poſſit; ex quo merito ancipites haeremus, quomodo nos in calculo hoc gerere debeamus.

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amus. Interim experimentum *Dollondi* nonnullis adhuc difficultatibus premi videtur, quas hic ob oculos ponere viſum eſt.

Digreſſio de refractione vitri chryſtallini ſecundum experimenta *Dollondi*.

Tab. III. Fig. 17.

I. Quo clarius theoriam, cui haec experimenta innituntur, ob oculos ponamus, rem aliquanto generalius, quam *Dollondus* fecit, conſideremus. Repraeſentet ergo triangulum ABC prisma triangulare ex vitro communi paratum, cuius angulus ad C, quem *Dollondus* 30° aſſumit, vocetur $= f$; alterum vero prisma ex vitro chryſtallino factum DBE ita inuerſe priori ſit adplicatum, vt ſit angulus $CBD = g$, ipſeque angulus huius prismatis $DBE = h$ vbi in experimento examinando erat $g = 0$ et $h = 19°$ tum vero refractio radiorum mediorum in prismate priori ſit vt $m : 1$, in poſteriori vero vt $n : 1$, ita vt facultates radios diſpergendi nobis per differentialia dm et dn repraeſententur. Quibus poſitis *Dollondus* ſe obſeruaſſe affirmat, ſi radius ſolis per haec duo prismata transmittatur, ſpectrum inde in parietem proiectum nullis coloribus inquinari. Vnde concluſit, fore $dm : dn = 2 : 3$, quam concluſionem adcuratius examinemus.

II. Hic primum notari conuenit, angulum, ſub quo radius ſolis Op per faciem prioris lentis AC immittitur, non definiri, quaſi perinde ſit, ſub quocunque

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cunque angulo haec immerſio fiat, id quod plenio rem expoſitionem poſtulat. Ponamus igitur angulum OPA $= a$, qui eſt complementum anguli incidentiae et anguli ſequentes quauis refractione orti ſint p, q, r, t, u, v, e et radius iterum emergens VZ; ex quo ſtatim perſpicitur, fore $q = p + f$, $r = g + t$ et $u = v + h$; tum vero ex ratione refractionis intelligimus I) Coſ. $a = m$. Coſ. p. II) Coſ. $r = m$. Coſ. q. III) Coſ. $t = n$. Coſ. u. IV) Coſ. $e = n$. Coſ. v.

III. Quia iam hoc tranſitu nulla colorum diſperſio obſeruata eſſe perhibetur, neceſſe eſt, vt in emerſione V omnes radii colorati redditi ſint inter ſe paralleli, ideoque angulus e conſtans perinde ac primus angulus a, etiamſi rationes refractionis m et n ſuis differentialibus dm et dn augeantur; quippe qua mutatione tantum anguli p, q, r, t, u, v, fiunt variabiles.

IV. Nunc igitur primo hanc angulorum mutationem in primo prismate euoluamus, differentiando ſcilicet aequationes ſupra notatas; vnde erit

$dq = dp$ deinde $0 = dm$. coſ. $p - m\,dp$. ſin. p ſeu

$dp = \frac{dm \cdot \text{coſ.} p}{m \cdot \text{ſin.} p} = dq$;

tum vero $-dr$. ſin. $r = dm$. coſ. $q - m\,dq$. ſin. q ſeu

$-dr$. ſin. $r = dm$. coſ. $q - \frac{dm \cdot \text{coſ.} p \cdot \text{ſin.} q}{\text{ſin.} p}$

$= \frac{dm}{\text{ſin.} p}(\text{coſ.} q \text{ ſin.} p - \text{coſ.} p \text{. ſin.} q) = \frac{dm \cdot \text{ſin.}(p-q)}{\text{ſin.} p}$

hincque tandem $dr = \frac{dm \cdot \text{ſin.} f}{\text{ſin.} p \cdot \text{ſin.} r}$ ob $p - q = -f$

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Simili modo euoluantur refractiones per alterum prisma, incipiendo ab angulo constante e vbi aequatio

$$\text{cos. } e = n.\ \text{cos. } v \text{ dat } o = dn.\ \text{cos. } v - n\,dv.\ \text{sin. } v \text{ siue}$$

$$dv = \frac{dn.\,\text{cos. } v}{n.\,\text{sin. } v} = du \text{ ob } u = v + b;$$

deinde vero aequatio cos. $t = n.$ cos. u dabit

$$-dt.\ \text{sin. } t = dn.\ \text{cos. } u - n\,du.\ \text{sin. } u = dn.\ \text{cos. } u - \frac{dn.\,\text{cos. } v.\,\text{sin. } u}{\text{sin. } v} = \frac{dn}{\text{sin. } v}$$

$$(\text{cos. } u.\ \text{sin. } v - \text{cos. } v.\ \text{sin. } u) = \frac{dn}{\text{sin. } v}.\ \text{sin. } (v-u)$$

ergo ob $u - v = b$ habebitur $dt = +\frac{dn.\,\text{sin. } b}{\text{sin. } v.\,\text{sin. } t}$; Cum autem sit

$r = g + t$, erit $dr = dt$; ergo $\frac{dm.\,\text{sin. } f}{\text{sin. } f.\,\text{sin. } r} = \frac{dn.\,\text{sin. } b}{\text{sin. } v.\,\text{sin. } t}$

ex qua aequatione haec relatio inter dm et dn concluditur

$$dm : dn = \frac{\text{sin. } b}{\text{sin. } v.\,\text{sin. } t} : \frac{\text{sin. } f}{\text{sin. } f.\,\text{sin. } r}$$

$$dm : dn = \frac{\text{sin. } f.\ \text{sin. } r}{\text{sin. } f} : \frac{\text{sin. } v.\,\text{sin. } t}{\text{sin. } b}$$

quae proportio generaliter locum habet, quoties angulus ad e prodit constans

V. Iam hanc conclusionem ad *Dollondi* experimentum adplicemus et quia ibi est $g = o$, hincque $r = t$ erit ratio inter vtramque dispersionem seu

$$dm : dn = \frac{\text{sin. } p}{\text{sin. } f} \cdot \frac{\text{sin. } v}{\text{sin. } b} = \frac{CQ}{PQ} : \frac{BU}{VU};$$

ex qua aequatione iam satis clare elucet, rationem $\frac{dm}{dn}$ ab angulo incidentiae a neutiquam esse independentem, vti *Dollondus* suppossuisse videtur; ita, vt etsi

diffu-

diffusio colorum pro certo quodam angulo a euanescens sit deprehensa, hanc conclusionem neutiquam ad omnes angulos incidentiae extendere liceat. Quoniam igitur hic angulus a *Dollondo* non est assignatus, ex hoc experimento nihil certi determinari poterit.

VI. Vt autem hic nullum dubium relinquatur, aliquot casus praecipuos pro angulo a calculo euolvamus, vt adpareat, quanta diuersitas inde in rationem $\frac{dm}{dn}$ ingrediatur, dum tamen plus vna vera esse nequit. Quare cum *Dollondus* inuenisset pro vtraque vitri specie a se adhibita I. $m = 1,53$ et II) $n = 1,58$, tum vero angulos $f = 30^\circ$ et $h = 19^\circ$ sequentia exempla hinc expediamus:

Exemplum I.

Angulum a ita constitutum concipiamus, vt fiat q angulus rectus, ideoque etiam r, t et u recti; erit sin. p = cos. f et sin. v = cos. h, ex quo colligitur $dm : dn$ = cotang. f : cotang. h; siue $\frac{dm}{dn} = 0,59639$, quae iam minor est, quam $\frac{2}{3}$

Exemplum II.

Sit $a = 90^\circ$, qui forte est ipse casus *Dollondi*; atque hinc inuenientur sequentes anguli $p = 90^\circ$; $q = 120^\circ$ $r = 139^\circ, 53' = t$. $u = 118^\circ 57'$. $v = 99^\circ 57'$. $e = 105^\circ 50'$. Vnde sequitur $\frac{dm}{dn} = \frac{\text{sin. } p. \text{ sin. } h.}{\text{sin. } e. \text{ sin. } v.} = 0,6615$. ideoque proxime $dm : dn = 2 : 3$ vt habet *Dollondus*.

Cete-

Ceterum hinc iam euidens est, hanc rationem pendere etiam ab angulo incidentiae, quae tamen in hoc experimento non est commemorata.

Si ergo hanc rationem cum *Dollondo* assumamus, fiet porro $\frac{d\,m}{m-1} : \frac{d\,n}{n-1} = 116 : 159 = 7 : 10$, ita, vt futurum sit pro lentibus obiectiuis duplicatis, quibus *Dollondus* vtitur, $\zeta = 7$, $\eta = 10$; quarum constructionem hic ex nostris principiis inuestigemus.

Lens Obiectiua Clariss. *Dollondi* duplicata prior, anteriore lente ex vitro communi, posteriore vero ex vitro chrystallino parata.

Cum igitur sit $n = 1,53$ et $n' = 1,58$, $\zeta = 7$, $\eta = 10$, erit statim $f = \frac{\eta}{\eta - \zeta} = \frac{10}{3}$ et $g = \frac{-\zeta}{\eta - \zeta} = -\frac{7}{3}$;

vnde distantiae determinatrices vtriusque lentis erunt

Pro prima a, et $\alpha = \frac{(\eta - \zeta)\,a\mathfrak{E}}{\eta a + \zeta \mathfrak{E}}$

Pro posteriori b et $\beta = \frac{-(\eta - \zeta)\,a\mathfrak{E}}{\eta a + \zeta \mathfrak{E}} = -\alpha$.

Dum ipsius lentis duplicatae distantiae determinatrices sunt a et $\mathfrak{E}$.

Tantum igitur restat, vt pro his binis lentibus numeri arbitrarii λ et λ' definiantur; et quoniam quaestio circa lentes obiectiuas versatur, statuamus statim $a = \infty$, vt adhuc resolui debeat haec aequatio

$$0 = \mu\lambda\eta^3 - \mu'\lambda'\zeta^3 + \eta(\eta - \zeta)\zeta\mu'\nu'.$$

seu $\mu'\lambda'\zeta^3 = \mu\lambda\eta^3 + \eta\zeta(\eta - \zeta)\mu'\nu'$. quae ob $\zeta = 7$ et $\eta = 10$ abit in hanc

$$343.\ \mu'\lambda' = 1000.\ \mu\lambda + 210.\ \mu'\nu'. \qquad \text{II.}$$

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II. Vt igitur *Dollondum* sequamur, pro his valoribus $n = 1,53$ et $n' = 1,58$ litteras inde deriuatas quaerere debemus.

Eruitur autem ex formulis superioribus

$\mu = 0,9875$	$\mu' = 0,8724$
$l\mu = 9.9945449.$	$l\mu' = 9.9407397$
$\nu = 0,2194$	$\nu' = 0,2529$
$l.\nu = 9.3413418$	$l.\nu' = 9.4030044$
$l.\mu\nu = 9.3358867.$	$l.\mu'\nu' = 9.3437441.$

Hinc coëfficientes trium terminorum nostrae aequationis computentur.

$l.343 = 2,5352941$	$l.1000 = 3,0000000$	$l.210 = 2,3222193$
$l.\mu' = 9.9407397$	$l.\mu = 9.9945449$	$l.\mu'\nu' = 9.3437441$
2.4760338	2.9945449	1.6659634
Subtrah. - - - -	2.4760338	2.4760338
	0.5185111	9.1899296
	$3,3000$	$0,1548$

ita, vt sit

$$\lambda' = 3,3000\,\lambda + 0,1548$$

quare sumto

$\lambda = 1$, fit $\lambda' = 3,4548$; ergo

$\lambda' - 1 = 2,4548$, et log. $\sqrt{\lambda' - 1} = 0,1950080$.

III. Iam ad radios facierum harum lentium definiendos, cum sit

$$a = \infty \text{ et } a = \frac{(n-\zeta)\varepsilon}{\eta} \text{ et } b = -\frac{(n-\zeta)\varepsilon}{\eta}$$

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erit pro lente priore radius faciei

anterioris $= \frac{\alpha}{\sigma} = \frac{3\varepsilon}{10.\sigma}$; posterioris $= \frac{\alpha}{\varrho} = \frac{3\varepsilon}{10.\varrho}$.

Pro lente posteriore radius faciei

$$\text{anter.} = \frac{-3\varepsilon}{10\varrho' - 3\sigma' \pm 7\tau'\sqrt{\lambda' - 1}}$$

$$\text{poster.} = \frac{-3\varepsilon}{10\sigma' - 3\varrho' \pm 7\tau'\sqrt{(\lambda' - 1)}}$$

quare pro duplici refractione $n = 1,53$ et $n' = 1,58$ valores litterarum ϱ, σ, τ quaeri oportet, qui ita se habebunt

$n = 1,53$	$n' = 1,58$
$\varrho = 0,2266$	$\varrho' = 0,1413$
$\sigma = 1,6602$	$\sigma' = 1,5827$
$\tau = 0,9251$	$\tau' = 0,8775.$

IV. Cum igitur ε denotet distantiam focalem ipsius lentis duplicatae, quam deinceps littera P indicabimus, pro lente priori ex vitro communi subviridi, cuius refractio est $n = 1,53$, reperiemus vtramque faciem; erit radius faciei.

Pro prima lente

anterioris $= 0,1807.$ P; posterioris $= 1,3239.$ P.

Pro altera ex vitro chrystallino, cuius refractio $n = 1,58$ facies ita erunt comparatae.

Pro secunda lente

$$\text{anterioris} = \frac{-3P}{-3,3351 \pm 9.6240}$$

$$\text{posterioris} = \frac{-3P}{15,4031 \pm 9.6240}$$

qua

quia nunc radios curuaturae minores euitari conuenit, signorum ambiguorum ea sunt sumenda, quae denominatores minores producunt; id quod fit, si signa superiora valeant; hinc ergo obtinebimus sequentes determinationes:

radius facici anterioris $= \frac{-3P}{6.2889} = 0.4770.P$

posterioris $= \frac{-3P}{5.7791} = -0,5191\,P$

quae lens obiectiua capax est aperturae, cuius semidiameter est $= 0.0452\,P$.

Lens obiectiua duplicata altera, anteriore lente ex vitro chrystallino, posteriori ex vitro communi parata.

I. Cum igitur hic sit

$n = 1,58$ et $n' = 1,53$ erit $\zeta = 10$ et $\eta = 7$; hinc

$f = \frac{\eta}{\eta - \zeta} = -\frac{7}{3}$. et

$g = -\frac{\zeta}{\eta - \zeta} = \frac{10}{3}$; Hincque

$a = \frac{-3.\alpha\varepsilon}{7\alpha + 10\varepsilon} = -\frac{3\varepsilon}{7}$ ob

$a = \infty$, et $b = \frac{3\varepsilon}{7}$.

Confusio autem primi generis vt euanescat, fieri debet

$$0 = 343\,\mu\lambda - 1000\,\mu'\lambda' - 210\,\mu'\nu' \text{ seu}$$
$$343\,\mu\lambda = 1000\,\mu'\lambda' + 210\,\mu'\nu'.$$

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II. Huius aequationis terni coëfficientes iam quaerantur:

$l.\ 343 = 2,5352941.$	$l.\ 1000 = 3,0000000$	$l.\ 210 = 2,3222193$
$l.\ \mu = 9,9407397$	$l.\ \mu' = 9,9945449$	$l.\ \mu'\nu' = 9.8358867$
2.4760338	2.9945449	$1,6581060$
	2.4760338	2.4760338
	0.5185111	9.1820722
	$3,3000$	$0,1520$

ita, vt sit

$\lambda = 3,3000\,\lambda' + 0,1520$; quare sumto $\lambda' = 1$ fiet $\lambda = 3.4520$ ac propterea $\lambda - 1 = 2,4520$ et $\log. \sqrt{(\lambda - 1)} = 0,3895205$.

III. Cum nunc sit

$a = \infty$ et $\alpha = \frac{-3\varepsilon}{7}$ et $b = \frac{3}{7}\varepsilon$

erunt radii facierum vtriusque lentis

Pro lente priore radius faciei

anterioris $= \frac{-3\varepsilon}{7(\sigma \pm \tau\sqrt{(\lambda-1)})}$; poster. $= \frac{-3\varepsilon}{7\varrho \pm 7\tau\sqrt{(\lambda-1)}}$

Pro lente posteriore radius faciei

anterioris $= \frac{3\varepsilon}{7\varrho' + 3\sigma'}$; posterioris $= \frac{3\varepsilon}{7\sigma' + 3\varrho'}$

est autem

$\varrho = 0,1413$; $\sigma = 1,5827$; $\tau = 0,8775$.
$\varrho' = 0,2266$; $\sigma' = 1,6602$:

quibus

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quibus substitutis erit pro priore lente radius faciei

anter. $\frac{-3\mathfrak{C}}{11.0789 \pm 9.6187}$; poster. $= \frac{-3\mathfrak{C}}{0.4631 \pm 9.6187}$

valeant autem ob rationes supra allegatas signa inferiora, eritque radius faciei

anterioris $= \frac{-3\mathfrak{C}}{1.4602} = -2,0545\,P$

posterioris $= \frac{-3\mathfrak{C}}{10.5078} = -0,2828\,P.$

Pro lente autem posteriore erit radius faciei

anterioris $= \frac{3\mathfrak{C}}{6,5668} = 0,4568\,P$

posterioris $= \frac{3\mathfrak{C}}{12.3012} = 0,2438\,P.$

Inter quos quatuor radios minimus est $0,2438\,P$ vnde haec lens duplicata aperturam admittere potest, cuius semidiameter $= 0,0609\,P$ ita, vt haec lens maiorem admittat aperturam, quam praecedens.

De Lentibus obiectiuis ex tribus lentibus compositis.

I. Sit pro prima lente ratio refractionis $= n$, pro secunda $= n'$, pro tertia $= n''$, deinde distantiae determinatrices primae a et α, secundae b et $\mathfrak{b}$ tertiae c et γ cum numeris arbitrariis λ, λ', λ'' eritque $\alpha + b = 0$, et $\mathfrak{b} + c = 0$, eruntque a et γ di-

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ſtantiae determinatrices ipſius lentis triplicatae. Iam vero ſtatuatur

$$\frac{1}{a}+\frac{1}{\alpha}=f\left(\frac{1}{\alpha}+\frac{1}{\gamma}\right);$$
$$\frac{1}{b}+\frac{1}{\beta}=g\left(\frac{1}{\alpha}+\frac{1}{\gamma}\right);$$
$$\frac{1}{c}+\frac{1}{\gamma}=h\left(\frac{1}{\alpha}+\frac{1}{\gamma}\right);$$

fietque

$$\frac{1}{\alpha}=\frac{f-1}{a}+\frac{f}{\gamma};\ \alpha=\frac{a\gamma}{\gamma(f-1)+af}=-b$$
$$\frac{1}{\beta}=\frac{g}{a}+\frac{g}{\gamma}+\frac{f-1}{a}+\frac{f}{\gamma}$$
$$\frac{1}{\beta}=\frac{g+f}{a}-1+\frac{g+f}{\gamma}=-\frac{1}{c}$$

ac proinde

$$\frac{-f-g+1}{a}-\frac{f-g+1}{\gamma}=\frac{b}{c}+\frac{b}{\gamma}$$

vnde ſequitur fore

$$\frac{-f-g-b+1}{a}-\frac{(f+g+b-1)}{\gamma}=0$$

hinc $f+g+b=1$.

II. Conditio vero confuſionem colorum penitus tollens poſtulat, vt ſupra vidimus, hanc aequationem

$$0=\frac{dn}{n-1}\left(\frac{1}{a}+\frac{1}{\alpha}\right)+\frac{dn'}{n'-1}\left(\frac{1}{b}+\frac{1}{\beta}\right)+\frac{dn''}{n''-1}\left(\frac{1}{c}+\frac{1}{\gamma}\right);$$

quae ponendo

$$\frac{dn}{n-1}=\zeta;\ \frac{dn'}{n'-1}=\eta;\ \frac{dn''}{n''-1}=\vartheta$$

abit in hanc formam $0=\zeta f+\eta g+\vartheta b$.

III. Vt autem confuſio ab apertura pendens deſtruatur, huic aequationi ſatisfieri debet:

$$0=\frac{\mu(A+1)(\lambda(A+1)^2+\nu A)}{A^3}-\frac{\mu'(B+1)(\lambda'(B+1)^2+\nu' B)}{A^3 B^3}$$
$$+\frac{\mu''(C+1)(\lambda''(C+1)^2+\nu'' C)}{A^3 B^3 C^3}.$$

vbi

vbi notetur esse

$\frac{A+1}{A} = af(\frac{1}{\alpha}+\frac{1}{\gamma})$ et

$\frac{B+1}{B} = bg(\frac{1}{\alpha}+\frac{1}{\gamma})$ et

$\frac{C+1}{C} = ch(\frac{1}{\alpha}+\frac{1}{\gamma})$;

vnde termini priores continentes λ erunt

$$\alpha^3(\tfrac{1}{\alpha}+\tfrac{1}{\gamma})^3(\mu\lambda f^3+\mu'\lambda' g^3+\mu''\lambda'' b^3)$$

termini vero tres posteriores fiunt

$$\alpha^3(\tfrac{1}{\alpha}+\tfrac{1}{\gamma})(\tfrac{\mu\nu f}{a\alpha}+\tfrac{\mu'\nu' g}{b\beta}+\tfrac{\mu''\nu'' b}{c\gamma})$$

adeoque prodit haec aequatio

$$0 = \mu\lambda f^3+\mu'\lambda' g^3+\mu''\lambda'' b^3 + \frac{1}{(\frac{1}{\alpha}+\frac{1}{\gamma})^2}(\tfrac{\mu\nu f}{a\alpha}+\tfrac{\mu'\nu' g}{b\beta}+\tfrac{\mu''\nu'' b}{c\gamma})$$

vbi notandum est, ex praecedentibus esse

$\frac{1}{a\alpha} = \frac{1}{a}(\frac{f-1}{a}+\frac{f}{\gamma})$;

$\frac{1}{b\beta} = -(\frac{f-1}{a}+\frac{f}{\gamma})(\frac{g+f-1}{a}+\frac{f+g}{\gamma})$

$\frac{1}{c\gamma} = -\frac{1}{\gamma}(\frac{f+g-1}{a}+\frac{f+g}{\gamma})$

IV. Quodsi nunc ponamus $a = \infty$, sequentes aequationes habebuntur resoluendae:

$a = \frac{\gamma}{f}$ et $b = \frac{-\gamma}{g}$; $\frac{1}{\beta} = \frac{g+f}{\gamma}$ seu

$\beta = \frac{\gamma}{f+g}$ et $c = \frac{-\gamma}{f+g}$.

Quibus

Quibus adiungi debent hae aequationes

$f+g+b=1$, et $\zeta f+\eta g+\vartheta b=0$ et

pro omni confusione tollenda

ob $\frac{1}{\alpha\alpha}=0$; $\frac{1}{\beta\beta}=\frac{-f(f+g)}{\gamma^2}$; $\frac{1}{\gamma\gamma}=\frac{-f-g}{\gamma^2}$

habebitur haec aequatio

$$0=\mu\lambda f^3+\mu'\lambda' g^3+\mu''\lambda'' b^3$$
$$-(f+g)(\mu'\nu' fg+\mu''\nu'' b).$$

V. Hic quidem assumsimus, omnes tres materias, ex quibus hae lentes sunt confectae, esse diuersas; in praxi autem nulla ratio suadet, vt tres diuersas vitri species adhibeamus; sed potius primam et tertiam lentem ex eadem materia parari conueniet; ex quo consequimur statim

$$n''=n;\ \mu''=\mu;\ \nu''=\nu \text{ et } \vartheta=\zeta,$$

ita, vt sit

$$f+b+\tfrac{\eta}{\zeta}\cdot g=0;$$

quae ab altera subtracta dat

$$g-\tfrac{\eta}{\zeta}\cdot g=1;\ \text{hincque } g=\frac{\zeta}{\zeta-\eta}$$

vnde porro fit

$$f+b=-\frac{\eta}{\zeta-\eta}.$$

Ponamus hunc in finem

$f-b=\frac{\alpha}{\zeta-\eta}$ eritque

$f=\frac{\alpha-\eta}{2(\zeta-\eta)}$ et $b=\frac{-\alpha-\eta}{2(\zeta-\eta)}$

ex

ex quibus valoribus prodit

$$a = \frac{2\gamma(\zeta-\eta)}{\alpha-\eta}, b = \frac{-2\gamma(\zeta-\eta)}{x-\eta};$$
$$c = \frac{2\gamma(\zeta-\eta)}{x+2\zeta-\eta}; e = \frac{-2\gamma(\zeta-\eta)}{x+2\zeta-\eta}.$$

VI. Aequatio ergo, cui adhuc satisfieri oportet, erit

$$0 = \tfrac{1}{2}\mu\lambda^{I}(x-\eta)^{II} + 2\mu^{I}\lambda^{I}\zeta^{II} - \tfrac{1}{2}\mu\lambda^{II}(x+\eta)^{II} - \tfrac{1}{2}(x+2\zeta-\eta)(\mu^{I}\nu^{I}(x-\eta)\zeta - \mu\nu(x+\eta)(\zeta-\eta))$$

ex qua aequatione vel vna litterarum λ vel etiam numerus x definiri potest; quo posteriori casu hoc commodi consequeremur, vt singulae litterae $\lambda = 1$ statui possent sicque singulae lentes vtpote ex principio minimi deductae facillime construi possent; hanc autem inuestigationem non aliter, nisi in exemplis particularibus suscipere licebit.

Ponamus igitur

$\lambda = 1$, $\lambda^{I} = 1$; $\lambda^{II} = 1$ fietque

$$0 = -\tfrac{1}{2}\mu(3\eta x^{2}+\eta^{3}) + 2\mu^{I}\zeta^{3} - \tfrac{1}{2}\mu^{I}\nu^{I}\zeta(x^{2}+2(\zeta-\eta)x-\eta(2\zeta-\eta)) + \tfrac{1}{2}\mu\nu(\zeta-\eta)(x^{2}+2\zeta x+\eta(2\zeta-\eta))$$

quae aequatio reducitur ad hanc

$$x^{2}(-3\mu\eta-\mu^{I}\nu^{I}\zeta+\mu\nu(\zeta-\eta)) + 2x.(\zeta-\eta)\zeta(\mu\nu-\mu^{I}\nu^{I}) - \mu\eta^{3} + 4\mu^{I}\zeta^{3} + \mu^{I}\nu^{I}\eta\zeta(2\zeta-\eta) + \mu\nu.\eta(\zeta-\eta)(2\zeta-\eta) = 0.$$

VII. Adplicemus haec ad illas duas species vitri, quibus *Dollondus* vsus est, ac duo casus euolvendi occurrunt. Primus igitur casus esto, quo tam prima, quam tertia lens ex vitro chrystallino conficitur;

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ficitur; media autem ex vitro communi ſubuiridi; ita, vt ſit $n=n''=1,58$ et $n'=1,53$, ideoque $\zeta=10$ et $\eta=7$ tum verò $\mu=0,8724$, $\nu=0,2529$ et $l.\mu\nu=9.3437441$ deinde $\mu'=0,9875$; $\nu'=0,2194$ et $l.\mu'\nu'=9.3358867$ vnde ſuperior aequatio reducetur ad hanc formam

$$+19,8255\,x^2=0,2340\,x+4177,6081, \text{ ſiue}$$

$$x^2=0.0118\,x+210,7190$$

cuius reſolutio praebet

$$x=0,0059\pm\sqrt{210,7190}$$

$$=0,0059\pm 14,5161$$

vnde ambo valores ipſius x ſunt

$$x=14,5220$$

$$x=-14,5102$$

Lens obiectiua triplicata perfecta prior cuius lens prima et tertia ex vitro chryſtallino, pro quo $n=1,58$, media vero ex vitro communi ſubuiridi, $n=1,53$, eſt confecta.

I. Ob duplicem valorem ipſius x duas etiam eiusmodi lentes exhibere poterimus; ſit igitur primo $x=14,5220$ et quia eſt $\zeta=10$ et $\eta=7$, erit $f=1,2537$; $g=3,3333$, $b=-3.5870$

$\frac{1}{a}=\frac{1,2537}{\gamma}$; $\frac{1}{b}=-\frac{1,2537}{\gamma}$; $\frac{1}{\beta}=\frac{4,5870}{\gamma}$ et $\frac{1}{c}=-\frac{4,5870}{\gamma}$;

quodſi

quodsi iam harum trium lentium radios facierum anteriorum ponamus F, F', F'', posteriorum vero

G, G', G'' ob $\lambda = \lambda' = \lambda'' = 1$,

habebimus

$\frac{1}{F} = \frac{\varrho}{\alpha} + \frac{\sigma}{\alpha} = \frac{\sigma}{\alpha} = \frac{1,2537 \cdot \sigma}{\gamma}$; $\frac{1}{G} = \frac{\varrho}{\alpha} + \frac{\sigma}{\alpha} = \frac{\varrho}{\alpha} = \frac{1,2537 \varrho}{\gamma}$ et

$\frac{1}{F'} = \frac{\varrho'}{b} + \frac{\sigma'}{c}$; $\frac{1}{G'} = \frac{\varrho'}{c} + \frac{\sigma'}{b}$

et substitutis valoribus

$\frac{1}{F'} = \frac{-1,2537 \cdot \varrho'}{\gamma} + \frac{4,5870 \cdot \sigma'}{\gamma}$

$\frac{1}{G'} = \frac{4,5870 \cdot \varrho'}{\gamma} - \frac{1,2537 \cdot \sigma'}{\gamma}$

simili modo pro lente tertia

$\frac{1}{F''} = \frac{\varrho}{c} + \frac{\sigma}{\gamma}$; $\frac{1}{G''} = \frac{\varrho}{\gamma} + \frac{\sigma}{c}$

$\frac{1}{F''} = \frac{-4,5870 \cdot \varrho + \sigma}{\gamma}$; $\frac{1}{G''} = \frac{\varrho - 4,5870 \cdot \sigma}{\gamma}$

existente

$\varrho = 0,1413$; $\sigma = 1,5827$ et

$\varrho' = 0,2266$; $\sigma' = 1,6602$

ac si ipsius lentis triplicatae distantia focalis ponatur $= P$, vt hic sit $\gamma = P$, reperiuntur valores sequentes

$\frac{1}{F} = \frac{1,0843}{P}$, siue $F = +0,5039 \cdot P.$

$\frac{1}{G} = \frac{0,1771}{P}$, hincque $G = +5,6450 \cdot P.$

$\frac{1}{F'} = \frac{7,3512}{P}$, siue $F' = +0,1364 \cdot P.$

$\frac{1}{G'} = \frac{-1,0420}{P}$, hincque $G' = -0,9597 \cdot P.$

$\frac{1}{F''} = \frac{0,9346}{P}$, hincque $F'' = +1,0699 \cdot P.$

$\frac{1}{G''} = \frac{-7,1186}{P}$, hincque $G'' = -0,1404 \cdot P.$

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II. Inter hos ſex radios cum minimus ſit 0, 1364. P, eius pars quarta $= 0,0341$. P dat ſemidiametrum maximae aperturae, cuius haec lens triplicata eſt capax. Contra vero haec lens hac praerogatiua gaudet, quod eius conſtructio ob $\lambda = 1$; $\lambda' = 1$; $\lambda'' = 1$ in praxi minimae difficultati ſit obnoxia.

Altera lens triplicata, pro qua pariter, vt ante, lens prima et tertia ex vitro chryſtallino, media vero ex vitro communi ſubviridi paratur.

I. Sumatur iam

$x = -14,5102$, ob $\zeta = 10$, et $\eta = 7$ erit

$f = -3,5850, g = 3,3333; b = +1,2517.$

$$\frac{1}{\alpha} = \frac{-3.5850}{\gamma}; \frac{1}{b} = \frac{3.5850}{\gamma}$$

$$\frac{1}{\mathfrak{b}} = \frac{-0.2517}{\gamma}; \frac{1}{c} = +\frac{0.2517}{\gamma}$$

Vnde pro radiis ſingularum facierum prodibunt ſequentes formulae:

$$\frac{1}{F} = \frac{\sigma}{\alpha} = \frac{-3.5850.\sigma}{\gamma}; \quad \frac{1}{G} = \frac{\rho}{\alpha} = \frac{-3.5850.\rho}{\gamma}$$

$$\frac{1}{F'} = \frac{\rho'}{b} + \frac{\sigma'}{\mathfrak{b}} = \frac{3.5850.\rho' - 0.2517.\sigma'}{\gamma}$$

$$\frac{1}{G'} = \frac{\rho'}{\mathfrak{b}} + \frac{\sigma'}{b} = \frac{-0.2517\rho' + 3.5850\sigma'}{\gamma}$$

$$\frac{1}{F''} = \frac{\rho}{c} + \frac{\sigma}{\gamma} = \frac{0.2517\rho + \sigma}{\gamma}$$

$$\frac{1}{G''} = \frac{\rho}{\gamma} + \frac{\sigma}{c} = \frac{\rho + 0.2517.\sigma}{\gamma}$$

Vnde

vnde sequentes valores determinati pro his radiis inveniuntur:

$\frac{1}{F} = \frac{-5,6739}{P}$ hincque $F = -0,1762.$ P.

$\frac{1}{G} = \frac{-0,5065}{P}$ hincque $G = -1,9741.$ P.

$\frac{1}{F'} = \frac{0,3045}{P}$ hincque $F' = +2,5349.$ P.

$\frac{1}{G'} = \frac{5,8648}{P}$, hincque $G' = +0,1696.$ P.

$\frac{1}{F''} = \frac{1,6182}{P}$, hincque $F'' = +0,6194.$ P.

$\frac{1}{G''} = \frac{0,5396}{P}$ hincque $G'' = +1,8532.$ P.

quorum sex radiorum minimus est 0,1696 P, cuius pars quarta 0,0424. P dat semidiametrum aperturae maximae, cuius haec lens triplicata est capax. Ideoque haec lens obiectiua praecedenti est anteferenda.

VIII. Secundum §. VII. adhuc alius casus euolui debet, quo lens prima et tertia ex vitro communi subuiridi $n = n'' = 1,53$; media autem ex Chrystallino $n = 1,58$ parari potest; sicque erit $\zeta = 7$, $\eta = 10$. Vnde cum fiat

$\mu = 0.9875$, $l.\mu\nu = 9.3358867$

$\mu' = 0.8724$. $l.\mu'\nu' = 9.3437441.$

aequatio nostra pro confusione tollenda sequentem induet formam:

$31,8198\,x^2 = +0,1638\,x + 245,2121$ siue

$x^2 = 0,0051.\,x + 7,7063$

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cuius resolutio dat

$$x = 0,0025 \pm 2,7760$$

vnde bini valores ipsius x erunt

I. $x = 2,7785$

II. $x = -2.7735.$

Lens triplicata perfecta prior, cuius lens prima et tertia ex vitro communi, $n = 1,53.$ media ex chrystallino, $n = 1,58$ est confecta.

Cum igitur sit

$x = 2,7785$ et $\xi = 7, \eta = 10$ erit

$$f = +1,2036; \quad g = -2,3333. \quad b = 2,1297;$$

hincque

$$\frac{1}{a} = \frac{1.2036}{\gamma}, \frac{1}{b} = \frac{-1.2036}{\gamma}, \frac{1}{c} = \frac{-1.1297}{\gamma}, \frac{1}{C} = \frac{1.1297}{\gamma}$$

ex quibus colligitur

$$\frac{1}{P} = \frac{\sigma}{a} = \frac{1.2036.\sigma}{\gamma}; \quad \frac{1}{C} = \frac{\rho}{a} = \frac{1.2036.\rho}{\gamma}$$

$$\frac{1}{P'} = \frac{\rho'}{b} + \frac{\sigma'}{c} = \frac{-1.2036.\rho' - 1.1297.\sigma'}{\gamma}$$

$$\frac{1}{C'} = \frac{\rho'}{c} + \frac{\sigma'}{b} = \frac{-1.1297.\rho' - 1.2036.\sigma'}{\gamma}$$

$$\frac{1}{P''} = \frac{\rho}{c} + \frac{\sigma}{\gamma} = \frac{1.1297.\rho + \sigma}{\gamma}$$

$$\frac{1}{C''} = \frac{\sigma}{c} + \frac{\rho}{\gamma} = \frac{1.1297.\sigma + \rho}{\gamma}$$

vbi

vbi $\varrho = 0,\ 2266$; $\sigma = 1,\ 6602$

$\varrho' = 0,\ 1413$; $\sigma' = 1,\ 5827$

Facto igitur calculo obtinebimus:

$\frac{1}{F} = \frac{1,9982}{P}$ hincque $F = 0,\ 5004.\ P.$

$\frac{1}{G} = \frac{0,2727}{P}$ hincque $G = 3,\ 6665.\ P.$

$\frac{1}{F'} = \frac{-1,9580}{P}$ hincque $F' = -0,\ 5107.\ P.$

$\frac{1}{G'} = \frac{-20,0645}{P}$ hincque $G' = -0,\ 4843.\ P.$

$\frac{1}{F''} = \frac{1,9161}{P}$ hincque $F'' = 0,\ 5219.\ P.$

$\frac{1}{G''} = \frac{20,1021}{P}$ hincque $G'' = 0,\ 4757.\ P.$

inter quos radios cum minimus sit 0, 4757 P eius pars quarta 0, 1189. P dat semidiametrum aperturae, quam haec lens admittit; ideoque haec lens duabus praecedentibus est anteferenda.

Lens triplicata perfecta altera, cuius lens prima et tertia ex vitro communi $n = 1,53$ media ex chrystallino, $n = 1,58$ est parata.

Sumatur iam ex binis ipsius x valoribus alter negatiuus $x = -2.\ 7735$; et cum sit $\zeta = 7$, $\eta = 10$, inuenitur

$f = +2,\ 1289$; $g = -2,\ 3333$ $b = +1,\ 2044$;

hincque

$\frac{1}{a} = \frac{2,1280}{\gamma}$; $\frac{1}{b} = \frac{-2,1289}{\gamma}$

$\frac{1}{c} = \frac{-0,1044}{\gamma}$; $\frac{1}{d} = \frac{0,1044}{\gamma}$

ex

ex quibus consequimur

$$\frac{1}{F} = \frac{\sigma}{\alpha} = \frac{2,1289.\sigma}{\gamma}; \quad \frac{1}{G} = \frac{\rho}{\alpha} = \frac{2,29.\rho}{\gamma}$$

$$\frac{1}{F'} = \frac{\rho'}{b} + \frac{\sigma'}{\beta} = \frac{-2,1289.\rho' - 0,1044.\sigma'}{\gamma}$$

$$\frac{1}{G'} = \frac{\rho'}{\beta} + \frac{\sigma'}{b} = \frac{-0,1044.\rho' - 2,1289.\sigma'}{\gamma}$$

$$\frac{1}{F''} = \frac{\rho}{c} + \frac{\sigma}{\gamma} = \frac{0,1044.\rho + \sigma}{\gamma}$$

$$\frac{1}{G''} = \frac{\sigma}{c} + \frac{\rho}{\gamma} = \frac{0,1044.\sigma + \rho}{\gamma}.$$

Vnde facto calculo pro radiis facierum singulatum lentium sequentes inuenientur radii.

$\frac{1}{F} = \frac{2,5344}{P}$; seu $F = 0,2829.\ P.$

$\frac{1}{G} = \frac{0,4824}{P}$; seu $G = 2,0729.\ P.$

$\frac{1}{F'} = \frac{-0,4660}{P}$; seu $F' = -2,1459.\ P.$

$\frac{1}{G'} = \frac{-3,3841}{P}$; seu $G' = -0,2955.\ P.$

$\frac{1}{F''} = \frac{1,6838}{P}$; seu $F'' = 0,5938.\ P.$

$\frac{1}{G''} = \frac{0,1999}{P}$; seu $G'' = 2,5006.\ P.$

Inter quos radios minimus est 0,2829. P, cuius pars quarta 0,0707. P, dat semidiametrum aperturae maximae, cuius haec lens triplicata est capax.

IX. Hae quatuor lentes triplicatae ideo prae ceteris, quas proponere liceret, sunt commendandae, quod in iis assumsimus $\lambda = 1$; $\lambda' = 1$; $\lambda'' = 1$. Quam ob caussam in praxi facillime construi possunt, ex iisdem vero principiis etiam constructio eiusmodi lentium com-

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compoſitarum deduci poſſet qua inter binas lentes vitreas aqua aliudue fluidum includeretur, ita, vt lens media ex fluido conſtaret: tum autem praeter conditiones ante tractatas duae nouae eſſent implendae; ſcilicet vt radii facierum mediae lentis aequales eſſent ſtatuendi et contrarii radiis facierum internarum primae ac tertiae lentis. Ob has igitur nouas conditiones non amplius liceret numeros λ, λ' et λ'' vnitati aequales aſſumere; qua poſitione conſtructioni practicae quam maxime conſulitur; deinde vero etiam radii facierum tam parui prodirent, vt tales lentes nimis exiguam aperturam eſſent admiſſurae; quam ob cauſſam operae pretium haud videtur, in earum conſtructionem adcuratius inquirere.

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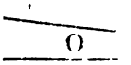

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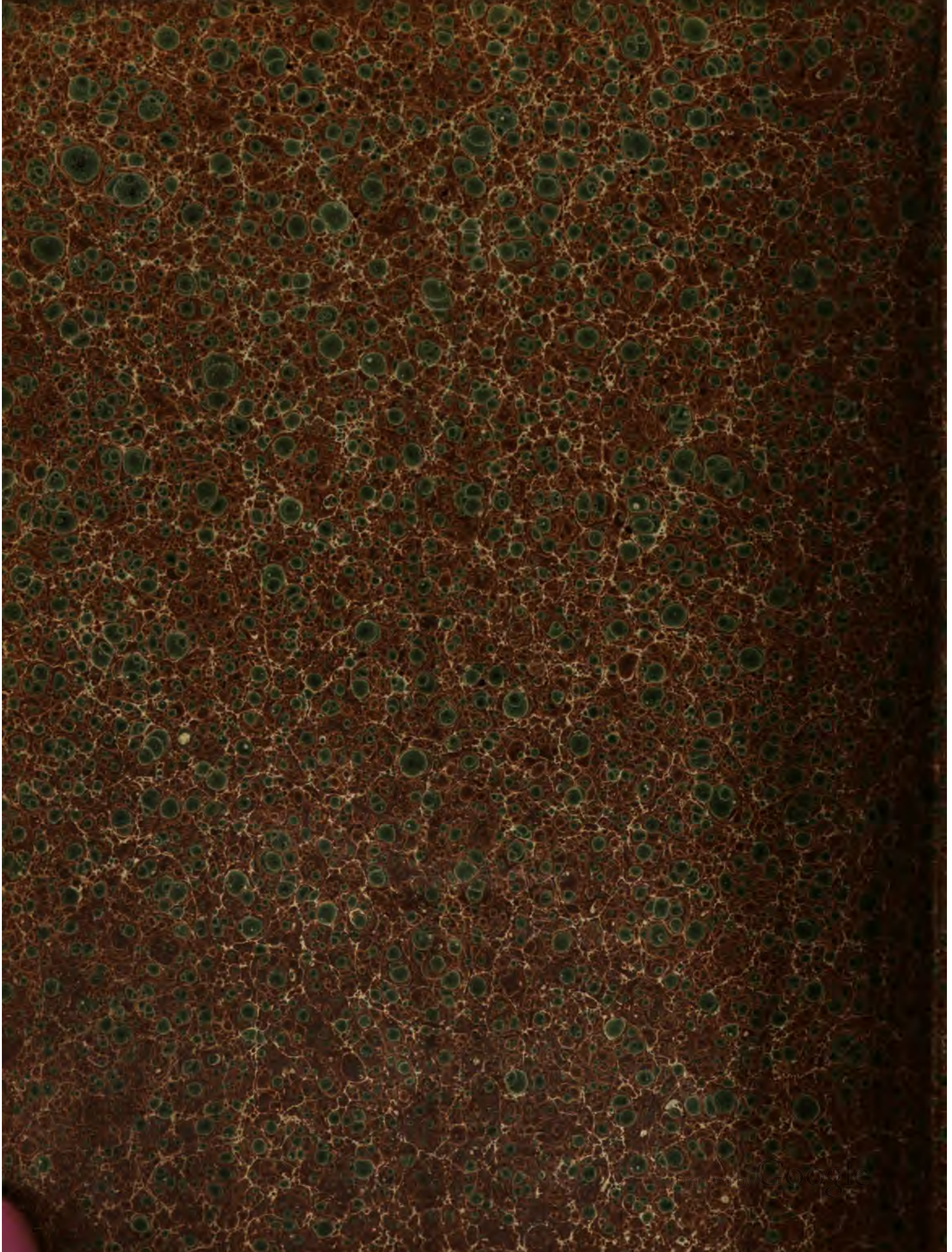

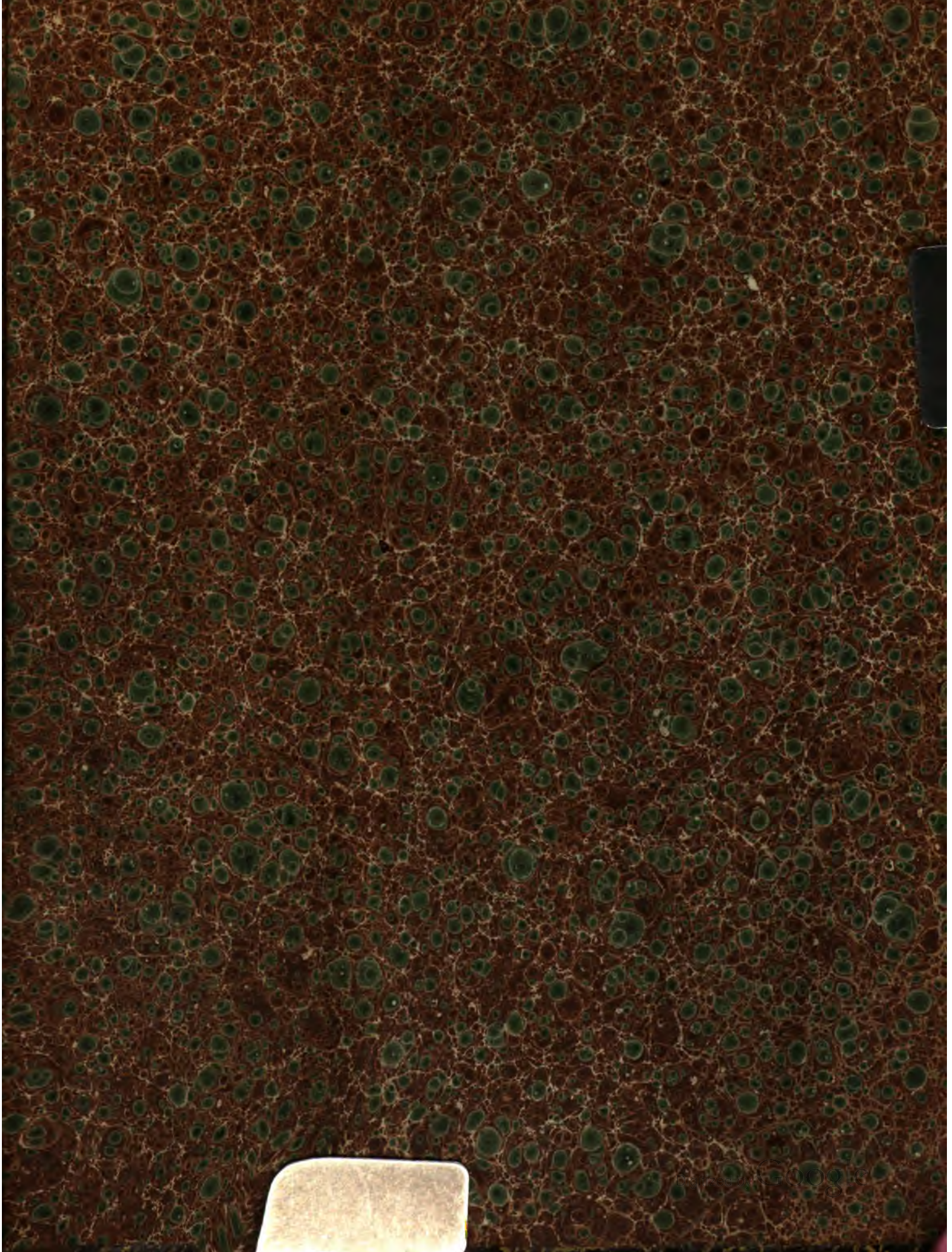

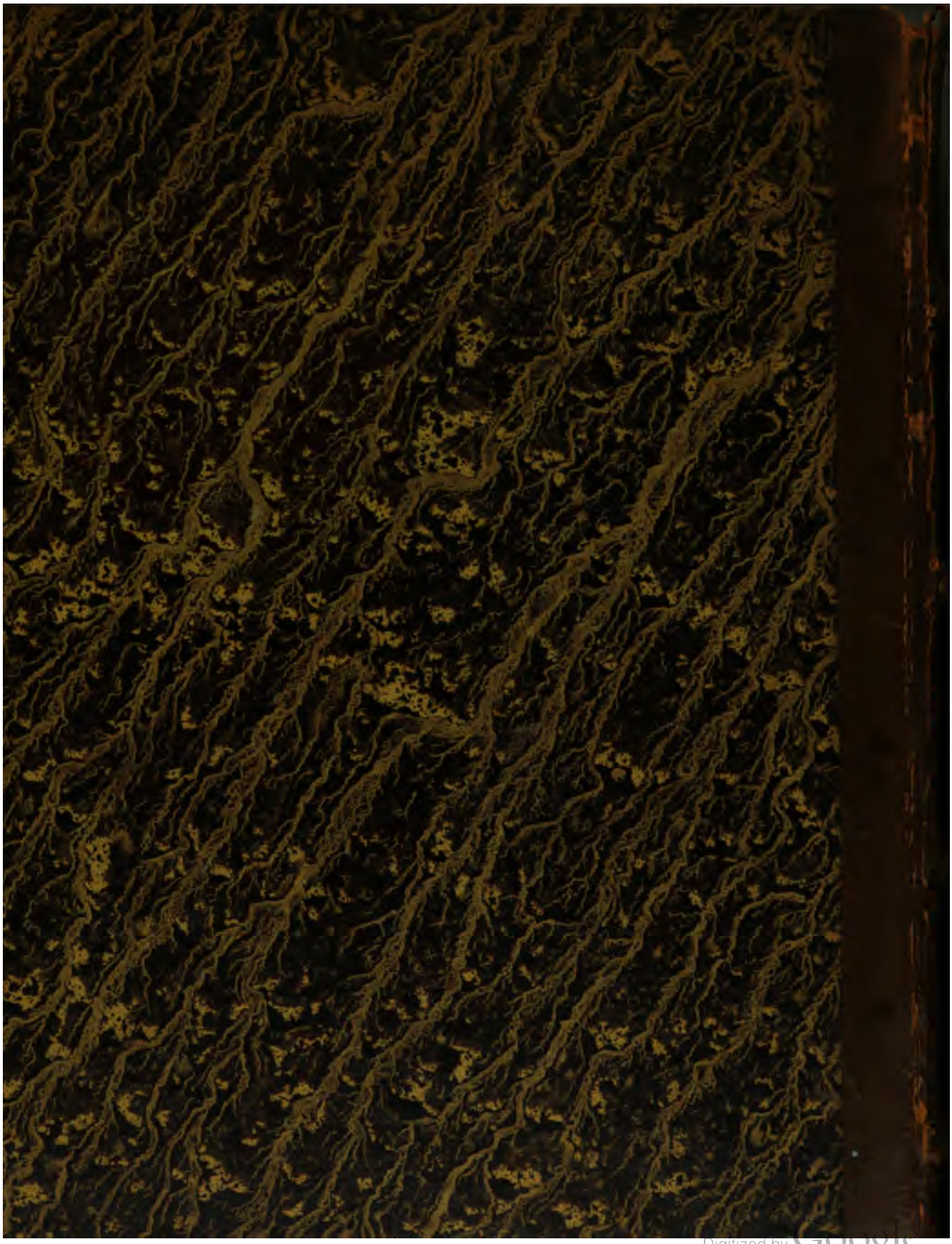
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